

# Paper A Paper 1

## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

### Scalar Product:

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

### Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

### Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

### Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$



1. A sequence is defined by the recurrence relation  $u_{n+1} = 2u_n + 1$ ,  $u_0 = 3$ .

What is the value of  $u_2$ ?

2. The line with equation  $kx - 2y + 9 = 0$  is parallel to the line with gradient 7.

What is the value of  $k$ ?

3. A circle has equation  $x^2 + y^2 - 8x + 2y - 1 = 0$ .

What is the radius of this circle?

4. What is the derivative of  $\frac{x^3 - 2}{3x}$  with respect to  $x$ ?

5. Find  $\int \frac{1}{2x^4} dx$ .

6. If  $x^2 - 12x + 37$  is written in the form  $(x - p)^2 + q$ , find the value of  $q$ .

7. A sequence is generated by the recurrence relation  $u_{n+1} = 0.8u_n + 16$ .

What is the limit of this sequence as  $n \rightarrow \infty$ ?

8. A circle with centre  $(-1, 5)$  passes through the point  $(2, 7)$ .

What is the equation of the circle?

9. The vectors  $\mathbf{p}$  and  $\mathbf{q}$  with components  $\mathbf{p} = \begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix}$  and  $\mathbf{q} = \begin{pmatrix} k \\ -3 \\ -2 \end{pmatrix}$  are perpendicular.

What is the value of  $k$ ?

10. Identify the nature of the roots of the equation  $2x^2 - 8x + 3 = 0$ .

11. What is the value of  $\cos \frac{5\pi}{3} - \tan \frac{7\pi}{4}$ ?

12. Given that  $\log_2 \frac{1}{8} = p$ , find the value of  $p$ .

13. Find  $\int (3x - 11)^5 dx$

14. K and L are the points with coordinates  $(0, -1, 4)$  and  $(3, -2, 5)$  respectively.

If  $\overrightarrow{KM} = 3\overrightarrow{KL}$ , find the coordinates of M.

15.  $h(x) = \frac{4}{x^2 - 2x - 8}$ .

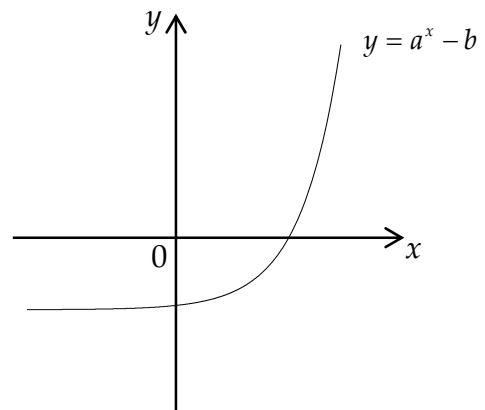
For what values of  $x$  is  $h(x)$  undefined?

16. Here are two statements about the graph with equation  $y = a^x - b$ , shown opposite.

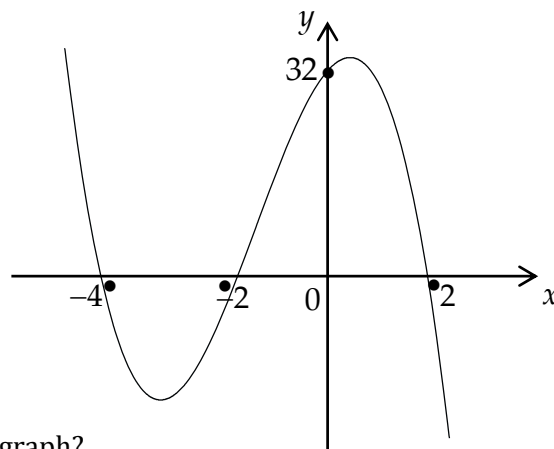
(1)  $0 < a < 1$ ;

(2)  $y$  is always increasing

Which of these statements are true?



