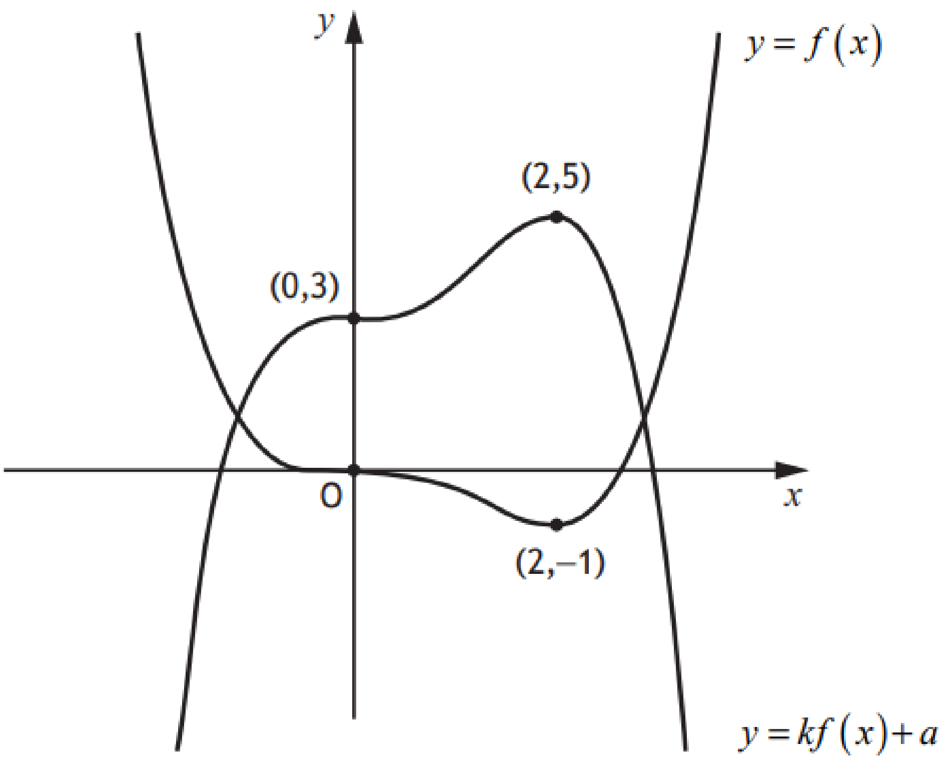
Functions & Graphs

Higher Maths Exam Questions

Source: 2019 P1 Q10 Higher Maths

(1) The diagram shows the graphs with equations $y = f(x)$ and $y = kf(x) + a$.

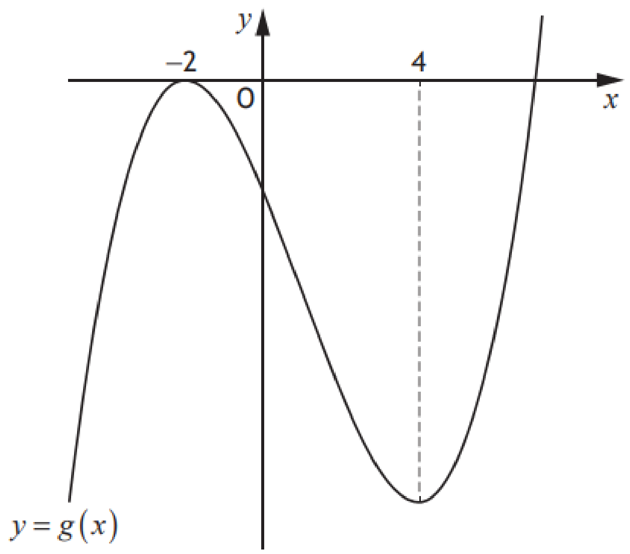
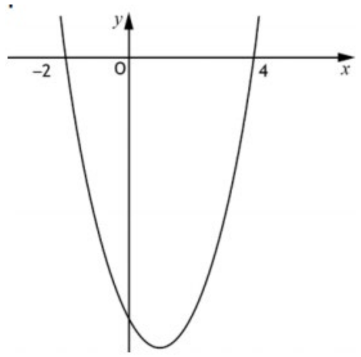


