



# Circles

## Higher Maths Exam Questions

Source: 2019 P1 Q3 Higher Maths

- (1) Circle  $C_1$  has equation  $x^2 + y^2 - 6x - 2y - 26 = 0$ .  
Circle  $C_2$  has centre  $(4, -2)$ .  
The radius of  $C_2$  is equal to the radius of  $C_1$ .  
Find the equation of circle  $C_2$ .

Answer:  $(x - 4)^2 + (y + 2)^2 = 36$

Source: 2019 P1 Q16 Higher Maths

- (2) The point P has coordinates  $(4, k)$ .  
C is the centre of the circle with equation  $(x - 1)^2 + (y + 2)^2 = 25$ .
- (a) Show that the distance between the points P and C is given by  $\sqrt{k^2 + 4k + 13}$ .
- (b) Hence, or otherwise, find the range of values of  $k$  such that P lies outside the circle.

Answers:

(a) •<sup>1</sup> identify centre

•<sup>2</sup> apply distance formula and demonstrate result

(b)  $k < -6, \quad k > 2$

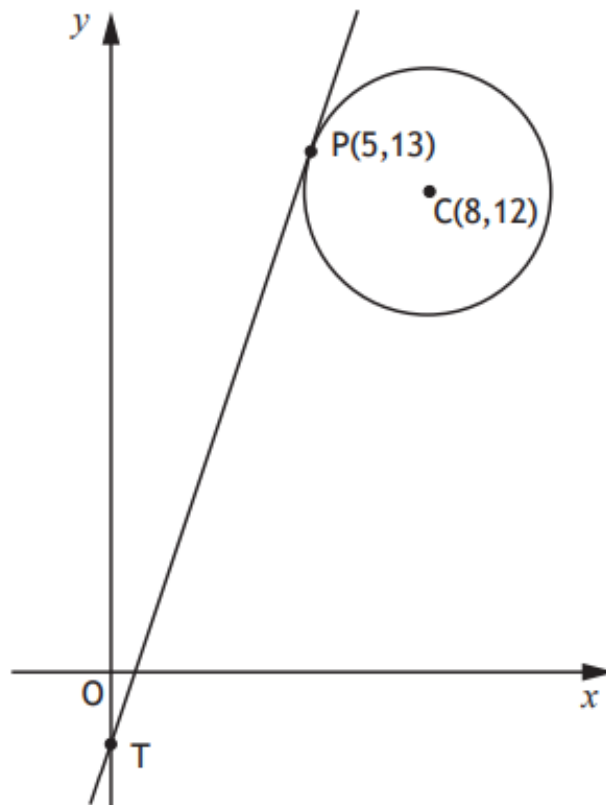
•<sup>1</sup>  $(1, -2)$  stated or implied by •<sup>2</sup>

•<sup>2</sup>  $\sqrt{(4-1)^2 + (k-(-2))^2}$  leading to  $\sqrt{k^2 + 4k + 13}$

(3)

A circle has centre  $C(8,12)$ .

The point  $P(5,13)$  lies on the circle as shown.



(a) Find the equation of the tangent at P.

The tangent from P meets the  $y$ -axis at the point T.

(b) (i) State the coordinates of T.

(ii) Find the equation of the circle that passes through the points C, P and T.

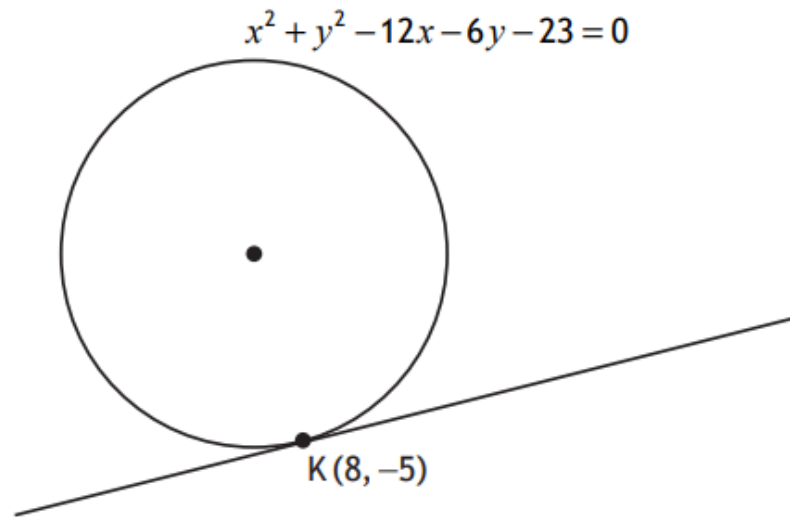
Answers: (a)  $y = 3x - 2$

(b) (i)  $T(0, -2)$  (ii)  $(x - 4)^2 + (y - 5)^2 = 65$

Source: 2018 P1 Q4 Higher Maths

(4)

The point  $K(8, -5)$  lies on the circle with equation  $x^2 + y^2 - 12x - 6y - 23 = 0$ .



