FORMULAE LIST

Circle:
The equation \(x^2 + y^2 + 2gx + 2fy + c = 0\) represents a circle centre \((-g, -f)\) and radius \(\sqrt{g^2 + f^2 - c}\).
The equation \((x - a)^2 + (y - b)^2 = r^2\) represents a circle centre \((a, b)\) and radius \(r\).

Scalar Product:  \[a \cdot b = |a||b| \cos \theta, \text{ where } \theta \text{ is the angle between } a \text{ and } b\]
or  \[a \cdot b = a_1b_1 + a_2b_2 + a_3b_3 \text{ where } a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.

Trigonometric formulae:
\[
\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B \\
\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B \\
\sin 2A = 2 \sin A \cos A \\
\cos 2A = \cos^2 A - \sin^2 A \\
= 2 \cos^2 A - 1 \\
= 1 - 2 \sin^2 A
\]

Table of standard derivatives:
\[
\begin{array}{|c|c|}
\hline
f(x) & f'(x) \\
\hline
\sin ax & a \cos ax \\
\cos ax & -a \sin ax \\
\hline
\end{array}
\]

Table of standard integrals:
\[
\begin{array}{|c|c|}
\hline
f(x) & \int f(x)dx \\
\hline
\sin ax & \frac{-1}{a} \cos ax + c \\
\cos ax & \frac{1}{a} \sin ax + c \\
\hline
\end{array}
\]
(a) Show that \((x + 3)\) is a factor of \(3x^4 + 10x^3 + x^2 - 8x - 6\).

(b) Hence, or otherwise, factorise \(3x^4 + 10x^3 + x^2 - 8x - 6\) fully.

The curve with equation \(y = x^3 - 3x^2 + 2x + 5\) is shown on the diagram.

(a) Write down the coordinates of \(P\), the point where the curve crosses the \(y\)-axis.

(b) Determine the equation of the tangent to the curve at \(P\).

(c) Find the coordinates of \(Q\), the point where this tangent meets the curve again.

(a) (i) Show that \((x - 2)\) is a factor of \(2x^3 - 3x^2 - 3x + 2\).

(ii) Hence, factorise \(2x^3 - 3x^2 - 3x + 2\) fully.

A cubic function, \(f\), is defined on the set of real numbers.

- \((x + 4)\) is a factor of \(f(x)\)
- \(x = 2\) is a repeated root of \(f(x)\)
- \(f'(2) = 0\)
- \(f'(x) > 0\) where the graph with equation \(y = f(x)\) crosses the \(y\)-axis

Sketch a possible graph of \(y = f(x)\) on the diagram in your answer booklet.
(a) Show that \((x - 1)\) is a factor of \(f(x) = 2x^3 - 5x^2 + x + 2\).

(b) Hence, or otherwise, solve \(f(x) = 0\).

The diagram below shows the graph with equation \(y = f(x)\), where
\(f(x) = k(x-a)(x-b)^3\).

(a) Find the values of \(a\), \(b\) and \(k\).

(a) (i) Show that \((x+1)\) is a factor of \(2x^3 - 9x^2 + 3x + 14\).

(ii) Hence solve the equation \(2x^3 - 9x^2 + 3x + 14 = 0\).

