2006 Mathematics

Higher – Paper 1

Finalised Marking Instructions

© The Scottish Qualifications Authority 2006

The information in this publication may be reproduced to support SQA qualifications only on a non-commercial basis. If it is to be used for any other purposes written permission must be obtained from the Assessment Materials Team, Dalkeith.

Where the publication includes materials from sources other than SQA (secondary copyright), this material should only be reproduced for the purposes of examination or assessment. If it needs to be reproduced for any other purpose it is the centre's responsibility to obtain the necessary copyright clearance. SQA's Assessment Materials Team at Dalkeith may be able to direct you to the secondary sources.

These Marking Instructions have been prepared by Examination Teams for use by SQA Appointed Markers when marking External Course Assessments. This publication must not be reproduced for commercial or trade purposes.
1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.

2. Award one mark for each ‘bullet’ point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.

3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made.
   This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (√). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (X or X√). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line.
   Work which is correct but inadequate to score any marks should be corrected with a double cross tick (XX).

5. • The total mark for each section of a question should be entered in red in the outer right hand margin, opposite the end of the working concerned.
   • Only the mark should be written, not a fraction of the possible marks.
   • These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked. Where a candidate has scored zero marks for any question attempted, “0” should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

8. Do not penalise:
   • working subsequent to a correct answer
   • legitimate variations in numerical answers
   • correct working in the “wrong” part of a question
   • omission of units
   • bad form
9. No piece of work should be scored through without careful checking - even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme - answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referral to the P.A. Please see the general instructions for P.A. referrals.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13. Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pr mark.

14. Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining the appropriate ic mark or pr mark.

15. Do not write any comments on the scripts. A revised summary of acceptable notation is given on page 4.

16. Working that has been crossed out by the candidate cannot receive any credit. If you feel that a candidate has been disadvantaged by this action, make a P.A. Referral.

17. Throughout this paper, unless specifically mentioned, a correct answer with no working receives no credit.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1. Tick correct working.
2. Put a mark in the outer right-hand margin to match the marks allocations on the question paper.
3. Do not write marks as fractions.
4. Put each mark at the end of the candidate's response to the question.
5. Follow through errors to see if candidates can score marks subsequent to the error.
6. Do not write any comments on the scripts.
Remember - No comments on the scripts. Please use the following and nothing else.

**Signs**

- **✓** The tick. You are not expected to tick every line but of course you must check through the whole of a response.

- **×** The cross and underline. Underline an error and place a cross at the end of the line.

- **✗** The tick-cross. Use this to show correct work where you are following through subsequent to an error.

- **∧** The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.

- **~~~** The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).

- **✘** The double cross-tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.

Bullets showing where marks are being allotted may be shown on scripts

<table>
<thead>
<tr>
<th>x² - 3x = 28</th>
<th>✓ •</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>x = 7</td>
<td>∧</td>
<td>1</td>
</tr>
</tbody>
</table>

**Note:** There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. These are all mistakes and as a consequence a mark is lost.

Page 5 lists the syllabus coding for each topic. This information is given in the legend underneath the question. The calculator classification is CN(calculator neutral), CR(calculator required) and NC(non-calculator).
<table>
<thead>
<tr>
<th></th>
<th>UNIT 1</th>
<th></th>
<th>UNIT 2</th>
<th></th>
<th>UNIT 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>determine range/domain</td>
<td>A15</td>
<td>use the general equation of a parabola</td>
<td>A18</td>
<td>use the laws of logs to simplify/find equiv. expression</td>
</tr>
<tr>
<td>A2</td>
<td>recognise general features of graphs: poly, exp, log</td>
<td>A16</td>
<td>solve a quadratic inequality</td>
<td>A20</td>
<td>sketch associated graphs</td>
</tr>
<tr>
<td>A3</td>
<td>sketch and annotate related functions</td>
<td>A17</td>
<td>find nature of roots of a quadratic</td>
<td>A30</td>
<td>solve equs of the form $A = Be^{at}$ for $A,B,k$ or $t$</td>
</tr>
<tr>
<td>A4</td>
<td>obtain a formula for composite functions</td>
<td>A18</td>
<td>given nature of roots, find a condition on coeffs</td>
<td>A31</td>
<td>solve equs of the form $\log(a) = c$ for $a,b,c$</td>
</tr>
<tr>
<td>A5</td>
<td>complete the square</td>
<td>A19</td>
<td>form an equation with given roots</td>
<td>A32</td>
<td>solve equations involving logarithms</td>
</tr>
<tr>
<td>A6</td>
<td>interpret equations and expressions</td>
<td>A20</td>
<td>apply A15-A19 to solve problems</td>
<td>A33</td>
<td>use relationships of the form $y = ax^2$ or $y = ab^x$</td>
</tr>
<tr>
<td>A7</td>
<td>determine function(poly,exp,log) from graph &amp; vv</td>
<td></td>
<td></td>
<td>A34</td>
<td>apply A28-A33 to problems</td>
</tr>
<tr>
<td>A8</td>
<td>sketch/annotate graph given critical features</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A9</td>
<td>interpret loci such as st. lines, para, poly, circle</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A10</td>
<td>use the notation $u_n$ for the $n$th term</td>
<td>A21</td>
<td>use Rem Th. For values, factors, roots</td>
<td>G16</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A11</td>
<td>evaluate successive terms of a RR</td>
<td>A22</td>
<td>solve cubic and quartic equations</td>
<td>G17</td>
<td>calculate the 3rd given two from A,B and vector AB</td>
</tr>
<tr>
<td>A12</td>
<td>decide when RR has limit/interpret limit</td>
<td>A23</td>
<td>find intersection of line and polynomial</td>
<td>G18</td>
<td>use unit vectors</td>
</tr>
<tr>
<td>A13</td>
<td>evaluate limit</td>
<td>A24</td>
<td>find if line is tangent to polynomial</td>
<td>G19</td>
<td>use: if $u, v$ are parallel then $v = ku$</td>
</tr>
<tr>
<td>A14</td>
<td>apply A10-A14 to problems</td>
<td>A25</td>
<td>find intersection of two polynomials</td>
<td>G20</td>
<td>add, subtract, find scalar mult. of vectors</td>
</tr>
<tr>
<td>A15</td>
<td>use C10 for problems</td>
<td>A26</td>
<td>confirm and improve on approx roots</td>
<td>G21</td>
<td>simplify vector pathways</td>
</tr>
<tr>
<td>A16</td>
<td>use A11 for problems</td>
<td>A27</td>
<td>apply A21-A36 to problems</td>
<td>G22</td>
<td>interpret 2D sketches of 3D situations</td>
</tr>
<tr>
<td>A17</td>
<td>use G1 to problems</td>
<td>A17</td>
<td></td>
<td>G23</td>
<td>find if 2 points in space are collinear</td>
</tr>
<tr>
<td>A18</td>
<td>use G2 to problems</td>
<td>G9</td>
<td>find C/R of a circle from its equation/other data</td>
<td>G24</td>
<td>find ratio which one point divides two others</td>
</tr>
<tr>
<td>A19</td>
<td>use G3 to problems</td>
<td>G10</td>
<td>find the equation of a circle</td>
<td>G25</td>
<td>given a ratio, find/interpret 3rd point/vector</td>
</tr>
<tr>
<td>A20</td>
<td>use G4 to problems</td>
<td>G11</td>
<td>find equation of a tangent to a circle</td>
<td>G26</td>
<td>calculate the scalar product</td>
</tr>
<tr>
<td>A21</td>
<td>use G5 to problems</td>
<td>G12</td>
<td>find intersection of line &amp; circle</td>
<td>G27</td>
<td>use: if $u, v$ are perpendicular then $v, u = 0$</td>
</tr>
<tr>
<td>A22</td>
<td>use G6 to problems</td>
<td>G13</td>
<td>find if/when line is tangent to circle</td>
<td>G28</td>
<td>calculate the angle between two vectors</td>
</tr>
<tr>
<td>A23</td>
<td>use G7 to problems</td>
<td>G14</td>
<td>find if two circles touch</td>
<td>G29</td>
<td>use the distributive law</td>
</tr>
<tr>
<td>A24</td>
<td>use G8 to problems</td>
<td>G15</td>
<td>apply G9-G14 to problems</td>
<td>G30</td>
<td>apply G16-G29 to problems</td>
</tr>
<tr>
<td>C1</td>
<td>differentiate sums, differences</td>
<td>C12</td>
<td>find integrals of $px^n$ and sums/diffs</td>
<td>C20</td>
<td>differentiate $\sin(ax+b), \cos(ax+b)$</td>
</tr>
<tr>
<td>C2</td>
<td>differentiate negative $&amp;$ fractional powers</td>
<td>C13</td>
<td>integrate with negative $&amp;$ fractional powers</td>
<td>C21</td>
<td>differentiate using the chain rule</td>
</tr>
<tr>
<td>C3</td>
<td>express in differentiable form and differentiate</td>
<td>C14</td>
<td>express in integrable form and integrate</td>
<td>C22</td>
<td>integrate $(ax + b)^n$</td>
</tr>
<tr>
<td>C4</td>
<td>find gradient at point on curve &amp; vv</td>
<td>C15</td>
<td>evaluate definite integrals</td>
<td>C23</td>
<td>integrate $\sin(ax+b), \cos(ax+b)$</td>
</tr>
<tr>
<td>C5</td>
<td>find equation of tangent to a polynomial/trig curve</td>
<td>C16</td>
<td>find area between curve and $x$-axis</td>
<td>C24</td>
<td>apply C20-C23 to problems</td>
</tr>
<tr>
<td>C6</td>
<td>find rate of change</td>
<td>C17</td>
<td>find area between two curves</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C7</td>
<td>find when curve strictly increasing etc</td>
<td>C18</td>
<td>solve differential equations(variables separable)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C8</td>
<td>find stationary points/values</td>
<td>C19</td>
<td>apply C12-C18 to problems</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C9</td>
<td>determine nature of stationary points</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C10</td>
<td>sketch curves given the equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C11</td>
<td>apply C1-C10 to problems to optimise, greatest/least</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T1</td>
<td>use general features of graphs of $f(x) = \sin(ax+b), \cos(ax+b)$</td>
<td>T7</td>
<td>solve linear $&amp;$ quadratic equations in radians</td>
<td>T12</td>
<td>solve sin,cos, equs of form $\cos(a)=p, \sin(a)=q$</td>
</tr>
<tr>
<td>T2</td>
<td>use radians inc conversion from degrees $&amp;$ vv</td>
<td>T8</td>
<td>apply compound and double angle $&amp;$ da formulae</td>
<td>T13</td>
<td>express $\cos(x) + \sin(x)$ in form $\cos x + \sin x$ etc</td>
</tr>
<tr>
<td>T3</td>
<td>know and use exact values</td>
<td>T9</td>
<td></td>
<td>T14</td>
<td>find max/min/zeros of $\cos(x) + \sin(x)$</td>
</tr>
<tr>
<td>T4</td>
<td>recognise form of trig. function from graph</td>
<td>T10</td>
<td>apply $c &amp;$ da formula when solving equations</td>
<td>T15</td>
<td>sketch graph of $y = \cos(x) + \sin(x)$</td>
</tr>
<tr>
<td>T5</td>
<td>interpret trig. equations and expressions</td>
<td>T11</td>
<td>apply T7-T10 to problems</td>
<td>T16</td>
<td>solve equ of the form $y = \cos(x) + \sin(x)$</td>
</tr>
<tr>
<td>T6</td>
<td>apply T2-T5 to problems</td>
<td></td>
<td></td>
<td>T17</td>
<td>apply T12-T16 to problems</td>
</tr>
</tbody>
</table>
1. Triangle ABC has vertices A(–1,12), B(–2, –5) and C(7, –2).
   (a) Find the equation of the median BD.
   (b) Find the equation of the altitude AE.
   (c) Find the coordinates of the point of intersection of BD and AE.