17. The diagram shows part of the graph of a cubic.

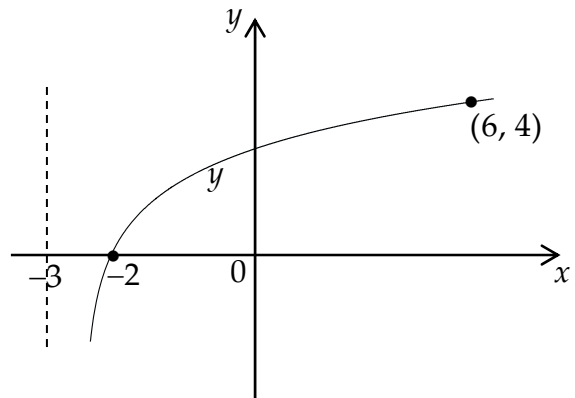


What is the equation of this graph?

18. Given that  $\log_4 y = 2 - \log_4 5x$ , express  $y$  in terms of  $x$ .

19. If  $p \cdot (p - q) = 18$  and  $|p| = 3$ , find the value of  $p \cdot q$

20. The diagram shows part of the curve with equation  $y = p \log_3(x + k)$ .  
What is the value of  $p$



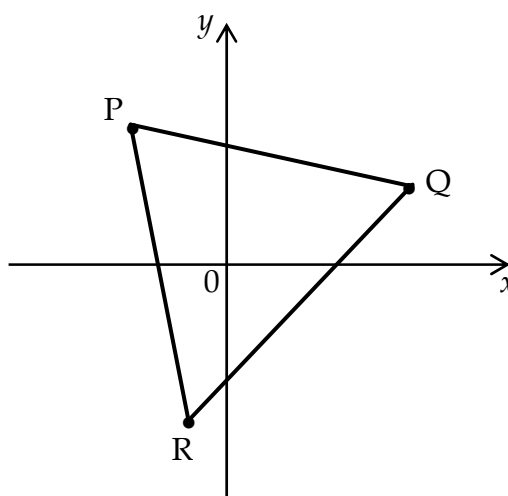
**End of Section A**

## SECTION B

ALL questions should be attempted.

Marks

21. Triangle PQR has vertices  $P(-3, 5)$ ,  $Q(7, 3)$  and  $R(-1, -5)$ , as shown.



- (a) Find the equation of the median RM. 3
- (b) Find the equation of the altitude AP. 3
- (c) Find the coordinates of the point of intersection of RM and AP. 2
22. Find the stationary points on the curve given by  $y = x^3 - 9x^2 + 24x - 2$  and determine their nature. 7
23. (a) Functions  $f$  and  $g$  are defined on suitable domains by
- $$f(x) = 2x^2 + 5 \text{ and } g(x) = x - 1$$
- Find  $f(g(x))$ . 2
- (b) Sketch the curve with equation  $y = f(g(x))$ . 3
24. (a) Show that  $2 \sin\left(x + \frac{\pi}{6}\right) - 2 \cos x = \sqrt{3} \sin x - \cos x$ . 2
- (b) Express  $\sqrt{3} \sin x - \cos x$  in the form  $k \sin(x - a)$  where  $k > 0$  and  $0 < a < \frac{\pi}{2}$ . 4
- (c) Hence, or otherwise, solve  $2 \sin\left(x + \frac{\pi}{6}\right) = 2 \cos x + \sqrt{3}$ , where  $0 \leq x \leq 2\pi$ . 4

End of question paper

## Paper A Paper 2

### FORMULAE LIST

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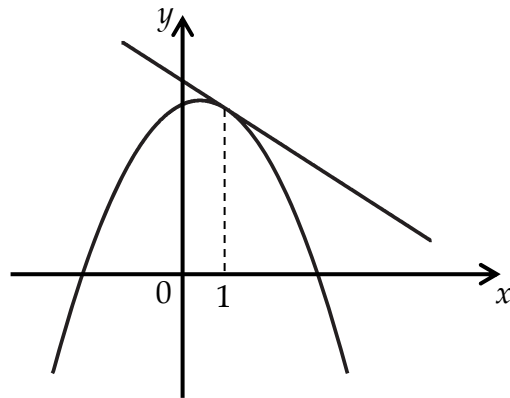
$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$

