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Paper H Paper 1

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

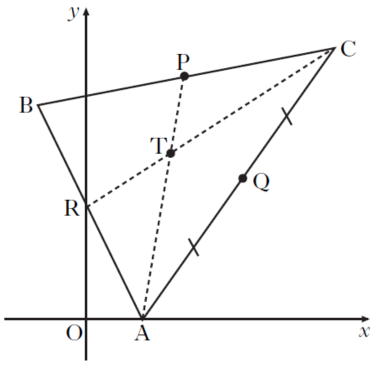
Practice Paper H - Higher

Paper 1

1 hour and 10 minutes – 60 marks

1. Triangle ABC has vertices A(4,0), B(-4,16) and C(18,20) as shown in the diagram opposite.

Medians AP and CR intersect at the point T(6, 12).



(a) Find the equation of median BQ **3**

(b) Verify that T lies on BQ. **1**

(c) Find the ratio in which T divides BQ. **2**

2. Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$ are defined on the set of real numbers.

(a) Find $h(x)$ where $h(x) = g(f(x))$. **2**

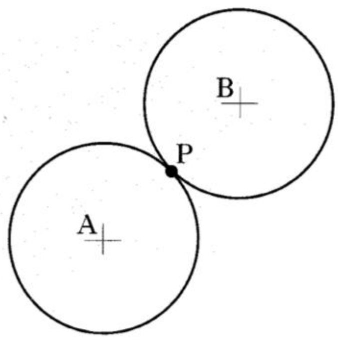
(b) (i) Write down the coordinates of the minimum turning point of $y = h(x)$.

(ii) Hence state the range of function h. **2**

3. Two congruent circles, with centres A and B, touch at P. Relative to suitable axes, their equations are

$x^2 + y^2 + 6x + 4y - 12 = 0$ and

$x^2 + y^2 - 6x - 12y + 20 = 0$.



(a) Find the coordinates of P. **3**

(b) Find the length of AB. **2**

4. Differentiate $(1+2\sin x)^4$ with respect to x. **2**

5. (a) The terms of a sequence satisfy $u_{n+1} = ku_n + 5$. Find the value of k which produces a sequence with a limit of 4. **2**

(b) A sequence satisfies the recurrence relation $u_{n+1} = mu_n + 5$, $u_o = 3$.

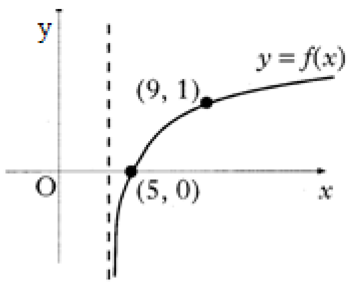
(i) Express u_1 and u_2 in terms of m.

(ii) Given that $u_2 = 7$, find the value of m which produces a sequence with no limit.

5

6. The function f is of the form $f(x) = \log_b(x-a)$.

The graph of $y = f(x)$ is shown in the diagram.



(a) Write down the values of a and b. **2**

(b) State the domain of f. **1**

7. A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.

(a) Show that $(x-3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully. **5**

(b) Find the co-ordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. **2**

(c) Find the greatest and least values of f in the interval $-2 \le x \le 2$. **5**

8. (a) Express $\sin x - \sqrt{3}\cos x$ in the form $k\sin(x-a)$ where $k > 0$ and $0 \le a \le 2\pi$. **4**

(b) Hence, or otherwise, sketch the curve with equation $y = 3 + \sin x - \sqrt{3}\cos x$ in the interval $0 \le x \le 2\pi$. **5**

9. (a) Diagram 1 shows a right angled triangle, where the line OA has equation $3x - 2y = 0$.

(i) Show that $\tan a = \frac{3}{2}$.

(ii) Find the value of $\sin a$.

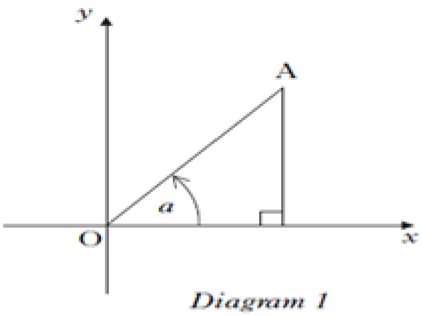


Diagram 1

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(b) A second right angled triangle is added as shown in Diagram 2.

The line OB has equation $3x - 4y = 0$.
Find the values of $\sin b$ and $\cos b$.

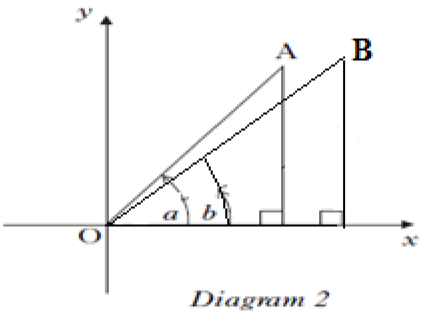

Diagram 2

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(i) Find the value of $\sin(a - b)$.

(ii) State the value of $\sin(b - a)$.

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[END OF QUESTION PAPER]