Notes
1. For candidates who find two medians (1,2,3) and (4,7,8) are available.
2. For candidates who find two altitudes (1,3,4) and (5,6,7) are available.
3. For candidates who find (a) altitude and (b) median see common error box number 3.
4. In (a) note that (4, 7) happens to lie on the median but does not qualify as a point to be used in •3.

Common Error 1
Finding two medians

\begin{align*}
1 & : D = (3,5) \\
2 & : m_{BD} = 2 \\
3 & : y - 5 = 2(x - 3) \quad \text{or} \quad y + 5 = 2(x - (-2)) \\
4 & : X \\
5 & : X \\
6 & : X \\
7 & : y = 2x - 1 \quad \& \quad 31x + 7y = 53 \\
8 & : x = \frac{4}{3} \\
9 & : y = \frac{5}{3} \\
\end{align*}

maximum of 6 marks

Common Error 2
Finding two altitudes

\begin{align*}
1 & : X \\
2 & : X \\
3 & : X \\
4 & : m_{BC} = \frac{1}{3} \\
5 & : m_{at} = -3 \\
6 & : y - 12 = -3(x - (-1)) \\
7 & : 4x - 7y = 27 \quad \& \quad y = -3x + 9 \\
8 & : x = \frac{18}{5} \\
9 & : y = -\frac{9}{5} \\
\end{align*}

maximum of 6 marks

Common Error 3
Finding (a) altitude and (b) median

\begin{align*}
1 & : m_{AC} = -\frac{7}{4} \\
X & : m_{BD} = \frac{4}{7} \\
2 & : y - 5 = \frac{1}{7}(x - (-2)) \\
X & : \text{midpt of } BC = \left(\frac{5}{2}, \frac{-1}{2}\right) \\
3 & : m_{AC} = \frac{-11}{7} \\
X & : y - 12 = \frac{-11}{7}(x - (-1)) \\
4 & : 4x - 7y = 27 \quad \& \quad 31x + 7y = 53 \\
X & : x = \frac{16}{7} \\
5 & : y = -\frac{125}{49} \\
X & : y = -\frac{125}{49} \\
\end{align*}

maximum of 5 marks
A circle has centre $C(-2, 3)$ and passes through $P(1, 6)$.

(a) Find the equation of the circle.
(b) PQ is a diameter of the circle. Find the equation of the tangent to this circle at Q.

**Notes**

1. In (a) $(\sqrt{18})^2$ is not acceptable for $r^2$.
2. In (b) if the coordinates of Q are estimated (i.e. guessed) then $\cdot^3$ can only be awarded if the coordinates are of the form $(a, 0)$ where $a < -2$.
3. In (b), $\cdot^6$ is only available if an attempt has been made to find a perpendicular gradient.

**General Notes applicable throughout the marking scheme**

There are many instances when follow throughs come into play and these will not always be highlighted for you. The following example is a reminder of what you have to look out for when you are marking.

**example**

At the $\cdot^3$ stage a candidate start with the wrong coordinates for Q.

Then

\[
\begin{align*}
X & \quad \cdot^3 \quad Q = (-4, 0) \\
X\sqrt{\cdot^4} & \quad m_{\text{diameter}} = \frac{6}{5} \\
X\sqrt{\cdot^5} & \quad m_{\text{tangent}} = -\frac{5}{6} \\
X\sqrt{\cdot^6} & \quad y - 0 = \frac{5}{6}(x + 4) \\
\end{align*}
\]

so the candidate loses $\cdot^3$ but gains $\cdot^4$, $\cdot^5$ and $\cdot^6$ as a consequence of following through.

Any error can be followed through and the subsequent marks awarded provided the working has not been eased.

Any deviation from this will be noted in the marking scheme.
Two functions $f$ and $g$ are defined on the set of real numbers by

$$f(x) = 2x + 3 \quad \text{and} \quad g(x) = 2x - 3.$$  

(a) Find an expressions for (i) $f\left(g(x)\right)$ (ii) $g\left(f(x)\right)$.

(b) Determine the least possible value of $f\left(g(x)\right) \times g\left(f(x)\right)$.

<table>
<thead>
<tr>
<th>Qu. part</th>
<th>marks</th>
<th>Grade</th>
<th>Syllabus Code</th>
<th>Calculator class</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>3a</td>
<td>3</td>
<td>C</td>
<td>A4</td>
<td>CN</td>
<td>06/07</td>
</tr>
<tr>
<td>3b</td>
<td>2</td>
<td>C</td>
<td>A6</td>
<td>CN</td>
<td></td>
</tr>
</tbody>
</table>

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

Primary Method : Give 1 mark for each ∙

- $f\left(g(x)\right) = f(2x - 3)$ stated or implied by ∙2
- $2(2x - 3) + 3$
- $g\left(f(x)\right) = 2(2x + 3) - 3$ stated explicitly
- $16x^2 - 9$
- $\min \text{value} = -9$

Alternative Marking 1 [Marks 1-3]

- $g\left(f(x)\right) = g(2x + 3)$
- $2(2x + 3) - 3$
- $f\left(g(x)\right) = 2(2x - 3) + 3$

Common Error No.1 for (a) “$g$ and $f$” transposed.

$$X$$

- $f\left(g(x)\right) = f(2x + 3)$
- $2(2x + 3) - 3$
- $g\left(f(x)\right) = 2(2x - 3) + 3$

Award 2 out of 3

Common Error No.2 for (a)

$$X$$

- $f\left(g(x)\right) = f(2x + 3)$
- $2(2x + 3) - 3$
- $g\left(f(x)\right) = 2(2x - 3) - 3$

Award 2 out of 3

Common Error No.3 for (a) Repeated error

$$X$$

- $f\left(g(x)\right) = f(2x - 3)$
- $2(2x + 3) - 3$
- $g\left(f(x)\right) = 2(2x - 3) + 3$

Award 2 out of 3

Notes

1. In (a) 2 marks are available for finding one of $f\left(g(x)\right)$ or $g\left(f(x)\right)$ and the third mark is for the other one.