Show that \((x + 3)\) is a factor of \(x^3 - 3x^2 - 10x + 24\) and hence factorise \(x^3 - 3x^2 - 10x + 24\) fully.
<table>
<thead>
<tr>
<th>Year</th>
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| 2014 | P1 Q22 | For the polynomial $6x^3 + 7x^2 + ax + b$,  
|      |       | • $x + 1$ is a factor  
|      |       | • $72$ is the remainder when it is divided by $x - 2$.  
|      |       | (a) Determine the values of $a$ and $b$.  
|      |       | (b) Hence factorise the polynomial completely.  |
| 2013 | P2 Q3a | Given that $(x - 1)$ is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic fully.  |
| 2012 | P1 Q21 | (a) (i) Show that $(x - 4)$ is a factor of $x^3 - 5x^2 + 2x + 8$.  
|      |       | (ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.  
|      |       | (iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$.  |
| 2011 | P2 Q3c | (c) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$.  
|      |       | (ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully.  |
| 2010 | P1 Q22 | (a) (i) Show that $(x - 1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$.  
|      |       | (ii) Hence factorise $f(x)$ fully.  
|      |       | (b) Solve $2x^3 + x^2 - 8x + 5 = 0$.  
|      |       | (c) The line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point $G$.  
|      |       | Find the coordinates of $G$.  
|      |       | (d) This tangent meets the curve again at the point $H$.  
|      |       | Write down the coordinates of $H$.  |
| 2009 | P2 Q3  | (a) (i) Show that $x = 1$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$.  
|      |       | (ii) Hence factorise $x^3 + 8x^2 + 11x - 20$ fully.  
|      |       | (b) Solve $\log_2(x + 3) + \log_2(x^2 + 5x - 4) = 3$.  |
21. A function \( f \) is defined on the set of real numbers by \( f(x) = x^3 - 3x + 2 \).

(a) Find the coordinates of the stationary points on the curve \( y = f(x) \) and determine their nature.

(b) (i) Show that \( (x - 1) \) is a factor of \( x^3 - 3x + 2 \).

(ii) Hence or otherwise factorise \( x^3 - 3x + 2 \) fully.

(c) State the coordinates of the points where the curve with equation \( y = f(x) \) meets both the axes and hence sketch the curve.

22. The diagram shows a sketch of the curve with equation \( y = x^3 - 6x^2 + 8x \).

(a) Find the coordinates of the points on the curve where the gradient of the tangent is \(-1\).

(b) The line \( y = 4 - x \) is a tangent to this curve at a point \( A \). Find the coordinates of \( A \).

8. The diagram shows a sketch of the graph of \( y = x^3 - 4x^2 + x + 6 \).

(a) Show that the graph cuts the \( x \)-axis at \((3, 0)\).

(b) Hence or otherwise find the coordinates of \( A \).

10. The diagram shows the graphs of a cubic function \( y = f(x) \) and its derived function \( y = f'(x) \).

Both graphs pass through the point \((0, 6)\).

The graph of \( y = f'(x) \) also passes through the points \((2, 0)\) and \((4, 0)\).

(a) Given that \( f'(x) \) is of the form \( k(x - a)(x - b) \):

(i) write down the values of \( a \) and \( b \);

(ii) find the value of \( k \).
8. A function $f$ is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where $x$ is a real number.
   
   (a) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully.
   
   (b) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the $x$- and $y$-axes.
   
   (c) Find the greatest and least values of $f$ in the interval $-2 \leq x \leq 2$.

11. (a) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$.
   
   (b) Hence find the range of values of $p$ for which all the roots of the cubic equation are real.

2. $f(x) = x^3 - x^2 - 5x - 3$.
   
   (a) (i) Show that $(x + 1)$ is a factor of $f(x)$.
       
   (ii) Hence or otherwise factorise $f(x)$ fully.
   
   (b) One of the turning points of the graph of $y = f(x)$ lies on the $x$-axis.
       Write down the coordinates of this turning point.

1. $f(x) = 6x^3 - 5x^2 - 17x + 6$.
   
   (a) Show that $(x - 2)$ is a factor of $f(x)$.
   
   (b) Express $f(x)$ in its fully factorised form.

5. Given that $(x - 2)$ and $(x + 3)$ are factors of $f(x)$ where $f(x) = 3x^3 + 2x^2 + cx + d$,
   find the values of $c$ and $d$.

1. (a) Given that $x + 2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of $k$.
   
   (b) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when $k$ takes this value.
<table>
<thead>
<tr>
<th>Specimen</th>
<th>1. Show that $x = 2$ is a root of the equation $y = 2x^3 + x^2 - 13x + 6 = 0$ and hence, or otherwise, find the other roots.</th>
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<tbody>
<tr>
<td>Specimen</td>
<td>3. (a) Show that $(x - 1)$ is a factor of $f(x) = x^3 - 6x^2 + 9x - 4$ and find the other factors.</td>
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<td>(b) Write down the coordinates of the points at which the graph of $y = f(x)$ meets the axes.</td>
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