Find the equation of the tangent to the circle at  $K$ .

Answer:  $y = \frac{1}{4}x - 7$

Source: 2017 P1 Q2 Higher Maths

(5)

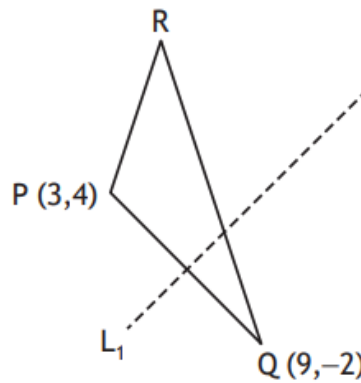
The point  $P(-2, 1)$  lies on the circle  $x^2 + y^2 - 8x - 6y - 15 = 0$ .

Find the equation of the tangent to the circle at  $P$ .

Answer:  $y = -3x - 5$

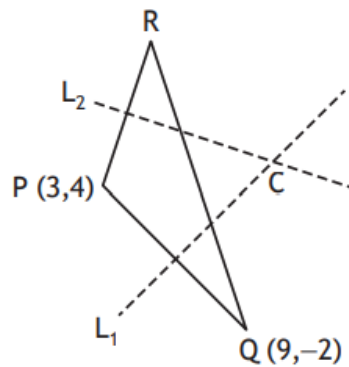
(6)

PQR is a triangle with  $P(3,4)$  and  $Q(9,-2)$ .



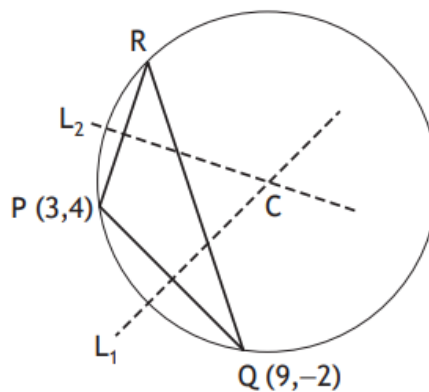
(a) Find the equation of  $L_1$ , the perpendicular bisector of  $PQ$ .

The equation of  $L_2$ , the perpendicular bisector of  $PR$  is  $3y + x = 25$ .



(b) Calculate the coordinates of  $C$ , the point of intersection of  $L_1$  and  $L_2$ .

$C$  is the centre of the circle which passes through the vertices of triangle  $PQR$ .

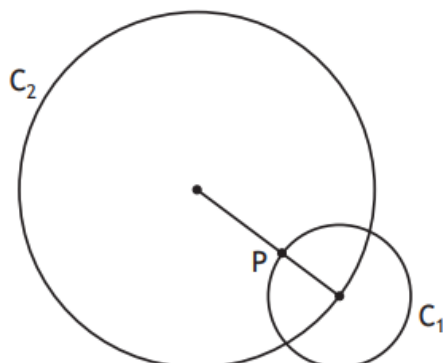


(c) Determine the equation of this circle.

(7)

Circle  $C_1$  has equation  $(x-13)^2 + (y+4)^2 = 100$ .

Circle  $C_2$  has equation  $x^2 + y^2 + 14x - 22y + c = 0$ .



- (a) (i) Write down the coordinates of the centre of  $C_1$ .  
 (ii) The centre of  $C_1$  lies on the circumference of  $C_2$ .  
 Show that  $c = -455$ .

The line joining the centres of the circles intersects  $C_1$  at P.

- (b) (i) Determine the ratio in which P divides the line joining the centres of the circles.  
 (ii) Hence, or otherwise, determine the coordinates of P.

P is the centre of a third circle,  $C_3$ .

$C_2$  touches  $C_3$  internally.

- (c) Determine the equation of  $C_3$ .

Answers:

(a) (i)  $(13, -4)$

(ii) *Substitute coordinates and process leading to  $c = -455$*

(b) (i)  $3 : 2$  or  $2 : 3$

(ii)  $P(5, 2)$

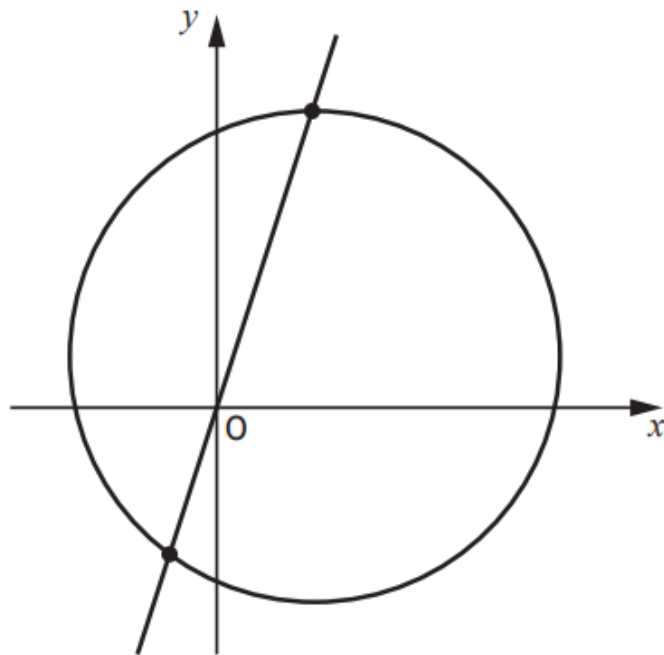
(c)  $(x - 5)^2 + (y - 2)^2 = 1600$

or  $x^2 + y^2 - 10x - 4y - 1571 = 0$

Source: 2017 P2 Q3 Higher Maths

(8)

The line  $y=3x$  intersects the circle with equation  $(x-2)^2 + (y-1)^2 = 25$ .