2. In (a) the finding of $f\left(f(x)\right)$ and $g\left(g(x)\right)$ earns no marks.

3. $\cdot^5$ is only available if $\cdot^4$ has been awarded.

4. In (b) for $\cdot^5$, no justification is necessary. Ignore any comments, rational or irrational.
A sequence is defined by the recurrence relation \( u_{n+1} = 0.8u_n + 12, \quad u_0 = 4 \).

(a) State why the recurrence relation has a limit.

(b) Find this limit.

### Notes

1. **For (a)**
   - Accept
     
     \[
     0.8 < 1
     \]
   - Do NOT accept
     
     \[
     -1 < 0.8 < 1
     \]
   - 0.8 lies between \(-1\) and \(1\)

2. **Do NOT accept**

   \[
   -1 < a < 1 \quad \text{unless } a \text{ is clearly identified/replaced by } 0.8 \text{ anywhere in the answer.}
   \]

3. **In (b)**

   \[
   L = \frac{b}{1 - a} \quad \text{and nothing else gains NO marks.}
   \]

   4. \[L = \frac{12}{0.2} \quad \text{or} \quad \frac{120}{2} \quad \text{or} \quad \frac{60}{1} \quad \text{etc does NOT gain \(x^3\).} \]

   5. An answer of 60 without any working gains NO marks.

6. Any calculations based on “wrong” formulae gain NO marks.

### Primary Method: Give 1 mark for each •

1. sequence has limit since \(-1 < 0.8 < 1\)  
2. \(L = 0.8L + 12\)  
3. limit = 60

### Alternative Method for (b)

1. \(L = \frac{12}{1 - 0.8}\)  
2. limit = 60

### Bad Form

1. \(L = \frac{12}{0.2}\)  
2. limit = 60

award 2 marks

### Common Error 1

1. \[X \quad *2 \quad L = \frac{4}{1 - 0.8}\]
2. \(X \sqrt{ } \quad *3 \quad \text{limit = 20} \]
A function \( f \) is defined by \( f(x) = (2x - 1)^3 \). Find the coordinates of the stationary point on the graph with equation \( y = f(x) \) and determine its nature.

### Marking Scheme

<table>
<thead>
<tr>
<th>Qu.</th>
<th>part</th>
<th>marks</th>
<th>Grade</th>
<th>Syllabus Code</th>
<th>Calculator class</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>6</td>
<td>C</td>
<td>C8, C9</td>
<td>NC</td>
<td>NC</td>
<td>06/76</td>
</tr>
</tbody>
</table>

The primary method is based on the following generic m/s. This generic m/s may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

**Primary Method:** Give 1 mark for each •

- •1 ss know to start to differentiate
- •2 pr differentiate
- •3 ss set derivative = 0
- •4 pr solve
- •5 pr evaluate
- •6 ic justification
- •7 ic state conclusion

#### Common Error No.1

1. The “= 0” shown at •3 must appear at least once somewhere in the working between •1 and •4 (but not necessarily at •3).
2. •4 is only available as a consequence of solving \( f'(x) = 0 \).
3. A wrong derivative which eases the working will preclude at least •4 from being awarded.
4. For marks •5, •6 and •7, a nature table is mandatory. The minimum amount of detail that is required is shown here:

\[
\begin{array}{c|ccc}
\quad & \frac{1}{2} & \frac{1}{2} & > \frac{1}{2} \\
\hline
f'(x) & + & 0 & + \\
\end{array}
\]

Candidates who use only \( f''(x) = 0 \) and try to draw conclusions from this cannot gain •5 or •7. \( f''(x) = 0 \) is a necessary but not sufficient condition for identifying points of inflexion.

5. •7 is **ONLY** available subsequent to a correct nature table for the candidate’s own derivative.

6. •4 is lost in each of the following cases for the candidate's solution to the equation at •3.
   (i) \( x = \frac{1}{2} \) and \( x = \text{something else} \)
   (ii) two wrong values for \( x \)
   (iii) guess a value for \( x \)

Only one value for \( x \) needs to be followed through for •5, •6 and •7.

#### Common Error No.2

\[
\begin{align*}
\sqrt{} & & f'(x) = \ldots \quad \text{(skipped)} \\
X & & 5(2x - 1)^4 \times 2 \\
\sqrt{} & & f'(x) = 0 \\
X \sqrt{} & & x = \frac{1}{2} \\
\end{align*}
\]

\[ f'(\frac{1}{2}) = 0 \]

•5, •6 and •7 are still available
The graph shown has equation \( y = x^3 - 6x^2 + 4x + 1 \).
The shaded area is bounded by the curve, the \( x \)-axis, the \( y \)-axis and the line \( x = 2 \).

(a) Calculate the shaded area labelled \( S \).
(b) Hence find the total shaded area.

<table>
<thead>
<tr>
<th>Qu. part</th>
<th>marks</th>
<th>Grade</th>
<th>Syllabus Code</th>
<th>Calculator class</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>6a</td>
<td>4C</td>
<td>C16</td>
<td>NC</td>
<td>06/40</td>
<td></td>
</tr>
<tr>
<td>b3</td>
<td>B</td>
<td>C16</td>
<td>NC</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The primary method m/s is based on the following generic m/s.
THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

Primary Method : Give 1 mark for each •

- 1. \( \int_{0}^{1} (x^3 - 6x^2 + 4x + 1) \, dx \) stated or implied by \( \sqrt{\cdot} \)
- 2. \( \frac{1}{4}x^4 - \frac{6}{3}x^3 + \frac{4}{2}x^2 + x \)
- 3. \( \left( \frac{1}{4}\cdot 1^4 - 2.1^3 + 2.1^2 + 1 \right) - 0 \)
- 4. \( \frac{5}{4} \) or equivalent
- 5. \( \int_{1}^{2} \ldots dx \)
- 6. \( \left( \frac{1}{4}\cdot 2^4 - 2.2^3 + 2.2^2 + 2 \right) - \left( \frac{1}{4}\cdot 1^4 - 2.1^3 + 2.1^2 + 1 \right) = -\frac{13}{4} \)
- 7. \( \frac{9}{2} \) or equivalent

Alternative Method 1 for (b)

- 5. \( \int_{1}^{2} \ldots dx \)
- 6. \( \left( \frac{1}{4}\cdot 2^4 - 2.2^3 + 2.2^2 + 2 \right) - \left( \frac{1}{4}\cdot 1^4 - 2.1^3 + 2.1^2 + 1 \right) \)
- 7. \( \frac{9}{2} \)

Alternative Method 2 for (b)

- 5. \( \int_{1}^{2} \ldots dx \)
- 6. \( \left( \frac{1}{4}\cdot 2^4 - 2.2^3 + 2.2^2 + 2 \right) + \left( \frac{1}{4}\cdot 1^4 - 2.1^3 + 2.1^2 + 1 \right) \)
- 7. \( \frac{9}{2} \)

Alternative Method 3 for (b)

- 5. \( \int_{1}^{2} \ldots dx \)
- 6. \( \left( \frac{1}{4}\cdot 2^4 - 2.2^3 + 2.2^2 + 2 \right) - \left( \frac{1}{4}\cdot 1^4 - 2.1^3 + 2.1^2 + 1 \right) \)
- 7. \( \frac{9}{2} \)

Common Error No.1

\[
\begin{align*}
\sqrt{1} & \quad \int_{0}^{1} (x^3 - 6x^2 + 4x + 1) \, dx \\
X & \quad 3x^2 - 12x + 4 \\
X & \quad 3(1)^2 - 12.1 + 4 \\
X & \quad -9 \\
\sqrt{5} & \quad \int_{1}^{2} \ldots dx \text{ or equivalent} \\
X \sqrt{6} & \quad (3.2^2 - 12.2 + 4) - (3.1^2 - 12.1 + 4) = -3 \\
X \sqrt{7} & \quad 12
\end{align*}
\]
Solve the equation \( \sin x^\circ - \sin 2x^\circ = 0 \) in the interval \( 0 \leq x \leq 360^\circ \).

**Notes**

1. An \( \approx 0 \) must appear somewhere between the start and \( \approx 2 \) evidence.
2. The inclusion of extra answers which would have been correct with a larger interval should be treated as bad form and NOT penalised.
3. The omission of a correct answer (e.g. 0) means the candidates loses a mark (\( \approx 4 \) in the Primary Method).
4. Candidates may embark on a journey with the wrong formula for \( \sin(2x^\circ) \). With an equivalent level of difficulty it may still be worth a maximum of 3 marks. See Common Error No.1.
5. Candidates who draw a sketch of \( y = \sin(x^\circ) \) and \( y = \sin(2x^\circ) \) giving 0,180,360 may be awarded \( \approx 1 \) and \( \approx 3 \).

