Circle:
The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.
The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a, b)$ and radius $r$.

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where $\theta$ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 2 \cos^2 A - 1$

$= 1 - 2 \sin^2 A$

Table of standard derivatives:

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>$f'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin ax$</td>
<td>$a \cos ax$</td>
</tr>
<tr>
<td>$\cos ax$</td>
<td>$-a \sin ax$</td>
</tr>
</tbody>
</table>

Table of standard integrals:

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>$\int f(x)dx$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin ax$</td>
<td>$-\frac{1}{a} \cos ax + c$</td>
</tr>
<tr>
<td>$\cos ax$</td>
<td>$\frac{1}{a} \sin ax + c$</td>
</tr>
</tbody>
</table>
Essential Skills 5

The skills in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Stationary Points

Find the co-ordinates and determine the nature of the stationary points:

1. \( y = x^3 - 3x^2 \)
2. \( f(x) = x^3 - 12x \)
3. \( f(x) = x^3 + 9x^2 + 24x - 18 \)
4. \( y = 2x^3 - 7x^2 + 4x + 4 \)
5. \( y = 2x^3 - 3x^2 - 36x + 17 \)
6. \( f(x) = x^2(2x - 3) \)
7. \( f(x) = x^3 - 2x^2 - 4x + 1 \)
8. \( y = (x - 1)(x - 2)^2 \)
9. \( y = x(27 - x^2) \)
10. \( f(x) = 2x^2(2 - x^2) \)

APPLYING QUESTIONS

1. An open top box measures \( x \) cm by \( 2x \) cm and has a depth of \( h \) cm. The outer surface has an area of 216cm².
   
   (a) Show that the volume of the cuboid is given by \( V(x) = 72x - \frac{2}{3}x^3 \)
   
   (b) Find the value of \( x \) for which the volume is a maximum and calculate the volume.

2. A function \( f \) is defined by \( f(x) = x(x^2 - 3) \), where \( 0 \leq x \leq 3 \).
   
   Find the maximum and minimum values of \( f \).