A manufacturer of chocolates is launching a new product in novelty shaped cardboard boxes.

The box is a cuboid with a cuboid shaped tunnel through it.
- The height of the box is \( h \) centimetres
- The top of the box is a square of side \( 3x \) centimetres
- The end of the tunnel is a square of side \( x \) centimetres
- The volume of the box is \( 2000 \text{ cm}^3 \)

(a) Show that the total surface area, \( A \text{ cm}^2 \), of the box is given by

\[
A = 16x^2 + \frac{4000}{x}.
\]

(b) To minimise the cost of production, the surface area, \( A \), of the box should be as small as possible.

Find the minimum value of \( A \).
A council is setting aside an area of land to create six fenced plots where local residents can grow their own food.

Each plot will be a rectangle measuring \( x \) metres by \( y \) metres as shown in the diagram.

\[
\begin{array}{ccc}
  & x & \\
  y &  & y \\
  & x & \\
\end{array}
\]

(a) The area of land being set aside is 108 m\(^2\).

Show that the total length of fencing, \( L \) metres, is given by

\[
L(x) = 9x + \frac{144}{x}.
\]

(b) Find the value of \( x \) that minimises the length of fencing required.
A design for a new grain container is in the shape of a cylinder with a hemispherical roof and a flat circular base. The radius of the cylinder is $r$ metres, and the height is $h$ metres.

The volume of the **cylindrical** part of the container needs to be 100 cubic metres.

(a) Given that the curved surface area of a hemisphere of radius $r$ is $2\pi r^2$ show that the surface area of metal needed to build the grain container is given by:

$$A = \frac{200}{r} + 3\pi r^2 \text{ square metres}$$

(b) Determine the value of $r$ which minimises the amount of metal needed to build the container.
A manufacturer is asked to design an open-ended shelter, as shown:

The frame of the shelter is to be made of rods of two different lengths:
- \( x \) metres for top and bottom edges;
- \( y \) metres for each sloping edge.

The total length, \( L \) metres, of the rods used in a shelter is given by:

\[
L = 3x + \frac{48}{x}
\]

To minimise production costs, the total length of rods used for a frame should be as small as possible.

(a) Find the value of \( x \) for which \( L \) is a minimum.

The rods used for the frame cost £8.25 per metre.
The manufacturer claims that the minimum cost of a frame is less than £195.

(b) Is this claim correct? Justify your answer.
The parabolas with equations \( y = 10 - x^2 \) and \( y = \frac{2}{5}(10 - x^2) \) are shown in the diagram below.

A rectangle PQRS is placed between the two parabolas as shown, so that:

- Q and R lie on the upper parabola;
- RQ and SP are parallel to the x-axis;
- T, the turning point of the lower parabola, lies on SP.

(a) (i) If TP = \( x \) units, find an expression for the length of PQ.

(ii) Hence show that the area, \( A \), of rectangle PQRS is given by

\[ A(x) = 12x - 2x^3. \]

(b) Find the maximum area of this rectangle.
(6) In the diagram, $Q$ lies on the line joining $(0, 6)$ and $(3, 0)$.

OPQR is a rectangle, where $P$ and $R$ lie on the axes and $OR = t$.

(a) Show that $QR = 6 - 2t$.

(b) Find the coordinates of $Q$ for which the rectangle has a maximum area.

(7) An open cuboid measures internally $x$ units by $2x$ units by $h$ units and has an inner surface area of 12 units$^2$.

(a) Show that the volume, $V$ units$^3$, of the cuboid is given by $V(x) = \frac{2}{3}x(6 - x^2)$.

(b) Find the exact value of $x$ for which this volume is a maximum.
An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.

The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length $x$ cm. The tank has a length of $l$ cm.

(a) Show that the surface area to be lined, $A$ cm$^2$, is given by $A(x) = x^2 + \frac{432000}{x}$.

(b) Find the value of $x$ which minimises this surface area.
The shaded rectangle on this map represents the planned extension to the village hall. It is hoped to provide the largest possible area for the extension.

The coordinate diagram represents the right angled triangle of ground behind the hall. The extension has length \( l \) metres and breadth \( b \) metres, as shown. One corner of the extension is at the point \((a, 0)\).

\[(a)\]
(i) Show that \( l = \frac{5}{4}a \).

(ii) Express \( b \) in terms of \( a \) and hence deduce that the area, \( A \) m\(^2\), of the extension is given by \( A = \frac{3}{4}a(8 - a) \).

\[(b)\] Find the value of \( a \) which produces the largest area of the extension.