**Common Error No.1**

\[
X \quad \approx 1 \sin(x^\circ) - \left(1 - 2 \sin^2(x^\circ)\right) = 0 \\
2 \sin^2(x^\circ) + \sin(x^\circ) - 1 = 0 \\
X \sqrt{2} \left(2 \sin(x^\circ) - 1\right)\left(\sin(x^\circ) + 1\right) = 0 \\
X \sqrt{3} \sin(x^\circ) = \frac{1}{2} \text{ or } \sin(x^\circ) = -1 \\
X \sqrt{4} \quad x = 30,150, \quad x = 270 \\
\text{award 3 marks}
\]

**Common Error No.2**

\[
\sin(x^\circ) - \sin^2(x^\circ) = 0 \\
X \quad \approx 1 \sin(x^\circ)(1 - \sin(x^\circ)) = 0 \\
X \sqrt{2} \sin(x^\circ) = 0 \text{ or } \sin(x^\circ) = 1 \\
X \sqrt{3} \quad x = 0,180,360, \quad 90 \\
\text{award 2 marks}
\]

**Common Error No.3**

\[
sin(x^\circ) - \sin(2x^\circ) = 0 \\
sin(x^\circ) = 0, \quad \sin(2x^\circ) = 0 \\
etc.
\text{gains NO marks}
\]
8 (a) Express \(2x^2 + 4x - 3\) in the form \(a(x+b)^2 + c\).

(b) Write down the coordinates of the turning point on the parabola with equation
\[ y = 2x^2 + 4x - 3. \]

The primary method m/s is based on the following generic m/s.
THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

**Primary Method : Give 1 mark for each**

- **1** ss know how to complete (deal with the “\(a\)“)
- **2** pr process the value of “\(b\)”
- **3** pr process the value of “\(c\)”
- **4** ic interpret equation of parabola

**Note**

1. Alternative Method 1 should be used for assessing part marks/follow throughs.

2. For \(x^4\), no justification is required.
   Candidates may choose to differentiate etc. but may still earn only one mark for the correct answer.

3. For \(x^4\), accept \((-b, c)\).

**Alternative Method 1 for (a)**

- **1** \(2(x^2 + 2x)\)
- **2** \(2(x + 1)^2\)
- **3** \(2(x + 1)^2 - 5\)
- **4** \((-1, -5)\)

**Alternative Method 2 for (a) : Comparing coefficients**

- **1** \(2x^2 + 4x - 3 = ax^2 + 2abx + ab^2 + c\) \(\Rightarrow a = 2\)
- **2** \(2ab = 4\) \(\Rightarrow b = 1\)
- **3** \(ab^2 + c = -3\) \(\Rightarrow c = -5\)
- **4** \((-1, -5)\)
9. \( u \) and \( v \) are vectors given by \( u = \begin{pmatrix} k^3 \\ 1 \\ k+2 \end{pmatrix} \) and \( v = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix} \), where \( k > 0 \).

(a) If \( u \cdot v = 1 \) show that \( k^3 + 3k^2 - k - 3 = 0 \).

(b) Show that \( (k+3) \) is a factor of \( k^3 + 3k^2 - k - 3 \) and hence factorise \( k^3 + 3k^2 - k - 3 \) fully.

(c) Deduce the only possible value of \( k \).

(d) The angle between \( u \) and \( v \) is \( \theta \). Find the exact value of \( \cos \theta \).

**Notes**

1. No explanation is required for \( k \) but the chosen value must follow from the working for \( \cdot 6 \) or \( \cdot 7 \). **Do not accept \( \sqrt{1} \).**

2. In primary method (\( \cdot 4 \)) and alternative (\( \cdot 5 \)) candidates must show some acknowledgement of the resulting “zero”. Although a statement w.r.t. the zero is preferable, accept something as simple as “underlining” the zero.

3. Only numerical values are acceptable for \( \cdot 9, \cdot 10 \) and \( \cdot 11 \); answers are acceptable in unsimplified form eg \( \cos \theta = \frac{1}{\sqrt{11} \times \sqrt{11}} \)

**Alternative method 1 (marks 3–7) Long Division**

\[
\begin{array}{c|cccc}
& k^2 & -1 & \hline
k & 3 & k^3 + 3k^2 - k - 3 \\
\hline
& -k & -3 & \\
\hline
\end{array}
\]

- \( \cdot 3 \) \( k^3 + 3k^2 - k - 3 \)
- \( \cdot 4 \) \( \text{remainder is zero so } (k+3) \text{ is a factor} \)
- \( \cdot 5 \) \( k^2 - 1 \)
- \( \cdot 6 \) \( (k+3)(k+1)(k-1) \) **stated explicitly**

**Primary Method: Give 1 mark for each**

\[
\begin{array}{c}
1. u \cdot v = k^3 + 1 + \left(3k^2\right) + \left(k + 2\right) \cdot (-1)^2 \text{ stated or implied by } \cdot 2 \\
2. k^3 + 3k^2 - k - 2 = 1 \text{ and complete } \\
3. \text{know to use } k = -3 \\
4. -27 + 27 - (3) - 3 = 0 \Rightarrow x + 3 \text{ is a factor} \\
5. (k + 3)(k^2 ... ) \\
6. (k + 3)(k^2 - 1) \\
7. (k + 3)(k + 1)(k - 1) \text{ stated explicitly} \\
8. k = 1 \\
9. u = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}, v = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} \text{ stated or implied by } \cdot 10 \\
10. |u| = \sqrt{11} \text{ and } |v| = \sqrt{11} \text{ } \\
11. \cos \theta = \frac{1}{11} \\
\end{array}
\]

**Alternative method 2 (marks 3–7) Synthetic Division**

\[
\begin{array}{c|cccc}
& & -3 \\
-3 & 1 & 3 & -1 & -3 \\
\hline
& -3 & 0 & 3 \\
\hline
1 & 0 & -1 & 0 \\
\end{array}
\]

- \( \cdot 3 \) \( (k^2 - 1) \)
- \( \cdot 4 \) \( f(-3) = 0 \text{ so } (k+3) \text{ is a factor} \)
- \( \cdot 5 \) \( (k+3)^2 = 0 \text{ so } (k+3) \text{ is a factor} \)
- \( \cdot 6 \) \( (k^2 - 1) \)
- \( \cdot 7 \) \( (k + 3)(k + 1)(k - 1) \) **stated explicitly**
Two variables, \( x \) and \( y \), are connected by the law \( y = a^x \). A graph of \( \log_4(y) \) against \( x \) is a straight line passing through the origin and the point \( A(6,3) \). Find the value of \( a \).

**Primary Method:** Give 1 mark for each •

1. \( \log_4(y) = \log_4(a^x) \)
2. \( 3 = \log_4(a^6) \)
3. \( a^6 = 4^3 \)
4. \( a = 2 \)  

**Alternative Method 1**

1. \( \log_4(y) = \log_4(a^x) \)
2. \( 3 = 6 \log_4(a) \)
3. \( \log_4(a) = \frac{1}{2} \)
4. \( a = 2 \)

**Alternative Method 2**

1. \( \log_4(y) = mx + c \)
2. \( m = \frac{1}{2}, \ c = 0 \)
3. \( y = 4^{\frac{x}{2}} \)
4. \( y = \left(4^\frac{1}{2}\right)^x = 2^x \Rightarrow a = 2 \)

**Alternative Method 3**

1. At \( A \) \( \log_4(y) = 3 \)
2. \( y = 4^3 \)
3. \( a^6 = 4^3 \)
4. \( a = 2 \)

**Alternative Method 4**

1. \( \log_4(y) = \log_4(a^x) \)
2. \( \log_4(y) = x \log_4(a) \)
3. \( \log_4(a) = \frac{1}{2} \)
4. \( a = 4^{\frac{1}{2}} = 2 \)
2006 Mathematics

Higher – Paper 2

Finalised Marking Instructions

© The Scottish Qualifications Authority 2006

The information in this publication may be reproduced to support SQA qualifications only on a non-commercial basis. If it is to be used for any other purposes written permission must be obtained from the Assessment Materials Team, Dalkeith.

Where the publication includes materials from sources other than SQA (secondary copyright), this material should only be reproduced for the purposes of examination or assessment. If it needs to be reproduced for any other purpose it is the centre's responsibility to obtain the necessary copyright clearance. SQA's Assessment Materials Team at Dalkeith may be able to direct you to the secondary sources.

These Marking Instructions have been prepared by Examination Teams for use by SQA Appointed Markers when marking External Course Assessments. This publication must not be reproduced for commercial or trade purposes.
1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.

2. Award one mark for each ‘bullet’ point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.

3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made.
   This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (√). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (X or X√). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line.
   Work which is correct but inadequate to score any marks should be corrected with a double cross tick (XX).