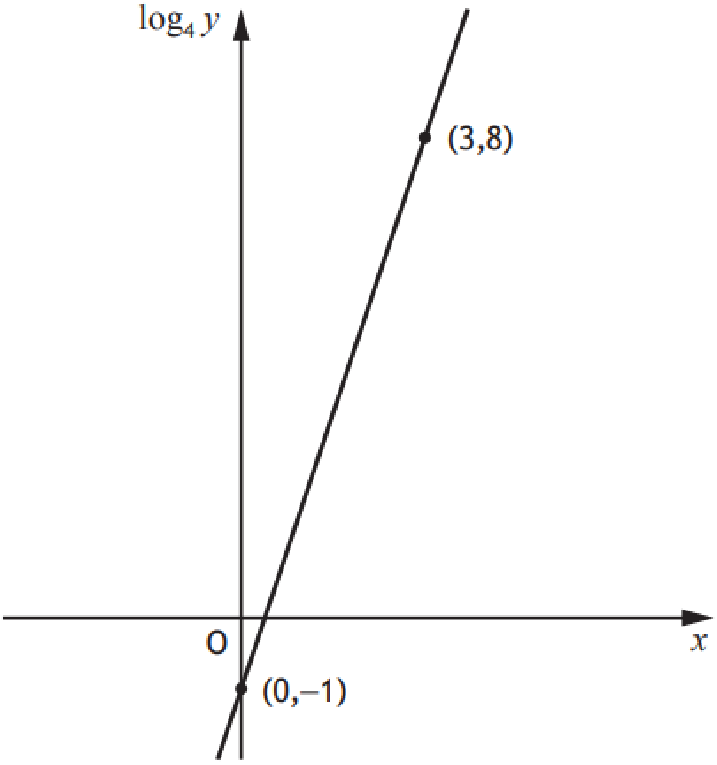
(a) State the value of a.

(b) Find the value of k.

Answers: $(a)\ a = 3 \quad (b)\ k = -2$

Source: 2019 P1 Q12 Higher Maths	
(2)	Functions f and g are defined by • $f(x)=\frac{1}{\sqrt{x}}$, where $x>0$ • $g(x)=5-x$, where $x\in\mathbb{R}$. (a) Determine an expression for $f(g(x))$. (b) State the range of values of x for which $f(g(x))$ is undefined.
Answers: $(a)\ \frac{1}{\sqrt{5-x}}$ $(b)\ x\geq 5$	

Source: 2019 P2 Q5 Higher Maths	
(3)	The diagram below shows the graph of a cubic function $y=g(x)$, with stationary points at $x=-2$ and $x=4$.  On the diagram in your answer booklet, sketch the graph of $y=g'(x)$.
Answer: 	

Source: 2019 P2 Q12 Higher Maths	
(4)	Two variables, x and y, are connected by the equation $y = ab^x$. The graph of $\log_4 y$ against x is a straight line as shown.  Find the values of a and b.
Answers: $a = \frac{1}{4}$ $b = 64$	

Source: 2018 P1 Q2 Higher Maths	
(5)	A function $g(x)$ is defined on $\mathbb{R}$, the set of real numbers, by $$g(x) = \frac{1}{5}x - 4.$$ Find the inverse function, $g^{-1}(x)$.
Answer: $g^{-1}(x) = 5(x + 4)$	

Source: 2018 P1 Q15 Higher Maths	
(6)	A cubic function, f, is defined on the set of real numbers. • $(x+4)$ is a factor of $f(x)$ • $x=2$ is a repeated root of $f(x)$ • $f'(-2)=0$ • $f'(x)>0$ where the graph with equation $y=f(x)$ crosses the y-axis Sketch a possible graph of $y=f(x)$ on the diagram in your answer booklet.

