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Paper A Paper 1

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Mathematics
Higher
Paper 1
Practice Paper A

SECTION A

ALL questions should be attempted.

Time allowed
1 hour 30 minutes

H

1. A sequence is defined by the recurrence relation $u_{n+1} = 2u_n + 1,\ u_0 = 3$.

   What is the value of u_2?

2. The line with equation $kx - 2y + 9 = 0$ is parallel to the line with gradient 7.

   What is the value of k?

3. A circle has equation $x^2 + y^2 - 8x + 2y - 1 = 0$.

   What is the radius of this circle?

4. What is the derivative of $\dfrac{x^3 - 2}{3x}$ with respect to x?

5. Find $\displaystyle\int \frac{1}{2x^4}\,dx$.

6. If $x^2 - 12x + 37$ is written in the form $(x - p)^2 + q$, find the value of q.

7. A sequence is generated by the recurrence relation $u_{n+1} = 0\cdot8u_n + 16$.

   What is the limit of this sequence as $n \to \infty$?

8. A circle with centre $(-1,\ 5)$ passes through the point $(2,\ 7)$.

   What is the equation of the circle?

9. The vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ with components $\boldsymbol{p} = \begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix} k \\ -3 \\ -2 \end{pmatrix}$ are perpendicular.

   What is the value of k?

10. Identify the nature of the roots of the equation $2x^2 - 8x + 3 = 0$.

11. What is the value of $\cos\frac{5\pi}{3} - \tan\frac{7\pi}{4}$?

12. Given that $\log_2 \frac{1}{8} = p$, find the value of *p*.

13. Find $\int (3x-11)^5 \, dx$

14. K and L are the points with coordinates $(0, -1, 4)$ and $(3, -2, 5)$ respectively.

If $\overrightarrow{KM} = 3\overrightarrow{KL}$, find the coordinates of M.

15. $h(x) = \dfrac{4}{x^2 - 2x - 8}$.

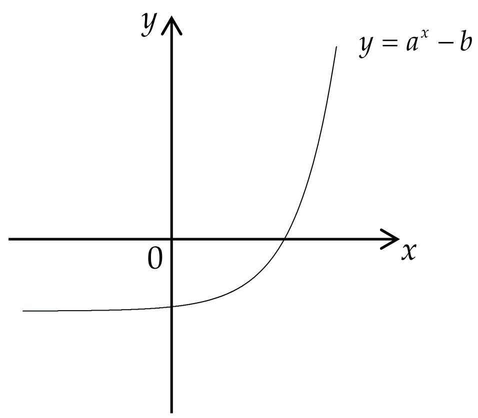
For what values of *x* is $h(x)$ undefined?

16. Here are two statements about the graph with equation $y = a^x - b$, shown opposite.

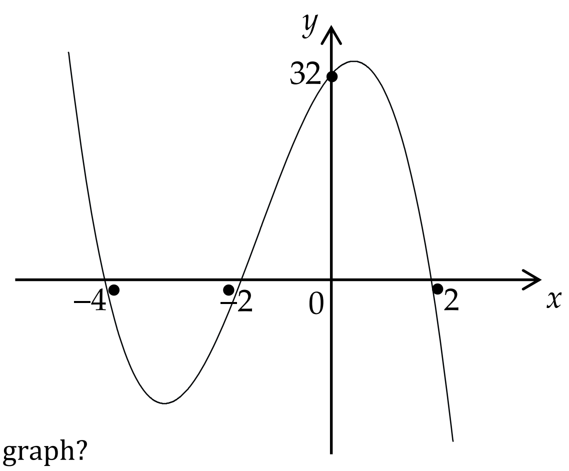
(1) $0 < a < 1$;

(2) y is always increasing

Which of these statements are true?



17. The diagram shows part of the graph of a cubic.



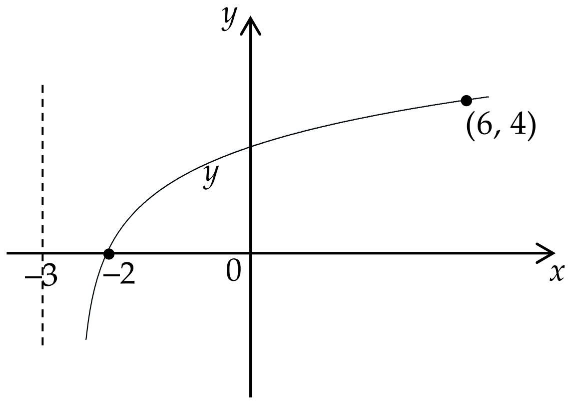
What is the equation of this graph?

18. Given that $\log_4 y = 2 - \log_4 5x$, express *y* in terms of *x*.

19. If $\boldsymbol{p} \cdot (\boldsymbol{p} - \boldsymbol{q}) = 18$ and $|\boldsymbol{p}| = 3$, find the value of $\boldsymbol{p} \cdot \boldsymbol{q}$

20. The diagram shows part of the curve with equation $y = p\log_3(x + k)$.

What is the value of p



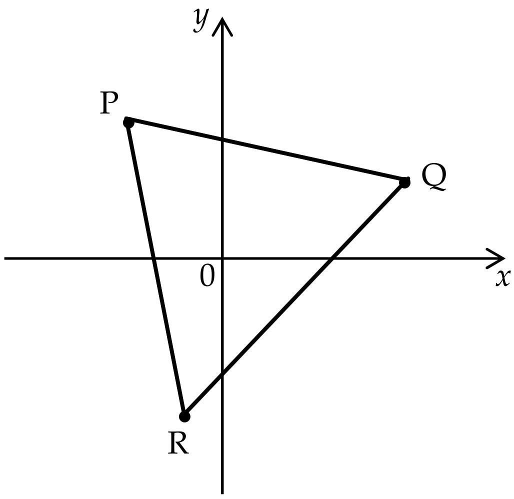
End of Section A

SECTION B

ALL questions should be attempted.

Marks

21. Triangle PQR has vertices P(–3, 5), Q(7, 3) and R(–1, –5), as shown.



(*a*) Find the equation of the median RM. **3**

(*b*) Find the equation of the altitude AP. **3**

(*c*) Find the coordinates of the point of intersection of RM and AP. **2**

22. Find the stationary points on the curve given by $y = x^3 - 9x^2 + 24x - 2$ and determine their nature. **7**

23. (*a*) Functions *f* and *g* are defined on suitable domains by

$$f(x) = 2x^2 + 5 \text{ and } g(x) = x - 1$$

Find $f(g(x))$. **2**

(*b*) Sketch the curve with equation $y = f(g(x))$. **3**

24. (*a*) Show that $2\sin\left(x + \frac{\pi}{6}\right) - 2\cos x = \sqrt{3}\sin x - \cos x$. **2**

(*b*) Express $\sqrt{3}\sin x - \cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. **4**

(*c*) Hence, or otherwise, solve $2\sin\left(x + \frac{\pi}{6}\right) = 2\cos x + \sqrt{3}$, where $0 \le x \le 2\pi$. **4**

End of question paper