5. • The total mark for each section of a question should be entered in red in the outer right hand margin, opposite the end of the working concerned.
   • Only the mark should be written, not a fraction of the possible marks.
   • These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks.
   Where appropriate, all summations for totals and grand totals must be carefully checked.
   Where a candidate has scored zero marks for any question attempted, “0” should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

8. Do not penalise:
   • working subsequent to a correct answer  • omission of units
   • legitimate variations in numerical answers  • bad form
   • correct working in the “wrong” part of a question
9. No piece of work should be scored through without careful checking - even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme - answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referral to the P.A. Please see the general instructions for P.A. referrals.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13 Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pr mark.

14 Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining the appropriate ic mark or pr mark.

15 **Do not write any comments on the scripts.** A revised summary of acceptable notation is given on page 4.

16 Working that has been crossed out by the candidate cannot receive any credit. If you feel that a candidate has been disadvantaged by this action, make a P.A. Referral.

17 Throughout this paper, unless specifically mentioned, a correct answer with no working receives no credit.

**Summary**

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1. **Tick** correct working.
2. Put a mark in the **outer right-hand margin** to match the marks allocations on the question paper.
3. Do **not** write marks as fractions.
4. Put each mark **at the end** of the candidate’s response to the question.
5. **Follow through** errors to see if candidates can score marks subsequent to the error.
6. Do **not** write any comments on the scripts.
Higher Mathematics: A Guide to Standard Signs and Abbreviations

Remember - No comments on the scripts. Please use the following and nothing else.

Signs

✔ The tick. You are not expected to tick every line but of course you must check through the whole of a response.

❌ The cross and underline. Underline an error and place a cross at the end of the line.

❌ The tick-cross. Use this to show correct work where you are following through subsequent to an error.

∧ The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.

~ The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).

❌ The double cross-tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.

Bullets showing where marks are being allotted may be shown on scripts

\[
\frac{du}{dx} = 4x - 7 \quad ✔ \quad \bullet
\]
\[
4x - 7 = 0 \quad ❌
\]
\[
x = \frac{7}{4}
\]
\[
y = 3 \frac{2}{4} \quad ❌ \quad ✔ \quad \bullet
\]
\[
C = (1, -1) \quad ❌
\]
\[
m = \frac{3 - (-1)}{4 - 1} \quad ❌
\]
\[
m_{rad} = \frac{1}{3} \quad ❌ \quad ✔ \quad \bullet
\]
\[
m_{grad} = \frac{1}{4} \quad ❌ \quad ✔ \quad \bullet
\]
\[
y - 3 = -\frac{2}{3}(x - 2) \quad ❌ \quad ✔ \quad \bullet
\]
\[
x^2 - 3x = 28 \quad ❌ \quad ✔ \quad \bullet
\]
\[
x = 7 \quad ∧ \quad ❌
\]
\[
\sin(x) = 0.75 = \sin(0.75) = 48.6^\circ \quad ✔ \quad \bullet
\]

Remember - No comments on the scripts. No abbreviations. No new signs. Please use the above and nothing else.

All of these are to help us be more consistent and accurate.

Note: There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. These are all mistakes and as a consequence a mark is lost.

Page 5 lists the syllabus coding for each topic. This information is given in the legend underneath the question. The calculator classification is CN (calculator neutral), CR (calculator required) and NC (non-calculator).
<table>
<thead>
<tr>
<th><strong>UNIT 1</strong></th>
<th><strong>UNIT 2</strong></th>
<th><strong>UNIT 3</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>A1 determine range/domain</td>
<td>A15 use the general equation of a parabola</td>
<td>A28 use the laws of logs to simplify/find equiv. expression</td>
</tr>
<tr>
<td>A2 recognise general features of graphs; poly., exp., log</td>
<td>A16 solve a quadratic inequality</td>
<td>A29 sketch associated graphs</td>
</tr>
<tr>
<td>A3 sketch and annotate related functions</td>
<td>A17 find nature of roots of a quadratic</td>
<td>A30 solve eqns of the form $A = Be^{kt}$ for $A, B, k$ or $t$</td>
</tr>
<tr>
<td>A4 obtain a formula for composite function</td>
<td>A18 given nature of roots, find a condition on coeffs</td>
<td>A31 solve eqns of the form $\log_a(a) = c$ for $a, b$ or $c$</td>
</tr>
<tr>
<td>A5 complete the square</td>
<td>A19 form an equation with given roots</td>
<td>A32 solve equations involving logarithms</td>
</tr>
<tr>
<td>A6 interpret equations and expressions</td>
<td>A20 apply $A_{15}-A_{19}$ to solve problems</td>
<td>A33 use relationships of the form $y = ax^2$ or $y = ab^2$</td>
</tr>
<tr>
<td>A7 determine function (poly., exp., log) from graph &amp; $\nu$</td>
<td>A21 use Rem. Th. For values, factors, roots</td>
<td>A34 apply $A_{28}-A_{33}$ to problems</td>
</tr>
<tr>
<td>A8 sketch/annotate graph given critical features</td>
<td>A22 solve cubic and quartic equations</td>
<td></td>
</tr>
<tr>
<td>A9 interpret loci such as st. lines, par., poly., circle</td>
<td>A23 find intersection of line and polynomial</td>
<td></td>
</tr>
<tr>
<td>A10 use the notation $u_n$ for the nth term</td>
<td>A24 find if line is tangent to polynomial</td>
<td></td>
</tr>
<tr>
<td>A11 evaluate successive terms of a RR</td>
<td>A25 find intersection of two polynomials</td>
<td></td>
</tr>
<tr>
<td>A12 decide when RR has limit/interpret limit</td>
<td>A26 confirm and improve on approx roots</td>
<td></td>
</tr>
<tr>
<td>A13 evaluate limit</td>
<td>A27 apply $A_{21}-A_{36}$ to problems</td>
<td></td>
</tr>
<tr>
<td>A14 apply $A_{10}-A_{14}$ to problems</td>
<td>G1 find the distance formula</td>
<td>G16 calculate the length of a vector</td>
</tr>
<tr>
<td>G2 find gradient from 2 pts., eqn., of line</td>
<td>G9 find C/R of a circle from its equation/other data</td>
<td>G17 calculate the 3rd given two from $A, B$ and vector $AB$</td>
</tr>
<tr>
<td>G3 find equation of a line</td>
<td>G10 find the equation of a circle</td>
<td>G18 use unit vectors</td>
</tr>
<tr>
<td>G4 interpret all equations of a line</td>
<td>G11 find equation of a tangent to a circle</td>
<td>G19 use: if $u$, $v$ are parallel then $v = ku$</td>
</tr>
<tr>
<td>G5 use property of perpendicular lines</td>
<td>G12 find intersection of line &amp; circle</td>
<td>G20 add, subtract, find scalar mult. of vectors</td>
</tr>
<tr>
<td>G6 calculate mid-point</td>
<td>G13 if/when line is tangent to circle</td>
<td>G21 simplify vector pathways</td>
</tr>
<tr>
<td>G7 find equation of median, altitude, perp. bisector</td>
<td>G14 find if two circles touch</td>
<td>G22 interpret 2D sketches of 3D situations</td>
</tr>
<tr>
<td>G8 apply $G_{1}-G_{7}$ to problems eg intersect., concur., collin.</td>
<td>G15 apply $G_{9}-G_{14}$ to problems</td>
<td>G23 find if 2 points in space are collinear</td>
</tr>
<tr>
<td>C1 differentiate sums, differences</td>
<td>C12 find integrals of $px^n$ and sums/diffs</td>
<td>G24 find ratio which one point divides two others</td>
</tr>
<tr>
<td>C2 differentiate negative $&amp;$ fractional powers</td>
<td>C13 integrate with negative $&amp;$ fractional powers</td>
<td>G25 given a ratio, find/interpret 3rd point/vector</td>
</tr>
<tr>
<td>C3 express in differentiable form and differentiate</td>
<td>C14 express in integrable form and integrate</td>
<td>G26 calculate the scalar product</td>
</tr>
<tr>
<td>C4 find gradient at point on curve $&amp;$ $\nu$</td>
<td>C15 evaluate definite integrals</td>
<td>G27 use: if $u$, $v$ are perpendicular then $u \cdot v = 0$</td>
</tr>
<tr>
<td>C5 find equation of tangent to a polynomial/trig curve</td>
<td>C16 find area between curve and $y$-axis</td>
<td>G28 calculate the angle between two vectors</td>
</tr>
<tr>
<td>C6 find rate of change</td>
<td>C17 find area between two curves</td>
<td>G29 use the distributive law</td>
</tr>
<tr>
<td>C7 find when curve strictly increasing etc</td>
<td>C18 solve differential equations(variables separable)</td>
<td>G30 apply $G_{16}-G_{29}$ to problems eg geometry provs.</td>
</tr>
<tr>
<td>C8 find stationary points/values</td>
<td>C19 apply $C_{12}-C_{18}$ to problems</td>
<td></td>
</tr>
<tr>
<td>C9 determine nature of stationary points</td>
<td>C10 sketch curve given the equation</td>
<td></td>
</tr>
<tr>
<td>C10 sketch curve given the equation</td>
<td>C11 apply $C_{1}-C_{10}$ to problems eg optimise, greatest/least</td>
<td></td>
</tr>
<tr>
<td>T1 use general features of graphs of $f(x) = \sin(ax+b)$, $f(x) = \cos(ax+b)$, identify period/amplitude</td>
<td>T7 solve linear $&amp;$ quadratic equations in radians</td>
<td>T12 solve sin., eqns of form $\cos(a) = p$, $\sin(a) = q$</td>
</tr>
<tr>
<td>T2 use radians inc conversion from degrees $&amp;$ $\nu$</td>
<td>T8 apply compound and double angle ($c &amp;$ du) formulae</td>
<td>T13 express $\cos(x) + \sin(x)$ in form $\cos(x \pm a) + \sin(x \pm b)$ etc</td>
</tr>
<tr>
<td>T3 know and use exact values</td>
<td>T9 apply $c &amp;$ du formulae in geometrical cases</td>
<td>T14 find max/min/zeros of $\cos(x) + \sin(x)$</td>
</tr>
<tr>
<td>T4 recognise form of trig. function from graph</td>
<td>T10 use $c &amp;$ du formulae when solving equations</td>
<td>T15 sketch graph of $y = \cos(x) + \sin(x)$</td>
</tr>
<tr>
<td>T5 interpret trig. equations and expressions</td>
<td>T11 apply $T_{7}-T_{10}$ to problems</td>
<td>T16 solve eqn of the form $y = \cos(\pi x) + \sin(\pi x)$</td>
</tr>
<tr>
<td>T6 apply $T_{2}-T_{5}$ to problems</td>
<td></td>
<td>T17 apply $T_{12}-T_{16}$ to problems</td>
</tr>
</tbody>
</table>
1 PQRS is a parallelogram. P is the point (2, 0), S is (4, 6) and Q lies on the $x$-axis, as shown.

