FORMULAE LIST

Circle:

The equation \( x^2 + y^2 + 2gx + 2fy + c = 0 \) represents a circle centre \((-g, -f)\) and radius \(\sqrt{g^2 + f^2 - c}\).

The equation \((x-a)^2 + (y-b)^2 = r^2\) represents a circle centre \((a, b)\) and radius \(r\).

Scalar Product:

\[
\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos \theta, \text{ where } \theta \text{ is the angle between } \mathbf{a} \text{ and } \mathbf{b}
\]

or

\[
\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.
\]

Trigonometric formulae:

\[
\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\sin 2A = 2 \sin A \cos A
\]

\[
\cos 2A = \cos^2 A - \sin^2 A
\]

\[= 2 \cos^2 A - 1\]

\[= 1 - 2 \sin^2 A\]

Table of standard derivatives:

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>( f'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin ax )</td>
<td>( a \cos ax )</td>
</tr>
<tr>
<td>( \cos ax )</td>
<td>( -a \sin ax )</td>
</tr>
</tbody>
</table>

Table of standard integrals:

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<thead>
<tr>
<th>( f(x) )</th>
<th>( \int f(x) , dx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin ax )</td>
<td>( -\frac{1}{a} \cos ax + c )</td>
</tr>
<tr>
<td>( \cos ax )</td>
<td>( \frac{1}{a} \sin ax + c )</td>
</tr>
</tbody>
</table>
Essential Skills 31

The skills in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Using the Natural Logarithm

Solve for $x$:

1. $3^x = 18$
2. $5^x = 90$
3. $12^x = 3000$
4. $4^{2x} = 35$
5. $2^{3x-1} = 11$
6. $0 \cdot 7^x = 0 \cdot 9$
7. $7^{2-3x} = 5$
8. $e^{0.6x} = 5 \cdot 2$
9. $e^{-0.3x} = 0 \cdot 16$
10. $50e^{-0.7x} = 45$

APPLYING QUESTIONS

1. Evaluate $\log_9 21$, giving your answer to 2 decimal places.

2. A radioactive element decays according to the formula $m_t = m_0 e^{-0.03t}$ where $m_0$ is the initial mass and $t$ is the time in years.
   (a) What mass remains of the initial 200mg of the element after 40 years?
   (b) What is the half-life of this element?

3. A colony of ants is estimated to be growing according to the formula $P = 420 e^{0.25t}$ where $P$ is the population after $t$ years.
   (a) What was the initial population of ants?
   (b) What is the population after 7 years?
   (c) How long will it take the population to increase by a factor of 10?