

# Paper D Paper 1

## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

### Scalar Product:

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

### Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

### Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

### Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$

## SECTION A

**ALL questions should be attempted.**

1. The midpoint of the line joining  $G(-1, 3, 7)$  to  $H(5, -1, p)$  is  $M(q, 1, 4)$ .

What are the values of  $p$  and  $q$ ?

2. Given that  $f(x) = \frac{1}{3x^5}$ , find  $f'(x)$ .

3. If  $x^2 + 12x + 7$  is written in the form  $(x + a)^2 + r$ , find the value of  $r$ .

4. A straight line passes through the points  $(4, 3)$  and  $(0, -1)$ .

What is the equation of the line?

5. Functions  $f$  and  $g$  are defined on the set of real numbers by

$$f(x) = x^2 + 1 \text{ and } g(x) = 3x - 5$$

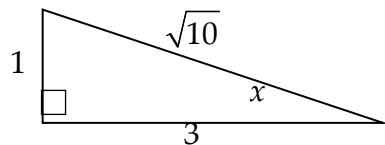
What is the value of  $g(f(-1))$ ?

6. The vectors with components  $\begin{pmatrix} 4 \\ 7 \\ -3 \end{pmatrix}$  and  $\begin{pmatrix} -5 \\ t \\ -2 \end{pmatrix}$  are perpendicular.

What is the value of  $t$ ?

7. The diagram shows a right-angled triangle with sides 1, 3 and  $\sqrt{10}$ .

What is the value of  $\cos 2x$ ?



8. Find  $\int_{-2}^0 6x^2 dx$

9. For what value of  $k$  does the equation  $2x^2 - 4x + k = 0$  have equal roots?

10.  $\overline{DE}$  and  $\overline{EF}$  have components  $\begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$  and  $\begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}$  respectively.

Given that D has coordinates  $(-2, 0, -2)$ , what are the coordinates of F?

11. What is the maximum value of  $8 - 3 \sin\left(x - \frac{7\pi}{9}\right)$ ?

12. Find  $\int (2x + 5)^3 dx$ .

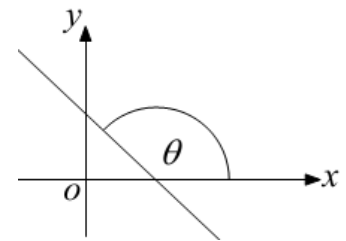
13. How many solutions does the equation  $(\sqrt{7} \cos x + 3)(4 \tan x - 9) = 0$  have in the interval  $0 \leq x < 2\pi$ ?

14. Given that  $f(x) = 4 \sin 3x$ , find  $f'\left(\frac{\pi}{6}\right)$ .

15. The diagram shows the line ST with equation  $2x + y = 0$ .

The angle between ST and the positive direction of the x-axis is  $\theta$ .

Find an expression for  $\theta$ .



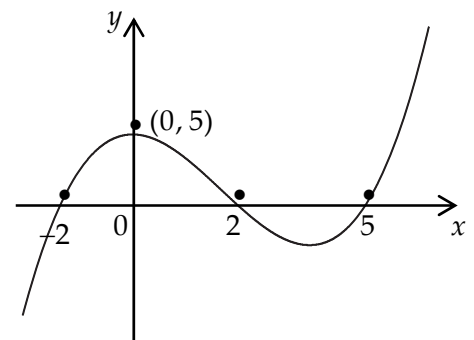
- A**  $\theta = \tan^{-1} \frac{1}{2}$       **B**  $\theta = \pi - \tan^{-1} \frac{1}{2}$       **C**  $\theta = \tan^{-1} 2$       **D**  
 $\theta = \pi - \tan^{-1} 2$

16. What is the value of  $\frac{\log_2 32}{\log_2 8}$ ?

17. The diagram shows a sketch of the curve with equation

$$y = k(x + 2)(x - 2)(x + a)$$

What are the values of  $a$  and  $k$ ?



18. Here are two statements about the function  $f(x) = \sqrt{x^2 - 4}$ .

(1) The largest possible domain is  $-2 \leq x \leq 2$ .

(2) The range is  $f(x) \geq 0$ .

Which of these statements is true?

19. Given that

$$f'(x) \begin{cases} > 0, & \text{for } x < 3 \\ = 0, & \text{for } x = 3 \\ > 0, & \text{for } x > 3 \end{cases}$$

Sketch a curve to represent  $y = f(x)$ ?

