

Paper F Paper 1

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product:

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$

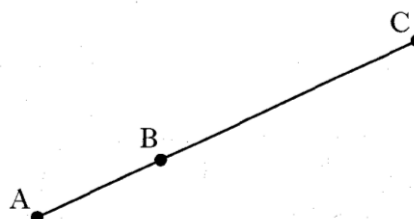
Practice Paper F Higher

Paper 1

1 hour and 10 minutes – 60 marks

1. Relative to a suitable coordinate system A and B are the points $(-2, 1, -1)$ and $(1, 3, 2)$ respectively.

A, B and C are collinear points and C is positioned such that $BC=2AB$.



Find the coordinates of C.

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2. A function is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

(a) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature.

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(b) (i) Show that $(x-1)$ is a factor of $x^3 - 3x + 2$.

(ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully.

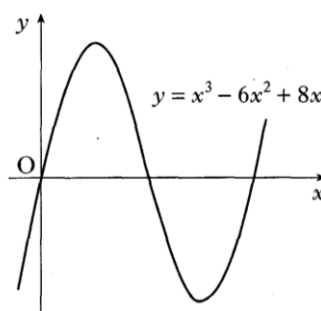
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(c) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axis and hence sketch the curve.

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3. The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.

(a) Find the coordinates of the points on the curve where the gradient of the tangent is -1 .



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(b) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A.

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4. A sequence is defined by the recurrence relation

$$U_{n+1} = \frac{1}{4}U_n + 16, U_0 = 0.$$

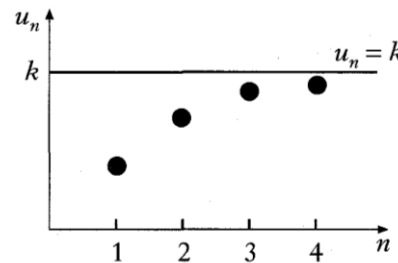
(a) Calculate the values U_1 , U_2 and U_3 .

Four terms of this sequence U_1 , U_2 , U_3 and U_4 are plotted as shown in the graph.

As $n \rightarrow \infty$, the points on the graph approach the line $U_n = k$, where k is the limit of the sequence.

(b) (i) Give a reason why this sequence has a limit.

(ii) Find the exact value of k .

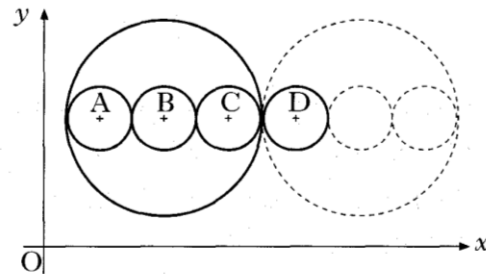


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5. The large circle has equation

$$x^2 + y^2 - 14x - 16y + 77 = 0.$$

Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the x-axis. This pattern is continued as shown in the diagram. Find the equation of the circle with centre D.



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6. Solve the equation $\sin 2x^\circ = 6 \cos x^\circ$ for $0 \leq x \leq 360$.

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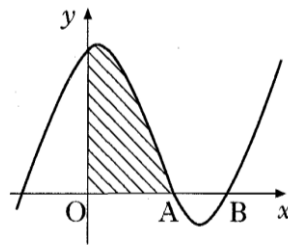
7. Given that $y = \sqrt{3x^2 + 2}$, find $\frac{dy}{dx}$.

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8. The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$. The graph cuts the x-axis at (3,0).

(a) Find the coordinates of A.

(b) Find the shaded area.



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9. Functions, f , g and h are defined on suitable domains by

$$f(x) = x^2 - x + 10, g(x) = 5 - x \text{ and } h(x) = \log_2 x.$$

(a) Find expressions for $h(f(x))$ and $h(g(x))$.

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(b) Hence solve $h(f(x)) - h(g(x)) = 3$.

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[END OF QUESTION PAPER]

Paper F Paper 2

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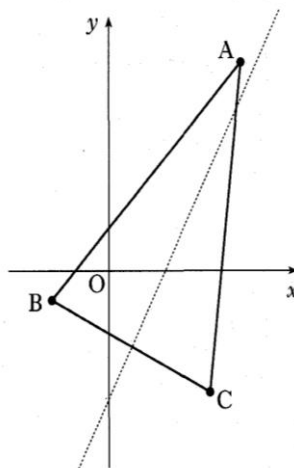
Practice Paper F Higher

Paper 2

1 hour and 30 minutes - 70 Marks

1. The vertices of triangle ABC are A(7,9), B(-3,-1) and C(5,-5) as shown in the diagram.

The broken line represents the perpendicular bisector of BC.



(a) Show that the equation of the perpendicular bisector of BC is $y = 2x - 5$.

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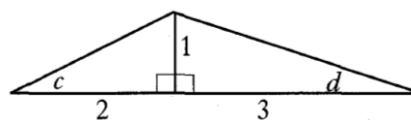
(b) Find the equation of the median from C.