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Marks

1. K is the point  $(3, -2, 3)$ , L $(5, 0, 7)$  and M $(7, -3, -1)$ .
- (a) Write down the components of  $\overrightarrow{KL}$  and  $\overrightarrow{KM}$ . 2
- (b) Calculate the size of angle LKM. 5
2. (a) (i) Show that  $(x-2)$  is a factor of  $f(x) = 2x^3 - 3x^2 - 3x + 2$ .
- (ii) Hence factorise  $f(x)$  fully. 4
- (b) Solve  $2(x^3 + 1) = 3x(x + 1)$ . 2

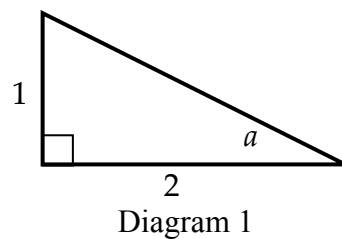
3. (a) Find the equation of the tangent to the parabola with equation
- $$y = 6 + x - x^2$$
- at the point  $(1, 6)$ .



- (b) Show that this line is also a tangent to the circle with equation
- $$x^2 + y^2 - 4x - 14y + 51 = 0$$
- 4
- 5

4. In the right-angled triangle shown in Diagram 1,  $\tan a = \frac{1}{2}$ .

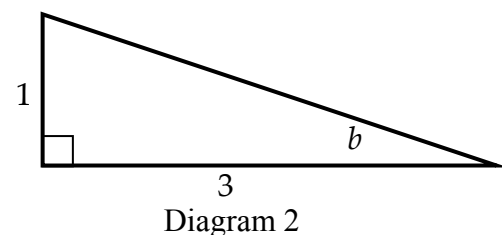
- (a) Find the exact values of
- (i)  $\cos a$ ;
- (ii)  $\cos 2a$ .



3

- In the right-angled triangle shown in Diagram 2,  $\tan b = \frac{1}{3}$ .

- (b) Find the exact value of  $\sin(2a + b)$ .