20. If  $5^x = a^2$ , find an expression for  $x$ .

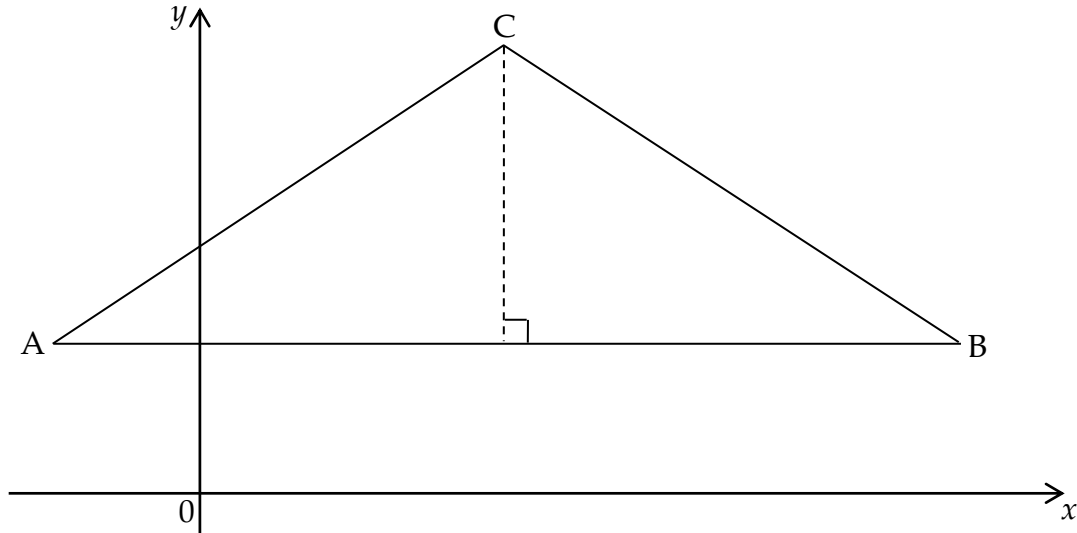
**End of Section A**

SECTION B

ALL questions should be attempted.

Marks

21.  $A(-2, 4)$ ,  $B(10, 4)$  and  $C(4, 8)$  are the vertices of triangle ABC shown in the diagram.

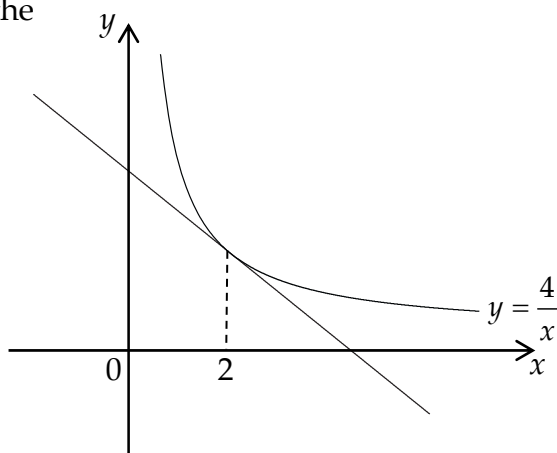


- (a) Write down the equation of the altitude from C. 1
- (b) Find the equation of the perpendicular bisector of BC. 4
- (c) Find the point of intersection of the lines found in (a) and (b). 2
- 
22. P is the point  $(4, 1, -2)$ , Q is  $(5, 2, 0)$  and R is  $(7, 4, 4)$ .
- (a) Show that P, Q and R are collinear. 3
- (b) Find the ratio in which Q divides PR. 1

23. Find the equation of the tangent to the curve with equation

$$y = \frac{4}{x}$$

at the point where  $x = 2$ .



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24. (a) Given that  $f'(x) = 3x^2 + 2x - 10$  and  $(x - 2)$  is a factor of  $f(x)$ , find a formula for  $f(x)$ .

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(b) Hence factorise  $f(x)$  fully.

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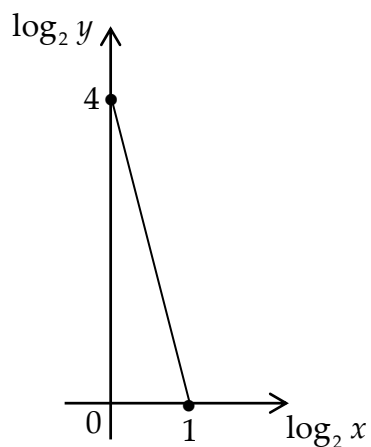
(c) Solve  $f(x) = 0$ .

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25. The graph illustrates the law  $y = ax^b$ .

The straight line joins the points  $(0, 4)$  and  $(1, 0)$ .

Find the values of  $a$  and  $b$ .



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End of question paper