Answer:

•1 root at $x=-4$ identifiable from graph	•1
•2 stationary point touching x-axis when $x=2$ identifiable from graph	•2
•3 stationary point when $x=-2$ identifiable from graph	•3
•4 identify orientation of the cubic curve and $f'(0)>0$ identifiable from graph	•4 

Source: 2018 P2 Q6 Higher Maths	
(7)	Functions, f and g, are given by $f(x)=3+\cos x$ and $g(x)=2x$, $x\in\mathbb{R}$. (a) Find expressions for (i) $f(g(x))$ and (ii) $g(f(x))$. (b) Determine the value(s) of x for which $f(g(x))=g(f(x))$ where $0\le x<2\pi$.

Answers: $(a)\ (i)\ 3+cos2x \quad (ii)\ 2(3+cosx) \qquad (b)\ x=\pi$

Source: 2017 P1 Q1 Higher Maths	
(8)	Functions f and g are defined on suitable domains by $f(x) = 5x$ and $g(x) = 2\cos x$. (a) Evaluate $f(g(0))$. (b) Find an expression for $g(f(x))$.
Answers: $(a)\ 10 \quad (b)\ 2cos5x$	

Source: 2017 P1 Q6 Higher Maths	
(9)	A function, h, is defined by $h(x) = x^3 + 7$, where $x \in \mathbb{R}$. Determine an expression for $h^{-1}(x)$.
Answer: $h^{-1}(x) = \sqrt[3]{x-7} \quad or \quad (x-7)^{\frac{1}{3}}$	

Source: 2016 P1 Q6 Higher Maths	
(10)	Functions f and g are defined on $\mathbb{R}$, the set of real numbers. The inverse functions f^{-1} and g^{-1} both exist. (a) Given $f(x) = 3x + 5$, find $f^{-1}(x)$. (b) If $g(2) = 7$, write down the value of $g^{-1}(7)$.
Answers: $(a)\ g^{-1}(x) = \frac{x-5}{3} \qquad (b)\ g^{-1}(7) = 2$	

Source: 2017 P1 Q15a Higher Maths

(11) A quadratic function, f, is defined on $\mathbb{R}$, the set of real numbers.

Diagram 1 shows part of the graph with equation $y = f(x)$.

The turning point is $(2, 3)$.

Diagram 2 shows part of the graph with equation $y = h(x)$.

The turning point is $(7, 6)$.

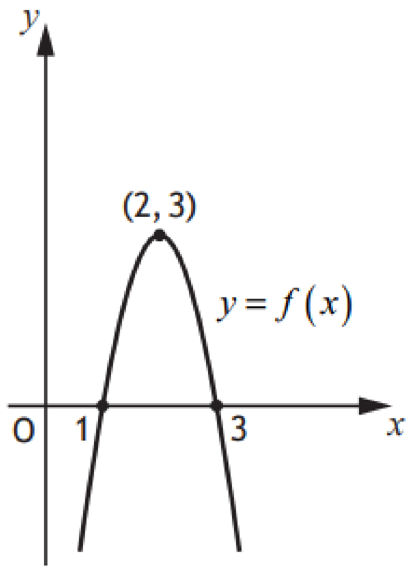


Diagram 1

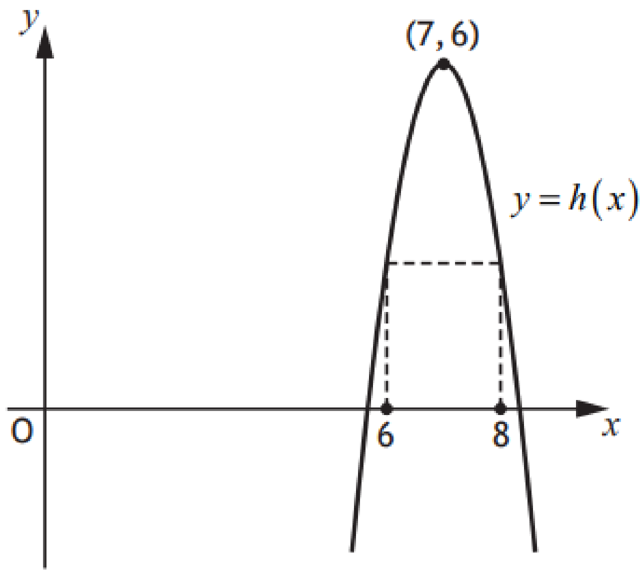


Diagram 2

(a) Given that $h(x) = f(x + a) + b$.

