FORMULAE LIST

Circle:
The equation \( x^2 + y^2 + 2gx + 2fy + c = 0 \) represents a circle centre \((-g, -f)\) and radius \( \sqrt{g^2 + f^2 - c} \).
The equation \((x-a)^2 + (y-b)^2 = r^2\) represents a circle centre \((a, b)\) and radius \(r\).

Scalar Product: \[ \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta, \text{ where } \theta \text{ is the angle between } \mathbf{a} \text{ and } \mathbf{b} \]
or \[ \mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}. \]

Trigonometric formulae:
\[
\begin{align*}
\sin (A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos (A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2 \cos^2 A - 1 \\
&= 1 - 2 \sin^2 A
\end{align*}
\]

Table of standard derivatives:

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>( f'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin ax )</td>
<td>( a \cos ax )</td>
</tr>
<tr>
<td>( \cos ax )</td>
<td>( -a \sin ax )</td>
</tr>
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</table>

Table of standard integrals:

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>( \int f(x)dx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin ax )</td>
<td>( -\frac{1}{a} \cos ax + c )</td>
</tr>
<tr>
<td>( \cos ax )</td>
<td>( \frac{1}{a} \sin ax + c )</td>
</tr>
</tbody>
</table>
## Functions

<table>
<thead>
<tr>
<th>Year</th>
<th>Paper</th>
<th>Question</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>2019</td>
<td>P2 Q8</td>
<td>A function, ( f ), is given by ( f(x) = \frac{3}{x} + 8 ). The domain of ( f ) is ( 1 \leq x \leq 1000, \ x \in \mathbb{R} ). The inverse function, ( f^{-1} ), exists. (a) Find ( f^{-1}(x) ). (b) State the domain of ( f^{-1} ).</td>
<td>3</td>
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<tr>
<td>2019</td>
<td>P1 Q12</td>
<td>Functions ( f ) and ( g ) are defined by ( f(x) = \frac{1}{\sqrt{x}}, ) where ( x &gt; 0 ) and ( g(x) = 5 - x, ) where ( x \in \mathbb{R} ). (a) Determine an expression for ( f(g(x)) ). (b) State the range of values of ( x ) for which ( f(g(x)) ) is undefined.</td>
<td>2 1</td>
</tr>
<tr>
<td>2018</td>
<td>P1 Q2</td>
<td>A function ( g(x) ) is defined on ( \mathbb{R} ), the set of real numbers, by ( g(x) = \frac{1}{5} x - 4 ). Find the inverse function, ( g^{-1}(x) ).</td>
<td>3</td>
</tr>
<tr>
<td>2018</td>
<td>P2 Q6</td>
<td>Functions, ( f ) and ( g ), are given by ( f(x) = 3 + \cos x ) and ( g(x) = 2x, \ x \in \mathbb{R} ). (a) Find expressions for (i) ( f(g(x)) ) and (ii) ( g(f(x)) ).</td>
<td>2 1</td>
</tr>
<tr>
<td>2017</td>
<td>P1 Q1</td>
<td>Functions ( f ) and ( g ) are defined on suitable domains by ( f(x) = 5x ) and ( g(x) = 2 \cos x ). (a) Evaluate ( f(g(0)) ). (b) Find an expression for ( g(f(x)) ).</td>
<td>3</td>
</tr>
<tr>
<td>2017</td>
<td>P1 Q6</td>
<td>A function, ( h ), is defined by ( h(x) = x^3 + 7 ), where ( x \in \mathbb{R} ). Determine an expression for ( h^{-1}(x) ).</td>
<td>1 2</td>
</tr>
<tr>
<td>Year</td>
<td>Question</td>
<td></td>
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| **2016 P1 Q6** | Functions $f$ and $g$ are defined on $\mathbb{R}$, the set of real numbers. The inverse functions $f^{-1}$ and $g^{-1}$ both exist.  
(a) Given $f(x) = 3x + 5$, find $f^{-1}(x)$.  
(b) If $g(2) = 7$, write down the value of $g^{-1}(7)$. |
| **2016 P1 Q12** | The functions $f$ and $g$ are defined on $\mathbb{R}$, the set of real numbers by $f(x) = 2x^2 - 4x + 5$ and $g(x) = 3 - x$.  
(a) Given $h(x) = f(g(x))$, show that $h(x) = 2x^2 - 8x + 11$.  
(b) Express $h(x)$ in the form $p(x+q)^2 + r$. |
| **2015 P1 Q5** | A function $g$ is defined on $\mathbb{R}$, the set of real numbers, by $g(x) = 6 - 2x$.  
(a) Determine an expression for $g^{-1}(x)$.  
(b) Write down an expression for $g(g^{-1}(x))$. |
| **2015 P2 Q2** | Functions $f$ and $g$ are defined on suitable domains by $f(x) = 10 + x$ and $g(x) = (1 + x)(3 - x) + 2$.  
(a) Find an expression for $f(g(x))$.  
(b) Express $f(g(x))$ in the form $p(x+q)^2 + r$.  
(c) Another function $h$ is given by $h(x) = \frac{1}{f(g(x))}$. What values of $x$ cannot be in the domain of $h$? |
| **2014 P2 Q3** | Functions $f$ and $g$ are defined on suitable domains by $f(x) = x(x - 1) + q$ and $g(x) = x + 3$.  
(a) Find an expression for $f(g(x))$.  
(b) Hence, find the value of $q$ such that the equation $f(g(x)) = 0$ has equal roots. |
### Functions and Roots