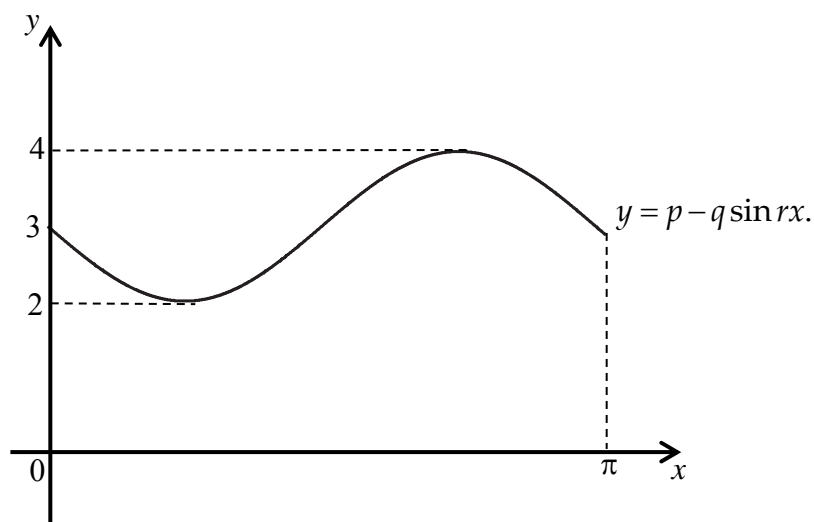


5

5. Solve  $\log_9(x+2) = \frac{1}{2} + \log_9(x-5)$ ,  $x > 5$ .

5

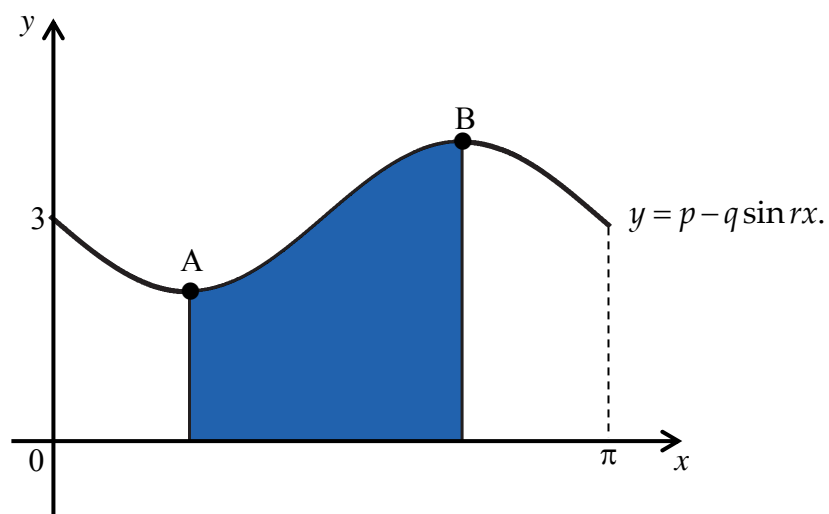
6. The diagram below shows part of the graph of  $y = p - q \sin rx$ .



(a) Write down the values of  $p$ ,  $q$  and  $r$ .

3

The graph of  $y = p - q \sin rx$  has a minimum turning point at A and a maximum turning point at B.



(b) Calculate the shaded area in the diagram above.

7



7. Cobalt-60 is used in food irradiation and decays to Nickel-60, which is a stable substance.

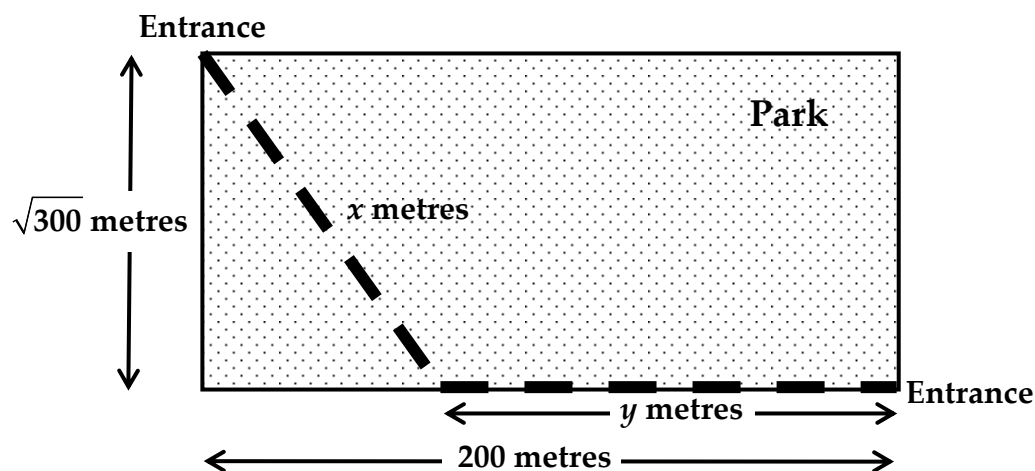
Cobalt-60 decays according to the law  $m_t = m_0 e^{kt}$ , where  $m_0$  is the initial mass of Cobalt-60 present and  $m_t$  is the mass remaining after  $t$  years.

The time taken for half the mass of Cobalt-60 to decay to Nickel-60 is 5 years.

- (a) Find the value of  $k$ , giving your answer correct to 3 significant figures. 3
- (b) In a sample of Cobalt-60 what percentage has decayed to Nickel-60 after 2 years? 3

8. A rectangular park measures 200 metres by  $\sqrt{300}$  metres.

A path connecting the two entrances, at opposite corners of the park, is to be laid through the park as shown.



The cost per metre of laying the path through the park is twice the cost, per metre, of laying the path along the perimeter.

- (a) Show that the total cost of laying this path can be modelled by

$$C(x) = 2x + 200 - \sqrt{x^2 - 300} \quad 3$$

- (b) Find the value of  $x$  which would minimise the cost of laying the path. 6

**End of Question Paper**