

Paper B Paper 2

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product:

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$

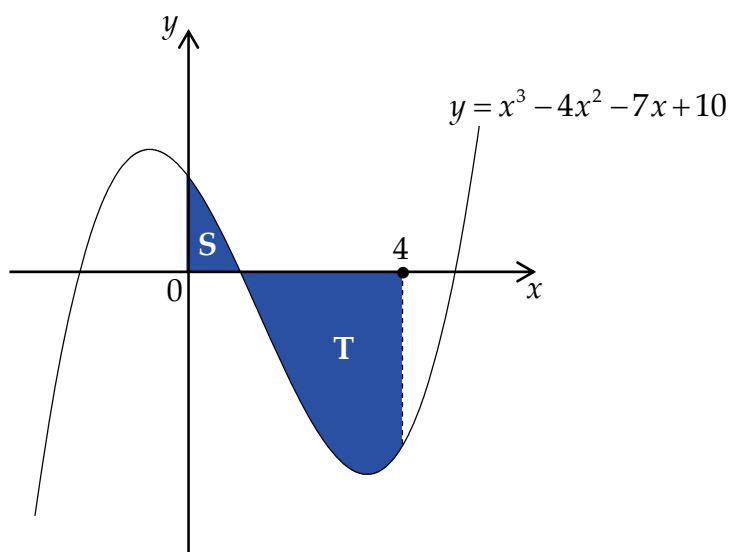
ALL questions should be attempted.

Marks

1. (a) A sequence is defined by the recurrence relation $u_{n+1} = 0.4u_n + 6$, $u_0 = 0$.
Determine the values of u_1 , u_2 and u_3 . 2
- (b) Why does this sequence have a limit as $n \rightarrow \infty$? 1
- (c) A second sequence, generated by $v_{n+1} = pv_n + 4$, has the same limit as the sequence in (a).
Find the value of p . 3

2. A function f is defined on the set of real numbers by $f(x) = x^3 - 4x^2 - 7x + 10$.
(a) Show that $(x - 1)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully. 4

The graph shown has equation of the form $y = x^3 - 4x^2 - 7x + 10$.



- (b) Calculate the shaded area labelled S. 4
- (c) Find the total shaded area. 3
3. D has coordinates $(7, -2, 1)$ and F is $(-1, 2, 5)$.
(a) Find the coordinates of E which divides DF in the ratio 1 : 3. 3
G has coordinates $(6, -2, 5)$.
(b) Show that EG is perpendicular to DF. 3

4. P, Q and R have coordinates $(-4, 6)$, $(8, 10)$ and $(2, 28)$ respectively.

(a) Show that PQ is perpendicular to QR. 2

(b) Hence find the equation of the circle which passes through P, Q and R. 3

5. Two functions f and g are defined on the set of real numbers by

$$f(x) = 2x + k \text{ and } g(x) = x^2 - 2k, \text{ where } k \neq 0$$

(a) Find (i) $f(g(x))$;

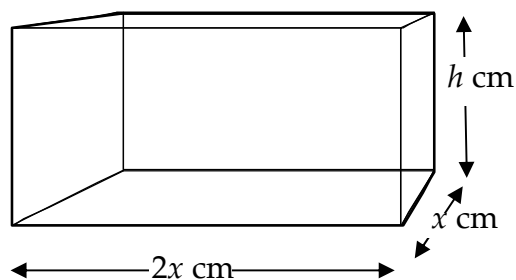
(ii) $g(f(x))$. 3

(b) Find the value of k for which $f(g(x)) = g(f(x))$ has equal roots. 5

6. A closed wooden box, in the shape of a cuboid, is constructed from a sheet of wood of area 600 cm^2 .

The base of the box measures $2x \text{ cm}$ by $x \text{ cm}$.

The height of the box is $h \text{ cm}$.



(a) Assuming the thickness of the sides of the box are negligible, show that the volume (in cubic centimetres) of the box is given by

$$V(x) = 200x - \frac{4}{3}x^3 \quad 3$$

(b) (i) Calculate the value of x for which this volume is a maximum.

(ii) Find the maximum volume of the box. 7

7. Whilst carrying out an experiment a scientist gathered some data.

The table shows the data collected by the scientist.

x	4.3	4.7	5.2	6.1
y	0.027	0.018	0.011	0.004



The variables x and y , in the table, are connected by a relationship of the form $y = ae^{bx}$.

Find the values of a and b .

6

8. Solve $2 - 3\cos x - 4\sin x = 0$ for $0 \leq x \leq 2\pi$.

8

End of Question Paper