Functions $f$ and $g$ are defined on the set of real numbers by

- $f(x) = x^2 + 3$
- $g(x) = x + 4$.

(a) Find expressions for:
   (i) $f(g(x))$
   (ii) $g(f(x))$.

(b) Show that $f(g(x)) + g(f(x)) = 0$ has no real roots.

### Functions and Equations

Functions $f$, $g$, and $h$ are defined on the set of real numbers by

- $f(x) = x^3 - 1$
- $g(x) = 3x + 1$
- $h(x) = 4x - 5$.

(a) Find $g(f(x))$.

(b) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$.

(c) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$.
    (ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully.

(d) Hence solve $g(f(x)) + xh(x) = 0$.

### Functions and Derivatives

Functions $f$ and $g$ are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$.

(a) (i) Find $p(x)$ where $p(x) = f(g(x))$.
    (ii) Find $q(x)$ where $q(x) = g(f(x))$.

(b) Solve $p'(x) = q'(x)$.

### Functions and Logarithms

Functions $f$, $g$, and $h$ are defined on suitable domains by

- $f(x) = x^2 - x + 10$, $g(x) = 5 - x$ and $h(x) = \log_2 x$.

(a) Find expressions for $h(f(x))$ and $h(g(x))$.

### Functions and Composite Functions

Functions $f$ and $g$, defined on suitable domains, are given by $f(x) = x^2 + 1$ and $g(x) = 1 - 2x$.

Find:

(a) $g(f(x))$;

(b) $g(g(x))$. 
### 3. Two functions $f$ and $g$ are defined by $f(x) = 2x + 3$ and $g(x) = 2x - 3$, where $x$ is a real number.
   
   (a) Find expressions for:
      
      (i) $f(g(x))$;
      
      (ii) $g(f(x))$.
   
   (b) Determine the least possible value of the product $f(g(x)) \times g(f(x))$.

### 4. Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$ are defined on the set of real numbers.
   
   (a) Find $h(x)$ where $h(x) = g(f(x))$.
   
   (b) (i) Write down the coordinates of the minimum turning point of $y = h(x)$.
      
      (ii) Hence state the range of the function $h$.

### 9. Functions $f(x) = \frac{1}{x - 4}$ and $g(x) = 2x + 3$ are defined on suitable domains.
   
   (a) Find an expression for $h(x)$ where $h(x) = f(g(x))$.
   
   (b) Write down any restriction on the domain of $h$.

### 9. The function $f$, defined on a suitable domain, is given by $f(x) = \frac{3}{x + 1}$.
   
   (a) Find an expression for $h(x)$ where $h(x) = f(f(x))$, giving your answer as a fraction in its simplest form.
   
   (b) Describe any restriction on the domain of $h$.

### 3. Functions $f$ and $g$ are defined on suitable domains by $f(x) = \sin (x^\circ)$ and $g(x) = 2x$.
   
   (a) Find expressions for:
      
      (i) $f(g(x))$;
      
      (ii) $g(f(x))$.

### 7. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.
   
   (a) Find expressions for:
      
      (i) $f(h(x))$;
      
      (ii) $g(h(x))$.

### 3. $f(x) = 3 - x$ and $g(x) = \frac{3}{x}$, $x \neq 0$.
   
   (a) Find $p(x)$ where $p(x) = f(g(x))$.
   
   (b) If $q(x) = \frac{3}{3 - x}$, $x \neq 3$, find $p(g(x))$ in its simplest form.
8. Functions $f$ and $g$ are defined on the set of real numbers by

\[
\begin{align*}
    f(x) &= x - 1 \\
    g(x) &= x^2.
\end{align*}
\]

(a) Find formulae for

(i) $f(g(x))$

(ii) $g(f(x))$. 