The diagonal QS is perpendicular to the side PS.

(a) Show that the equation of QS is $x + 3y = 22$.

(b) Hence find the coordinates of Q and R.

---

### General Notes applicable throughout the marking scheme

There are many instances when follow throughs come into play and these will not always be highlighted for you. The following example is a reminder of what you have to look out for when you are marking.

**example**

At the $^5$ stage a candidate may switch the coordinates round so we have

- $^5$ $X \quad Q(0, 22)$
- $^6$ $X\sqrt{r} \quad R(2, 28)$ repeated error

so the candidate loses $^5$ for switching the coordinates but gains $^6$ as a consequence of following through.

Any error can be followed through and the subsequent marks awarded provided the working has not been eased.

Any deviation from this will be noted in the marking scheme.

---

Notes

1. In (a), $^3$ is only available if an attempt has been made to find and use a perpendicular gradient.

2. In the Primary method and the Alt. method 1, $^4$ is only available for reaching the required equation.

3. To gain $^4$, some evidence of completion needs to be shown.

   e.g. $y - 6 = -\frac{1}{3}(x - 4)$
   
   $3(y - 6) = -(x - 4)$
   
   $x + 3y = 22$

4. Sometimes candidates manage to find R first. Provided the coordinates of R are of the form $(?, 6)$, only then is $^6$ available as a follow through.

5. $^5$ and $^6$ are available to candidates who use their own erroneous equation for QS.

---

### Primary Method : Give 1 mark for each •

- $^1 m_{PS} = 3$
- $^2 m_{QS} = -\frac{1}{3}$
- $^3 y - 6 = -\frac{1}{3}(x - 4)$
- $^4$ completes proof
- $^5 Q = (22, 0)$
- $^6 R = (24, 6)$

### Alternative Method 1

- $^1 m_{PS} = 3$
- $^2 m_{QS} = -\frac{1}{3}$
- $y = -\frac{1}{3}x + c$
- $^3 6 = -\frac{1}{3}\times 4 + c$
- $^4$ completes proof
- $^5 Q = (22, 0)$
- $^6 R = (24, 6)$

### Alternative Method 2

Let $Q = (q, 0)$

- $^1 (q - 2)^2 = 2^2 + 6^2 + (q - 4)^2 + 6^2$
- $^2 q = 22$
- $^3 Q = (22, 0) \ and \ R = (24, 6)$
- $^4 m_{QS} = -\frac{1}{3}$
- $^5 y - 0 = -\frac{1}{3}(x - 22)$
- $^6 leading \ to \ 3y + x = 22$

N.B.

The coordinates of Q can also be arrived at by right-angled trig.

Use the alt. method 2 marking scheme with $^1$ replaced by appropriate trig. work.

The only acceptable value for q is 22.
2 Find the value of \( k \) such that the equation \( kx^2 + kx + 6 = 0 \), \( k \neq 0 \), has equal roots.

<table>
<thead>
<tr>
<th>Qu. part</th>
<th>marks</th>
<th>Grade</th>
<th>Syllabus Code</th>
<th>Calculator class</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>C</td>
<td>A18 CN</td>
<td>CN</td>
<td>06/new</td>
</tr>
</tbody>
</table>

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME.

- Primary Method: Give 1 mark for each •
  - 1 ss know to use “discriminant = 0”
  - 2 ic interpret \( a, b, c \)
  - 3 pr substitute & factorise
  - 4 ic interpret solution

Notes

1. The evidence for •1 and/or •2 may not appear until the working immediately preceding the evidence for •3, i.e. a candidate may simply start
   \[
   \sqrt{\cdot1} \cdot \sqrt{\cdot2} \quad k^2 - 4 \times k \times 6 = 0
   \]
   or
   \[
   \sqrt{\cdot2} \quad k^2 - 4 \times k \times 6
   \]
   \[
   \sqrt{\cdot3} \quad k(k - 24) = 0
   \]

2. The “= 0” has to appear at least once, at the •1 stage or at the •2 stage.

3. In the Primary method, candidates who do not deal with the root \( k = 0 \) cannot obtain •4. [see Common Errors 1 and 2]

4. Minimum evidence for •4 would be scoring out “\( k = 0 \)” or “\( k = 24 \)” underlined.

5. Some candidates may start with the quadratic formula. Apply the marking scheme to the part underneath the square root sign.

Alternative Method 1 (completing the square)

- 1 \( (x + \frac{1}{2})^2 + \ldots \)
- 2 \( (x + \frac{1}{2})^2 - \frac{6}{k} = 0 \)
- 3 equal roots \( \Rightarrow -\frac{1}{4} + \frac{6}{k} = 0 \)
- 4 \( k = 24 \)

Acceptable alternative for •4

- 1 \( "b^2 - 4ac" = 0 \)
- 2 \( a = k, b = k, c = 6 \)
- 3 \( k(k - 24) \)
- 4 \( k = 0 \) or 24

Common Error 1 at the •4 stage

- 1 \( "b^2 - 4ac" = 0 \)
- 2 \( a = k, b = k, c = 6 \)
- 3 \( k(k - 24) \)
- 4 \( k = 0 \) or 24

Common Error 2 at the •4 stage

- 1 \( "b^2 - 4ac" = 0 \)
- 2 \( a = k, b = k, c = 6 \)
- 3 \( k(k - 24) \)
- 4 \( k = 24 \)

Common Error 3 Division by \( k \)

- 1 \( "b^2 - 4ac" = 0 \)
- 2 \( a = k, b = k, c = 6 \)
- 3 \( k^2 - 24k = 0 \)
- 4 \( k^2 = 24k \)
- 5 \( k = 24 \)
3 The parabola with equation \( y = x^2 - 14x + 53 \) has a tangent at the point \( P(8,5) \).

(a) Find the equation of this tangent.

(b) Show that the tangent found in (a) is also a tangent to the parabola with equation \( y = -x^2 + 10x - 27 \) and find the coordinates of the point of contact \( Q \).