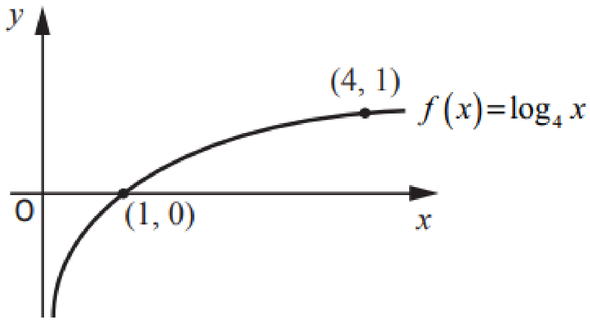
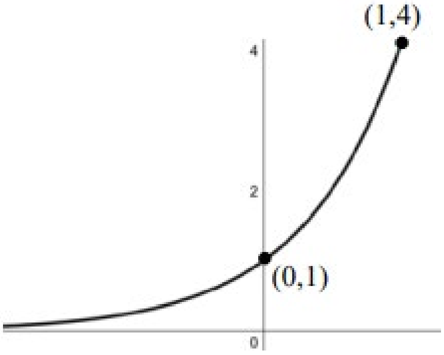
Write down the values of a and b.

(b) It is known that $\int_1^3 f(x)\,dx = 4$.

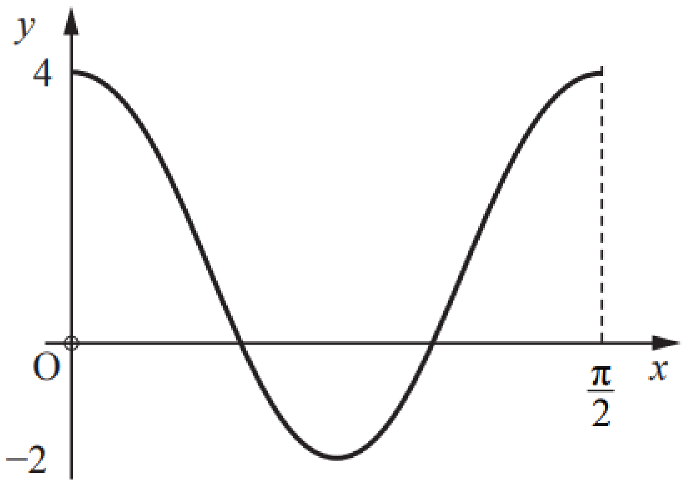
Determine the value of $\int_6^8 h(x)\,dx$.

(c) Given $f'(1) = 6$, state the value of $h'(8)$.

Answers: $(a)\ a = -5\ \ b = 3 \quad (b)\ 10 \quad (c) - 6$

Source: 2016 P1 Q10 Higher Maths	
(12)	The diagram below shows the graph of the function $f(x)=\log_4 x$, where $x>0$.  The inverse function, f^{-1}, exists. On the diagram in your answer booklet, sketch the graph of the inverse function.
Answer: 	

Source: 2016 P1 Q12 Higher Maths	
(13)	The functions f and g are defined on $\mathbb{R}$, the set of real numbers by $f(x)=2x^2-4x+5$ and $g(x)=3-x$. (a) Given $h(x)=f(g(x))$, show that $h(x)=2x^2-8x+11$. (b) Express $h(x)$ in the form $p(x+q)^2+r$.
Answers: $(a)\ Proof \quad (b)\ 2(x-2)^2+3$	

Source: 2015 P1 Q4 Higher Maths	
(14)	The diagram shows part of the graph of the function $y = p\cos qx + r$  Write down the values of p, q and r.
Answers: $p = 3,\quad q = 4,\quad r = 1$	

Source: 2015 P1 Q5 Higher Maths	
(15)	A function g is defined on $\mathbb{R}$, the set of real numbers, by $g(x) = 6 - 2x$. (a) Determine an expression for $g^{-1}(x)$. (b) Write down an expression for $g(g^{-1}(x))$.
Answers: $(a)\ \ g^{-1}(x) = \frac{6-x}{2} \quad = 3 - \frac{x}{2} \quad = \frac{x-6}{-2}$ $(b)\ x$	

Source: 2015 P2 Q2 Higher Maths

(16)

Functions f and g are defined on suitable domains by

$$f(x) = 10 + x \text{ and } g(x) = (1 + x)(3 - x) + 2.$$

(a) Find an expression for $f(g(x))$.