Find the coordinates of the points of intersection.

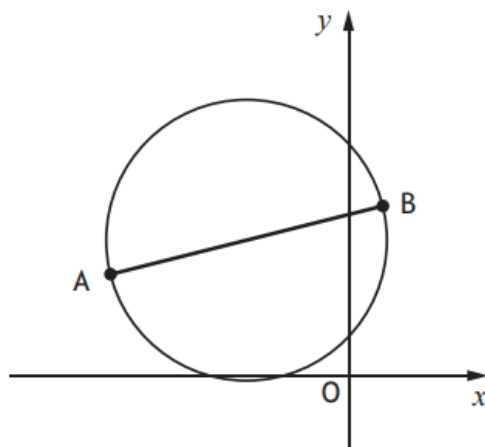
Answers:  $(2, 6)$  &  $(-1, -3)$

Source: 2016 P1 Q4 Higher Maths

(9)

A and B are the points  $(-7, 3)$  and  $(1, 5)$ .

AB is a diameter of a circle.



Find the equation of this circle.

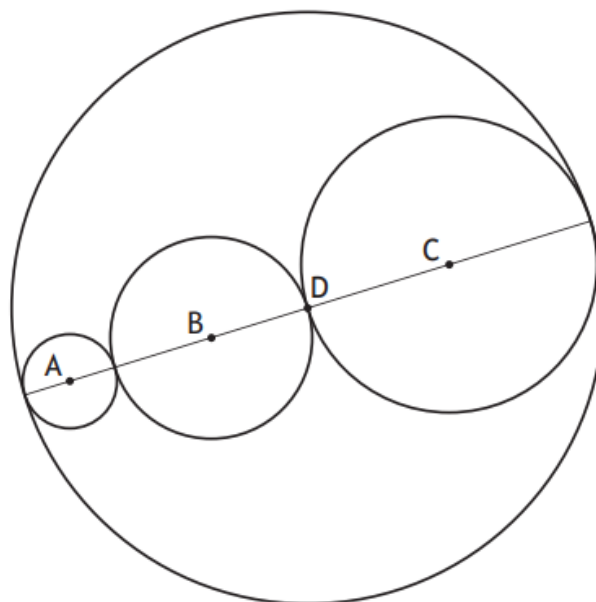
Answer:  $(x + 3)^2 + (y - 4)^2 = 17$

Source: 2017 P2 Q10 Higher Maths

(10)

(a) Show that the points  $A(-7, -2)$ ,  $B(2, 1)$  and  $C(17, 6)$  are collinear.

Three circles with centres  $A$ ,  $B$  and  $C$  are drawn inside a circle with centre  $D$  as shown.



The circles with centres  $A$ ,  $B$  and  $C$  have radii  $r_A$ ,  $r_B$  and  $r_C$  respectively.

- $r_A = \sqrt{10}$
- $r_B = 2r_A$
- $r_C = r_A + r_B$

(b) Determine the equation of the circle with centre  $D$ .

Answers: (a) *Proof* (b)  $(x - 8)^2 + (y - 3)^2 = 360$

Source: 2016 P1 Q8 Higher Maths

(11)

Show that the line with equation  $y = 3x - 5$  is a tangent to the circle with equation  $x^2 + y^2 + 2x - 4y - 5 = 0$  and find the coordinates of the point of contact.

Answers: *Substitute for y and one solution means tangency.*  
*Point of contact (2, 1)*

Source: 2016 P2 Q4 Higher Maths

(12)

Circles  $C_1$  and  $C_2$  have equations  $(x+5)^2 + (y-6)^2 = 9$  and  $x^2 + y^2 - 6x - 16 = 0$  respectively.

- (a) Write down the centres and radii of  $C_1$  and  $C_2$ .
- (b) Show that  $C_1$  and  $C_2$  do not intersect.

Answer: (a)  $C_1 (-5, 6) r = 3$   $C_2 (3, 0) r = 5$

(b)

- |   |   |
|---|---|
| • <sup>5</sup> calculate the distance between the centres | • <sup>1</sup> 10   |
| • <sup>6</sup> calculate the sum of the radii             | • <sup>2</sup> 8  |
| • <sup>7</sup> interpret significance of calculations     | • <sup>3</sup> $8 < 10 \therefore$ the circles do not intersect |