### Primary Method

- **Give 1 mark for each •**

1. \( \frac{dy}{dx} = \)
2. \( 2x - 14 \)
3. \( m = 2 \) stated or implied by •4
4. \( y - 5 = 2(x - 8) \)
5. \( y = 2x - 11 \)
6. \( 2x - 11 = -x^2 + 10x - 27 \)
7. \( x^2 - 8x + 16 = 0 \)
8. \( (x - 4)^2 = 0 \Rightarrow equal roots so tgt \)
9. \( Q = (4,-3) \)

### Alternative Marking 1 [Marks 8]

- \( b^2 - 4ac = 64 - 4 \times 16 = 0 \Rightarrow line \ is \ a \ tangent \)

### Alternative Method 1 for (b)

- \( 2x = y + 11 \)
- \( 4y = -(y^2 + 22y + 121) + 20y + 220 - 108 \)
- \( y^2 + 6x + 9 = 0 \)
- \( (y + 3)^2 = 0 \Rightarrow equal \ roots \ so \ tgt \)
- \( Q = (4,-3) \)

### Alternative Method 2 for (b)

- \( Find \ the \ equ. \ of \ the \ tgt \ to \ 2nd \ curve \ with \ grad. \ 2 \) stated or implied by •6
- \( -2x + 10 = 2 \)
- \( Q = (4,-3) \)
- \( y = (-3) = 2(x - 4) \)
- \( y = 2x - 11 \ which \ is \ the \ same \ equ. \ as \ (a) \) stated explicitly

### Notes

- **In (a)**
  1. •4 is only available if an attempt has been made to find the gradient from differentiation.
  2. •6 is only available for a numerical value of \( m \).
  3. An "= 0" must occur somewhere in the working between •7 and •8.
  4. •8 is awarded for drawing a conclusion from the candidate’s quadratic equation.
  5. Candidates may substitute the equation of the parabola into the equation of the line. This is a perfectly acceptable approach.
4 The circles with equations \((x - 3)^2 + (y - 4)^2 = 25\) and \(x^2 + y^2 - kx - 8y - 2k = 0\)
have the same centre. Determine the radius of the larger circle.

### Notes

1. \(*^2\) requires no justification.
2. Evidence for \(*^3\) may appear for the first time at the \(*^5\) stage.
3. If \(R_1 = 5\) is clearly stated at the \(*^3\) stage, then it does not have
to appear at the \(*^5\) stage for the conclusion to be drawn.
4. For any formula masquerading as the radius formula
   (e.g. see Common Error 2), \(*^4\) and \(*^5\) are NOT available.

### Primary Method: Give 1 mark for each •

- \(C_1 = (3, 4)\)
- \(k = 6\)
- \(R_1 = 5\)
- \(R_2 = \sqrt{(-3)^2 + (-4)^2 - 12}\) or equivalent
- \(\sqrt{37} > 5\) or "2nd circle"

### Alternative Method 1

- \(x^2 + y^2 - 6x - 8y + 25 = 25\)
- \(k = 6\)
- \(R_1 = 5\)
- \(R_2 = \sqrt{(-3)^2 + (-4)^2 - 12}\) or equivalent
- \(\sqrt{37} > 5\) or "2nd circle"

### Common Error 1

\[\sqrt{\bullet^1} C_1 = (3, 4) \]
\[\sqrt{\bullet^2} k = 6 \]
\[\sqrt{\bullet^3} R_1 = 5 \]
\[X \bullet^4 R_2 = \sqrt{(-3)^2 + (-4)^2 - 12} \]
\[X \sqrt{\bullet^5} \sqrt{13} < 5\) or "1st circle"

### Common Error 2

\[\sqrt{\bullet^1} C_1 = (3, 4) \]
\[\sqrt{\bullet^2} k = 6 \]
\[\sqrt{\bullet^3} R_1 = 5 \]
\[X \bullet^4 R_2 = \sqrt{(-3)^2 + (-4)^2 + (12)^2} \]
\[X \sqrt{\bullet^5} 13 > 5\) or "2nd circle"
5 The curve \( y = f(x) \) is such that \( \frac{dy}{dx} = 4x - 6x^2 \). The curve passes through the point \((-1, 9)\). Express \( y \) in terms of \( x \).

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME.

Primary Method: Give 1 mark for each.

- **1** ss know to integrate
- **2** pr integrate
- **3** ic substitute values
- **4** pr process constant

Notes

1 The equation “\( y = \ldots \)” must appear somewhere in the solution.

---

Common Error 1 Missing “equation”

\[
\sqrt{\bullet^1} \quad y = \int \ldots \\
\sqrt{\bullet^2} \quad \frac{1}{2} x^2 - \frac{6}{3} x^3
\]

\[X\]

\[
\bullet^3 \quad 9 = 2(-1)^2 - 2(-1)^3 + c \\
\bullet^4 \quad y = 2x^2 - 2x^3 + 5 \quad \text{stated explicitly}
\]

Award 3 marks

Common Error 2: Not using \((-1, 9)\)

\[
\sqrt{\bullet^1} \quad y = \int \ldots \\
\sqrt{\bullet^2} \quad \frac{1}{2} x^2 - \frac{6}{3} x^3
\]

\[X\]

\[
\bullet^3 \quad 2(-1)^2 - 2(-1)^3 + c = 0 \\
\bullet^4 \quad y = 2x^2 - 2x^3 - 4
\]

Award 2 marks

---

Alternative Marking

\[
\bullet^1 \quad y = \int \ldots \\
\bullet^2 \quad \frac{1}{2} x^2 - \frac{6}{3} x^3
\]

\[
y = 2x^2 - 2x^3 + c \\
\text{and} \\
9 = 2(-1)^2 - 2(-1)^3 + c
\]

\[
\bullet^1 \quad c = 5
\]
6 P is the point \((-1, 2, -1)\) and Q is \((3, 2, -4)\).

(a) Write down \(\overrightarrow{PQ}\) in component form.  
1

(b) Calculate the length of \(\overrightarrow{PQ}\).  
1

(c) Find the components of a unit vector which is parallel to \(\overrightarrow{PQ}\).  
1

The primary method m/s is based on the following generic m/s. 
THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

• 1 ic state vector components
• 2 pr find the length of a vector
• 3 ic state unit vector

Primary Method : Give 1 mark for each •

\[
\begin{align*}
\text{• 1} & \quad \overrightarrow{PQ} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} & 1 \text{ mark} \\
\text{• 2} & \quad |\overrightarrow{PQ}| = 5 & 1 \text{ mark} \\
\text{• 3} & \quad \begin{pmatrix} \frac{4}{5} \\ 0 \\ -\frac{3}{5} \end{pmatrix} & 1 \text{ mark}
\end{align*}
\]

Note

In (a)
1. It is perfectly acceptable to write the components as a row vector eg \(\overrightarrow{PQ} = \begin{pmatrix} 4 & 0 & -3 \end{pmatrix}\).

Treat \(\overrightarrow{PQ} = (4,0,-3)\) as bad form (i.e. not penalised).

In (b)
2. \(\cdot 2\) is not awarded for an unsimplified \(\sqrt{25}\).

3. Beware of misappropriate use of the scalar product where, by coincidence, \(\overrightarrow{p} \cdot \overrightarrow{q} = 5\).

In (c)
4. Accept \(\begin{pmatrix} 1/5 \\ 0 \\ -3/5 \end{pmatrix}\) for \(\cdot 3\).
7 The diagram shows the graph of a function \( y = f(x) \).

Copy the diagram and on it sketch the graphs of

(a) \( y = f(x - 4) \)
(b) \( y = 2 + f(x - 4) \)

Notes

For (a)

1. A translation of \( \begin{pmatrix} -4 \\ 0 \end{pmatrix} \) earns a maximum of 1 mark with both points clearly annotated and \( f(x) \) retaining its shape.

2. Any other translation gains no marks.

In the Primary method

For (b)

3. \( a^2 \) and \( a^4 \) are only available for applying the translation to the resultant graph from (a).

4. A translation of \( \begin{pmatrix} 0 \\ -2 \end{pmatrix} \) earns a maximum of 1 mark with both points clearly annotated and the resultant graph from (a) retaining its shape.

5. Any other translation gains no marks.

In the Alternative method

For (b)

6. A translation of \( \begin{pmatrix} 4 \\ -2 \end{pmatrix}, \begin{pmatrix} -4 \\ 2 \end{pmatrix} \text{ or } \begin{pmatrix} -4 \\ -2 \end{pmatrix} \) applied to the original graph earns a maximum of 1 mark with both points clearly annotated and the resultant graph retaining its original shape.

7. Any other translation gains no marks.

In either method

For (a) and (b)

8. For the annotated points, accept a superimposed grid or clearly labelled axes.

9. A candidate may choose to use two separate diagrams. This is acceptable.
The diagram shows a right-angled triangle with height 1 unit, base 2 units and an angle of $a^\circ$ at A.

(a) Find the exact values of

(i) $\sin a^\circ$

(ii) $\sin 2a^\circ$.

(b) By expressing $\sin 3a^\circ$ as $\sin(2a + a)^\circ$, find the exact value of $\sin 3a^\circ$.

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- ic interpret diagram for $\sin(a^\circ)$
- ss use double angle formula for $\sin(2A)$
- ic interpret diagram for $\cos(a^\circ)$
- pr substitute and complete
- ss use compound angle formula
- pr use double angle formula for $\cos(2A)$
- ic substitute
- pr complete

Note

1 Calculating approximate angles using $\arcsin$ and $\arccos$ gains no credit.

2 There are 3 processing marks 4, 6 and 8. None of these are available for an answer > 1.

3 $\sin(2a) = 0.8$ and $\cos(2a) = 0.6$ are the only two decimal fractions which may receive any credit.

