

Paper E Paper 1

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product:

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$

SECTION A

ALL questions should be attempted.

1. K and L have position vectors $\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$ respectively.

What is the magnitude of \overline{KL} ?

2. If $f(x) = x^3 - 4x + 7$, find $f'(-2)$.

3. Find $\int \left(x^{\frac{1}{4}} - x^{-2} \right) dx$

4. A function f is defined on the set of real numbers by $f(x) = 4x + 5$.

Find an expression for $f(f(x))$.

5. Evaluate $4\sqrt{2} \sin \frac{\pi}{4} \cos \frac{2\pi}{3}$.

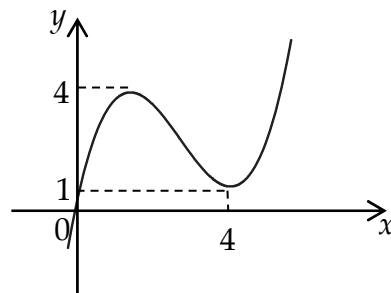
6. A circle with centre $(-3, 4)$ passes through the point $(-2, 2)$.

What is the equation of the circle?

7. $f(x) = 2x^3 - x^2 - 5x + 4$.

What is the remainder when $f(x)$ is divided by $(x + 2)$?

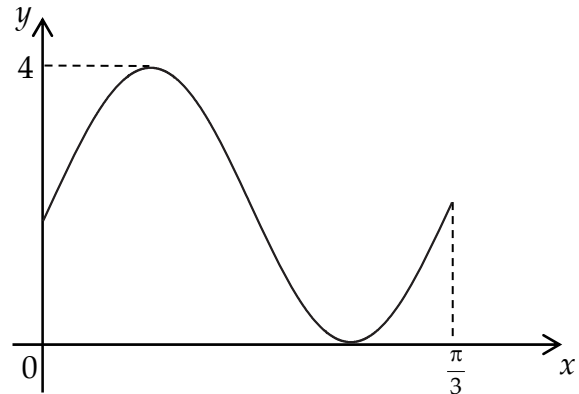
8. The diagram shows the part of the graph of the cubic $y = f(x)$.



Sketch the graph of $y = 4 - f(x)$?

9. The graph shown in the diagram has equation $y = p + 2\sin(qx)$.

What are the values of p and q ?



10. A sequence is generated by the recurrence relation $u_{n+1} = 7 - 2u_n$.

If $u_2 = 5$, what is the value of u_0 ?

11. For what value of k does the equation $kx^2 - 6x + 1 = 0$ have equal roots?

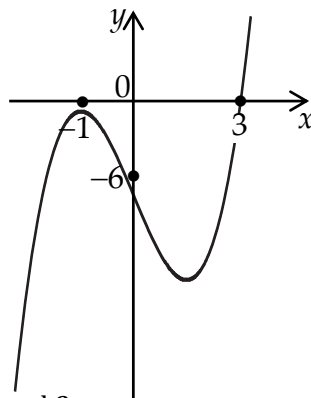
12. Find $\int (2x + 7)^4 dx$.

13. Given that $f'(x) = 6x^2$ and $f(1) = 5$, find a formula for $f(x)$ in terms of x .

14. What are the coordinates of the centre of the circle with equation

$$3x^2 + 3y^2 - 6x + 18y - 5 = 0?$$

15. The diagram shows part of the graph of a cubic function.

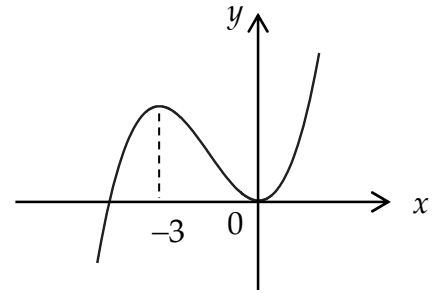


What is the equation of this graph?

16. The diagram shows part of the graph of the cubic $y = f(x)$.

There are stationary points at $x = 0$ and $x = -3$.

Sketch the graph of $y = f'(x)$?



17. If $4x^2 + 8x - 1$ is expressed in the form $4(x + p)^2 + q$, what is the value of q ?

18. If $3 \log_2 t - \log_2 5 = 3$, find the value of t .

19. If $p = 4x^{-3}$ find the rate of change of p with respect to x when $x = 2$.

20. Find the solutions for $8 - 2x - x^2 < 0$?

End of Section A

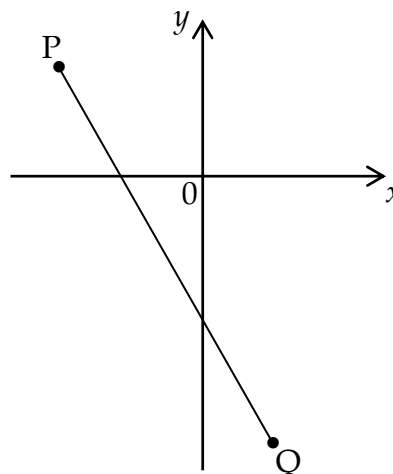
SECTION B

ALL questions should be attempted.

Marks

21. A line joins the points $P(-4, 3)$ and $Q(2, -7)$.

Find the equation of the perpendicular bisector of PQ .



4

22. Show that the line with equation $y = 2x + 10$ is a tangent to the circle with equation $x^2 + y^2 - 2x - 4y - 15 = 0$ and find the coordinates of the point of contact of the tangent and circle.

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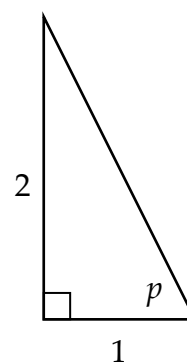
23. The diagram shows a right-angled triangle with height 2 units, base 1 unit and an angle of p .

(a) Find the exact values of:

(i) $\cos p$;

(ii) $\cos 2p$.

(b) By writing $3p = 2p + p$, find the exact value of $\cos 3p$.



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24. A function f is defined by $f(x) = x^3 - 2x^2 - 4x + 1$, where $0 \leq x \leq 3$.

Find the maximum and minimum values of f .

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25. (a) Express $2\sqrt{2} \cos x^\circ - 2\sqrt{2} \sin x^\circ$ in the form $k \cos(x - a)^\circ$, where $k > 0$
and $0 \leq a < 360$. 4

(b) Find:

(i) the maximum value of $3 + 2\sqrt{2} \sin x^\circ - 2\sqrt{2} \cos x^\circ$;

(ii) a value of x where this maximum value occurs in the interval $0 \leq x < 360$. 4

End of question paper