(b) Express $f(g(x))$ in the form $p(x + q)^2 + r$.

(c) Another function h is given by $h(x) = \frac{1}{f(g(x))}$.

What values of x cannot be in the domain of h?

Answers: $(a) 10 + (1 + x)(3 - x) + 2 \quad (b)\ -x^2 + 2x + 15 \quad (c)\ 5\ \&\ -3$

Source: Specimen P1 Q8 Higher Maths

(17)

$f(x)$ and $g(x)$ are functions, defined on the set of real numbers, such that

$f(x) = 1 - \frac{1}{2}x$ and $g(x) = 8x^2 - 3$.

(a) Given that $h(x) = g(f(x))$, show that $h(x) = 2x^2 - 8x + 5$.

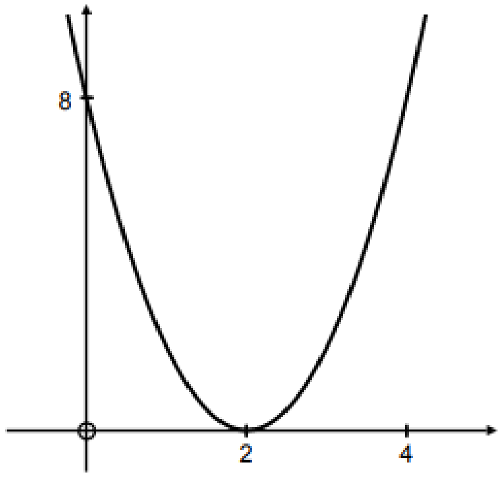
(b) Express $h(x)$ in the form $a(x + p)^2 + q$.

(c) Hence, or otherwise, state the coordinates of the turning point on the graph of $y = h(x)$.

(d) Sketch the graph of $y = h(x) + 3$, showing clearly the coordinates of the turning point and the y-axis intercept.

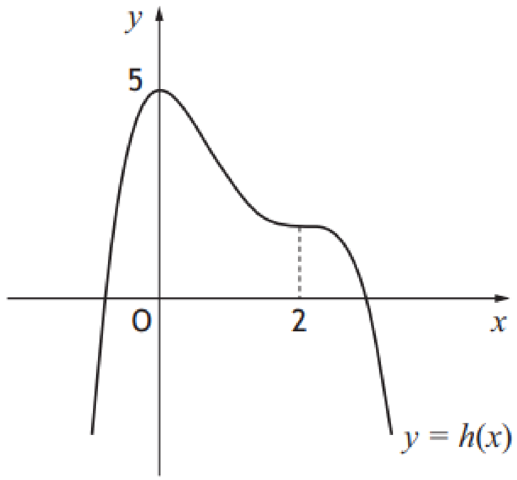
Answers: $(a)\ Proof \quad (b)\ 2(x - 2)^2 - 3 \quad (c)\ (2, -3)$

(d)



Source: Exemplar P1 Q8 Higher Maths

(18) The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $x = 0$ and $x = 2$.



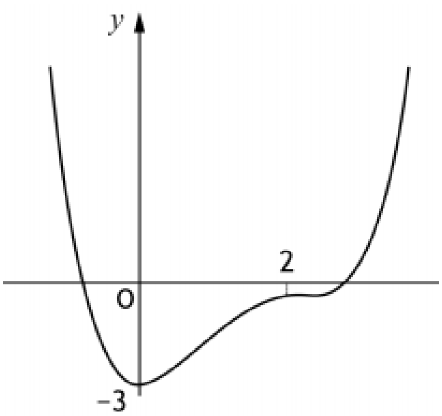
On separate diagrams sketch the graphs of:

(a) $y = 2 - h(x)$.

(b) $y = h'(x)$.

Answers:

(a)



(b)