4 Some candidates may double the height of the triangle and then call the base angle $2a$. This error is equivalent to Common Error 1 illustrated on the right.

Common Error 2
An example based on a numerical error in Pythagoras

| $X$ | $\sin(a^\circ) = \frac{1}{\sqrt{3}}$ | $\sin(2a^\circ) = 2\sin(a^\circ)\cos(a^\circ)$ | $\cos(a^\circ) = \frac{2}{\sqrt{3}}$ | $\sin(2a^\circ) = \frac{4}{3}$ | $\sin(3a^\circ) = \sin(2a^\circ)\cos(a^\circ) + \cos(2a^\circ)\sin(a^\circ) | $\cos(2a^\circ) = \frac{2}{\sqrt{5}}$ | $\sin(3a^\circ) = \frac{4}{\sqrt{5}} + \frac{2}{\sqrt{5}}$ | $\sin(3a^\circ) = \frac{8}{5}$ |

| $X^\sqrt{3}$ | $\cos(a^\circ) = \frac{2}{\sqrt{3}}$ | $\cos(2a^\circ) = 2\cos^2(a^\circ) - 1 = \frac{5}{3}$ or equivalent | $\cos(2a^\circ) = \frac{2}{\sqrt{5}}$ | $\cos(2a^\circ) = \frac{4}{\sqrt{5}}$ | $\cos(2a^\circ) = \frac{2}{\sqrt{5}}$ | $\cos(2a^\circ) = \frac{4}{\sqrt{5}}$ |

$X^\sqrt{7}$ $\sin(3a^\circ) = \frac{4}{3} \cdot \frac{2}{\sqrt{3}} + \frac{5}{3} \cdot \frac{1}{\sqrt{3}}$

$X^\sqrt{8}$ $\sin(3a^\circ) = \frac{13}{3\sqrt{3}}$

$X^\sqrt{5}$ $\sin(2a^\circ) = \frac{2}{\sqrt{5}}$
9 \[ y = \frac{1}{x^3} - \cos 2x, \ x \neq 0, \] find \( \frac{dy}{dx} \).

<table>
<thead>
<tr>
<th>Qu. part</th>
<th>marks</th>
<th>Grade</th>
<th>Syllabus Code</th>
<th>Calculator class</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>4</td>
<td>C/B</td>
<td>C3,C20</td>
<td>CN</td>
<td>06/79</td>
</tr>
</tbody>
</table>

Notes

1. For clearly integrating, correctly or otherwise, only •1 is available.

2. If you cannot decide whether a candidate has attempted to differentiate or integrate, assume they have attempted to differentiate.
10 A curve has equation \( y = 7 \sin x - 24 \cos x \).

(a) Express \( 7 \sin x - 24 \cos x \) in the form \( k \sin(x - a) \) where \( k > 0 \) and \( 0 \leq a \leq \frac{\pi}{2} \).  

(b) Hence find, in the interval \( 0 \leq x \leq \pi \), the x-coordinate of the point on the curve where the gradient is 1.

<table>
<thead>
<tr>
<th>Qu.</th>
<th>part</th>
<th>marks</th>
<th>Grade</th>
<th>Syllabus Code</th>
<th>Calculator class</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>a</td>
<td>4</td>
<td>C</td>
<td>T13 CR</td>
<td>06/97</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>3</td>
<td>A/B</td>
<td>T17 CR</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Primary Method : Give 1 mark for each •

- \( k \sin(x)\cos(a) - \cos(x)\sin(a) \) stated explicitly
- \( k \cos(a) = 7, k\sin(a) = 24 \) stated explicitly
- \( k = 25 \)
- \( a = 1.29 \) 4 marks
- \( 25\sin(x - 1.29) \)
- \( \frac{dy}{dx} = 25\cos(x - 1.29) = 1 \)
- \( x = 2.82 \) 3 marks

Notes

\( k \sin(x)\cos(a) - \cos(x)\sin(a) \) is acceptable for •1.

Treat \( k\sin(x)\cos(a) - \cos(x)\sin(a) \) as bad form if •2 is gained.

No justification is required for •3.

\( 25\sin(x)\cos(a) - \cos(x)\sin(a) \) is acceptable evidence for •1 and •3.

Candidates may use any form of the wave equation to start with as long as their final answer is in the form \( k\sin(x - a) \). If it is not, then •4 is not available.

•4 is only available for

(i) an answer in radians which rounds to 1.3 OR
(ii) an answer given as a multiple of \( \pi \) e.g. \( \frac{\pi}{4} \pi \).

\( k\cos(a) = 7 \) and \( k\sin(a) = -24 \) leading to \( a = 4.99 \) can only gain •4 if a comment intimating that this answer is not in the given interval is given.

In (b)

In (b) candidates have a choice of two starting points.

They can either start from \( y = 25\sin(x - 1.29) \) as shown in the Primary method OR they can start from \( \frac{dy}{dx} = 7\cos(x) + 24\sin(x) \). Either of these starting positions may be awarded •5.

Candidates who work in degrees will lose •6 for attempting to differentiate.

•7 is only available as a consequence of solving \( \frac{dy}{dx} = 1 \). Do not penalise “extra” solutions at the •7 stage (e.g. 6.04).
It is claimed that a wheel is made from wood which is over 1000 years old. To test this claim, carbon dating is used. The formula $A(t) = A_0 e^{-0.000124t}$ is used to determine the age of the wood, where $A_0$ is the amount of carbon in any living tree, $A(t)$ is the amount of carbon in the wood being dated and $t$ is the age of the wood in years. For the wheel it was found that $A(t)$ was 88% of the amount of carbon in a living tree. Is the claim true?

**Primary Method**

Give 1 mark for each •

1. $A(t) = 0.88A_0$
   - Stated or implied by •2
2. $e^{-0.000124t} = 0.88$
3. $\ln(e^{-0.000124t}) = \ln(0.88)$
   - Stated or implied by •4
4. $-0.000124t = \ln(0.88)$
5. $t = 1031$ years so claim valid

**Alternative Method 1**

Graph and Calculator Solution

1. $A(1000) = A_0 e^{-0.000124 \times 1000}$
2. $0.883A_0$ and 1000 year old piece of wood contains 88.3% carbon.
3. Try a point where $t > 1030$
   - E.g. $A(1050)$ getting 0.878$A_0$
4. Sketch of $y=A_0 e^{-0.000124t}$ showing
   1. A monotonic decreasing function
   2. Points representing eg (1000, 88.3%) etc
5. Observation that the point lies between the two plotted values for $t$ and so claim valid.
12 PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- $P$ lies on the curve with equation $y = \frac{8}{x}$ between $(1, 8)$ and $(4, 2)$
- $R$ is the point $(6, 12)$.

(a) (i) Express the lengths of PS and RS in terms of $x$, the $x$-coordinate of $P$.

(ii) Hence show that the area, $A$ square units, of PQRS is given by $A = 80 - 12x - \frac{48}{x}$.

(b) Find the greatest and least possible values of $A$ and the corresponding values of $x$ for which they occur.

### Table

<table>
<thead>
<tr>
<th>Qu.</th>
<th>part</th>
<th>marks</th>
<th>Grade</th>
<th>Syllabus Code</th>
<th>Calculator class</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>a</td>
<td>3</td>
<td>A</td>
<td>C12</td>
<td>CN</td>
<td>06/20</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>9</td>
<td>A/B</td>
<td>C12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- $1^i$ ic interpret diagram to find PS
- $2^i$ ic interpret diagram to find RS
- $3^i$ ic complete proof
- $4^i$ ic express in differentiable form
- $5^i$ ss know to set derivative to zero
- $6^i$ pr differentiate
- $7^i$ pr process equation
- $8^i$ pr evaluate area at the turning point
- $9^i$ pr evaluate area at the end point
- $10^i$ pr evaluate area at the end point
- $11^i$ ic state conclusion

### Primary Method: Give 1 mark for each -

1. $PS = 6 - x$
2. $RS = 12 - \frac{8}{x}$
3. $Area = (6 - x) \left(12 - \frac{8}{x}\right)$ and complete 3 marks
4. $48x^{-1}$
5. $\frac{dA}{dx} = 0$
6. $-12 + 48x^{-2}$
7. $x = 2$
8. $A(2) = 32$
9. $A(1) = 20$
10. $A(4) = 20$
11. max. $A = 32$ at $x = 2$ and $min. A = 20$ at $x = 1$ or $x = 4$

### Notes

1. For $3^i$ there needs to be clear evidence that candidates have multiplied out the brackets in order to complete the proof.
2. An * = 0 * must appear somewhere in the working between $4^i$ and $5^i$.
3. At the $7^i$ stage, ignore the omission or inclusion of $x = -2$.
4. $8^i$ has to be as a consequence of solving $\frac{dA}{dx} = 0$.
5. $11^i$ is only available if both end points have been considered.