Integration – Area under a Curve

Higher Maths Exam Questions

Source: 2019 P1 Q8 Higher Maths

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<td>The graphs of ( y = x^2 + 2x + 3 ) and ( y = 2x^2 + x + 1 ) are shown below.</td>
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The graphs intersect at the points where \( x = -1 \) and \( x = 2 \).

(a) Express the shaded area, enclosed between the curves, as an integral.

(b) Evaluate the shaded area.

Answers:  
(a) \( \int_{-1}^{2} (-x^2 + x + 2) \)  
(b) \( \text{Area} = \frac{9}{2} \)
The diagram shows the curve with equation \( y = 3 + 2x - x^2 \).

Calculate the shaded area.

Answer: \( \frac{32}{3} \text{ units}^2 \)
Two curves with equations $y = x^3 - 4x^2 + 3x + 1$ and $y = x^2 - 3x + 1$ intersect as shown in the diagram.

(a) Calculate the shaded area.

The line passing through the points of intersection of the curves has equation $y = 1 - x$.

(b) Determine the fraction of the shaded area which lies below the line $y = 1 - x$.

Answers: (a) Shaded Area $= \frac{8}{3}$ units$^2$  (b) Fraction $= \frac{1}{2}$
A quadratic function, $f$, is defined on $\mathbb{R}$, the set of real numbers.

Diagram 1 shows part of the graph with equation $y = f(x)$. The turning point is $(2, 3)$.

Diagram 2 shows part of the graph with equation $y = h(x)$. The turning point is $(7, 6)$.

(a) Given that $h(x) = f'(x + a) + b$.
   Write down the values of $a$ and $b$.

(b) It is known that $\int_{2}^{3} f(x) \, dx = 4$.
   Determine the value of $\int_{6}^{8} h(x) \, dx$.

(c) Given $f'(1) = 6$, state the value of $h'(8)$.

Answers:  
(a) $a = -5$, $b = 3$  
(b) 10  
(c) $-6$
(5) 

(a) (i) Show that \((x+1)\) is a factor of \(2x^3 - 9x^2 + 3x + 14\).

(ii) Hence solve the equation \(2x^3 - 9x^2 + 3x + 14 = 0\).

(b) The diagram below shows the graph with equation \(y = 2x^3 - 9x^2 + 3x + 14\).

The curve cuts the \(x\)-axis at \(A\), \(B\) and \(C\).

\[ y = 2x^3 - 9x^2 + 3x + 14 \]

(i) Write down the coordinates of the points \(A\) and \(B\).

(ii) Hence calculate the shaded area in the diagram.

\[ (a) \quad (i) \quad \text{Use synthetic division or substitution} \quad (ii) \quad x = -1, 2, 3.5 \]

\[ (b) \quad (i) \quad A(-1,0) \quad B(2,0) \quad (ii) \quad \text{Shaded area} = 27 \text{ units}^2 \]
The diagram shows part of the graph of \( y = a \cos bx \).

The shaded area is \( \frac{1}{2} \) unit\(^2\).

What is the value of \( \int_{0}^{\frac{3\pi}{4}} (a \cos bx) \, dx \)?

Answer: \( -\frac{1}{2} \)

The line with equation \( y = 2x + 3 \) is a tangent to the curve with equation \( y = x^3 + 3x^2 + 2x + 3 \) at \( A(0, 3) \), as shown.

The line meets the curve again at \( B(-3, -3) \). Find the area enclosed by the line and the curve.

Answer: \( \text{Area} = \frac{27}{4} \text{units}^2 \)
A wall plaque is to be made to commemorate the 150th anniversary of the publication of “Alice’s Adventures in Wonderland”.

The edges of the wall plaque can be modelled by parts of the graphs of four quadratic functions as shown in the sketch.

\[ f(x) = \frac{1}{4}x^2 - \frac{1}{2}x + 3 \]
\[ g(x) = \frac{1}{4}x^2 - \frac{3}{2}x + 5 \]
\[ h(x) = \frac{3}{8}x^2 - \frac{9}{4}x + 3 \]
\[ k(x) = \frac{3}{8}x^2 - \frac{3}{4}x \]

(a) Find the x-coordinate of the point of intersection of the graphs with equations \( y = f(x) \) and \( y = g(x) \).

The graphs of the functions \( f(x) \) and \( h(x) \) intersect on the y-axis.
The plaque has a vertical line of symmetry.

(b) Calculate the area of the wall plaque.

Answers: \( (a) \ x = 2 \quad (b) \ Area = \frac{19}{3} \text{ units}^2 \)
9. A sea-life visitor attraction has a new logo in the shape of a shark fin. The outline of the logo can be represented by parts of
   - the x axis
   - the curve with equation $y = \cos(2x)$
   - the curve with equation $y = \sin\left(\frac{1}{4}x - \frac{3}{2} \pi\right)$
as shown in the diagram.

Calculate the shaded area.

Answer: $Area = \frac{5}{6} \text{ units}^2$