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(c) Find the coordinates of the point of intersection of the perpendicular bisector of BC and the median from C.

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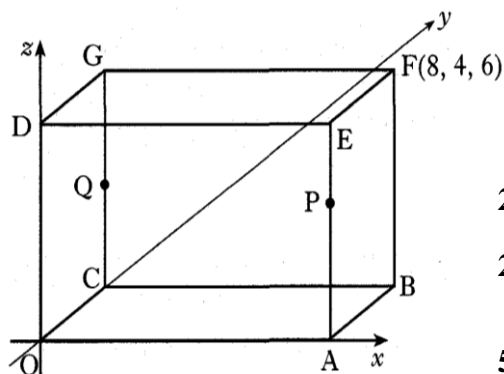
2. The diagram shows two right angled triangles with angles c and d marked as shown.



Find the exact value of $\sin(c + d)$.

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3. The diagram shows a cuboid OABC, DEFG. F is the point (8, 4, 6). P divides AE in the ratio 2:1. Q is the midpoint of CG.



(a) State the coordinates of P and Q.

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(b) Write down the components of PQ and PA.

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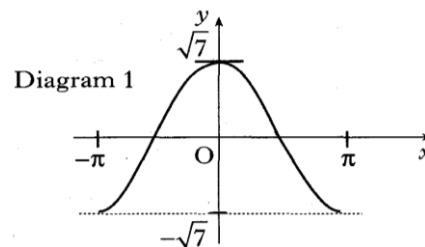
(c) Find the size of angle QPA.

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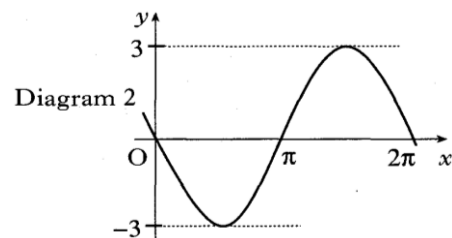
4. Find the value of $\int_0^2 \sin(4x+1)dx$.

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5. (a) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p \cos x$. Write down the value of p .



(ii) Diagram 2 shows part of the graph $y = g(x)$, where $f(x) = q \sin x$. Write down the value of q .



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(b) Write $f(x) + g(x)$ in the form $k \cos(x+a)$ where $k > 0$, and $0 < a < \frac{\pi}{2}$.

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(c) Hence find $f'(x) + g'(x)$ as a single trigonometric expression

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6. (a) Write down the centre and the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$.

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(b) A second circle has equation $(x-4)^2 + (y-6)^2 = 26$.

Find the distance between the centres of these two circles and hence show that the circles intersect.

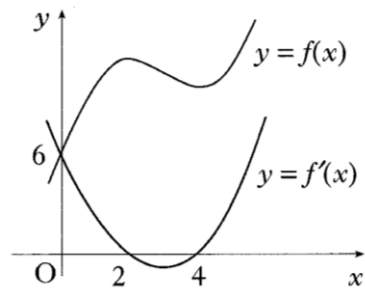
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(c) The line with equation $y = 4 - x$ is a common chord passing through the intersection of the two circles.

Find the coordinates of the points of intersection of the two circles

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7. The diagram shows the graph of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.



Both graphs pass through $(0, 6)$.

The graph of $y = f'(x)$ also passes through the points $(2, 0)$ and $(4, 0)$.

(a) Given that $f'(x)$ is of the form $k(x-a)(x-b)$:

(i) write down the values of a and b ;

(ii) find the value of k .

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(b) Find the equation of the graph of the cubic function $y = f(x)$.

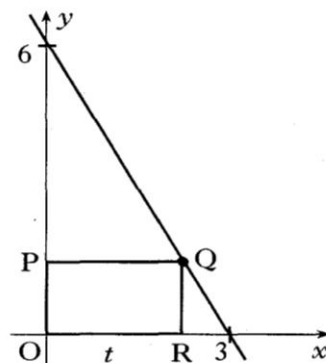
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8. In the diagram, Q lies on the line joining $(0, 6)$ and $(3, 0)$.

OPQR is a rectangle, where P and R lie on the axes and $OR = t$.

(a) Show that $QR = 6 - 2t$.

(b) Find the coordinates of Q for which the rectangle has a maximum area.



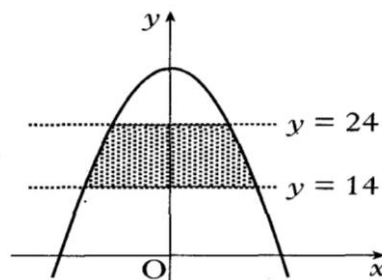
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9. The parabola shown in the diagram has equation $y = 32 - 2x^2$.

The shaded area lies between the lines $y = 14$ and $y = 24$.

Calculate the shaded area.



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[END OF QUESTION PAPER]