2007 Mathematics

Higher – Paper 1

Finalised Marking Instructions

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1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.

2. Award one mark for each ‘bullet’ point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.

3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made.
   This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (√). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (X or X√). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line.
   Work which is correct but inadequate to score any marks should be corrected with a double cross tick (XX).

5. • The total mark for each section of a question should be entered in red in the outer right hand margin, opposite the end of the working concerned.
   • Only the mark should be written, not a fraction of the possible marks.
   • These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked.
   Where a candidate has scored zero marks for any question attempted, “0” should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

8. Do not penalise:
   • working subsequent to a correct answer
   • legitimate variations in numerical answers
   • correct working in the “wrong” part of a question
   • omission of units
   • bad form
9. No piece of work should be scored through without careful checking - even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme - answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referral to the P.A. Please see the general instructions for P.A. referrals.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13. Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pd mark.

14. Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining the appropriate ic mark or pd mark.

15. Do not write any comments on the scripts. A revised summary of acceptable notation is given on page 4.

16. Working that has been crossed out by the candidate cannot receive any credit. If you feel that a candidate has been disadvantaged by this action, make a P.A. Referral.

17. Throughout this paper, unless specifically mentioned, a correct answer with no working receives no credit.

Summary

<table>
<thead>
<tr>
<th>Step</th>
<th>Instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Tick correct working.</td>
</tr>
<tr>
<td>2</td>
<td>Put a mark in the outer right-hand margin to match the marks allocations on the question paper.</td>
</tr>
<tr>
<td>3</td>
<td>Do not write marks as fractions.</td>
</tr>
<tr>
<td>4</td>
<td>Put each mark at the end of the candidate’s response to the question.</td>
</tr>
<tr>
<td>5</td>
<td>Follow through errors to see if candidates can score marks subsequent to the error.</td>
</tr>
<tr>
<td>6</td>
<td>Do not write any comments on the scripts.</td>
</tr>
</tbody>
</table>
Higher Mathematics : A Guide to Standard Signs and Abbreviations

Remember - No comments on the scripts. Please use the following and nothing else.

Signs

✓ The tick. You are not expected to tick every line but of course you must check through the whole of a response.

✗ The cross and underline. Underline an error and place a cross at the end of the line.

✗ The tick-cross. Use this to show correct work where you are following through subsequent to an error.

∧ The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.

~ The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).

✗ The double cross-tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.

Bullets showing where marks are being allotted may be shown on scripts.

<table>
<thead>
<tr>
<th>Marking</th>
<th>Description</th>
<th>Margins</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ •</td>
<td>( \frac{dy}{dx} = 4x - 7 )</td>
<td></td>
</tr>
<tr>
<td>✗</td>
<td>( 4x - 7 = 0 )</td>
<td></td>
</tr>
<tr>
<td>✗ •</td>
<td>( x = \frac{7}{4} )</td>
<td>2</td>
</tr>
<tr>
<td>✗ •</td>
<td>( y = 3 \frac{7}{8} )</td>
<td></td>
</tr>
<tr>
<td>✗</td>
<td>( C = (1,-1) )</td>
<td></td>
</tr>
<tr>
<td>✗</td>
<td>( m = \frac{3-(-1)}{4-1} )</td>
<td></td>
</tr>
<tr>
<td>✗ •</td>
<td>( m_{\text{rad}} = \frac{4}{3} )</td>
<td></td>
</tr>
<tr>
<td>✗ •</td>
<td>( m_{\text{tg}} = \frac{-1}{3} )</td>
<td></td>
</tr>
<tr>
<td>✗ •</td>
<td>( m_{\text{ctg}} = -\frac{2}{3} )</td>
<td></td>
</tr>
<tr>
<td>✗ •</td>
<td>( y - 3 = -\frac{2}{3}(x - 2) )</td>
<td>3</td>
</tr>
<tr>
<td>✗</td>
<td>( x^2 - 3x = 28 )</td>
<td></td>
</tr>
<tr>
<td>✗ •</td>
<td>( x = 7 )</td>
<td>1</td>
</tr>
<tr>
<td>✗</td>
<td>( \sin(x) = 0.75 = \text{invsin}(0.75) = 48.6^\circ )</td>
<td>1</td>
</tr>
</tbody>
</table>

Remember - No comments on the scripts. No abbreviations. No new signs. Please use the above and nothing else.

All of these are to help us be more consistent and accurate.

Note: There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. These are all mistakes and as a consequence a mark is lost.

Page 5 lists the syllabus coding for each topic. This information is given in the legend underneath the question. The calculator classification is CN(calculator neutral), CR(calculator required) and NC(non-calculator).
<table>
<thead>
<tr>
<th>UNIT 1</th>
<th>UNIT 2</th>
<th>UNIT 3</th>
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<td>A1</td>
<td>A15</td>
<td>A28</td>
</tr>
<tr>
<td>A2</td>
<td>A16</td>
<td>A29</td>
</tr>
<tr>
<td>A3</td>
<td>A17</td>
<td>A30</td>
</tr>
<tr>
<td>A4</td>
<td>A18</td>
<td>A31</td>
</tr>
<tr>
<td>A5</td>
<td>A19</td>
<td>A32</td>
</tr>
<tr>
<td>A6</td>
<td>A20</td>
<td>A33</td>
</tr>
<tr>
<td>A7</td>
<td>apply A15-A19 to solve problems</td>
<td>use the laws of logs to simplify/find equiv. expression</td>
</tr>
<tr>
<td>A8</td>
<td>apply A1-A14 to problems</td>
<td>use the notation ( u ) for the nth term</td>
</tr>
<tr>
<td>A9</td>
<td>use Rem Th. For values, factors, roots</td>
<td>use c &amp; da formulae when solving equations</td>
</tr>
<tr>
<td>A10</td>
<td>use the general equation of a parabola</td>
<td>use the general equation of a parabola</td>
</tr>
<tr>
<td>A11</td>
<td>use cubic and quartic equations</td>
<td>use the notation ( u ) for the nth term</td>
</tr>
<tr>
<td>A12</td>
<td>find intersection of line and polynomial</td>
<td>use exact values</td>
</tr>
<tr>
<td>A13</td>
<td>find if line is tangent to polynomial</td>
<td>find ( C/R ) of a circle from its equation/other data</td>
</tr>
<tr>
<td>A14</td>
<td>find intersection of two polynomials</td>
<td>use: ( \frac{u}{v} )</td>
</tr>
<tr>
<td>A15</td>
<td>confirm and improve on approx roots</td>
<td>find gradient from 2 pts./eqn. of line</td>
</tr>
<tr>
<td>A16</td>
<td>apply A21-A36 to problems</td>
<td>find ( C/R ) of a circle from its equation/other data</td>
</tr>
<tr>
<td>A17</td>
<td>apply G1-G7 to problems</td>
<td>use the equations of a tangent to a circle</td>
</tr>
<tr>
<td>A18</td>
<td>apply G16-G29 to problems ( \text{eq geometry probs.} )</td>
<td></td>
</tr>
<tr>
<td>A19</td>
<td>apply G1-G7 to problems ( \text{eq intersect.,concur.,collin.} )</td>
<td>apply G16-G29 to problems ( \text{eq geometry probs.} )</td>
</tr>
<tr>
<td>A20</td>
<td>determine range/domain</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A21</td>
<td>determine function(poly,exp,log) from graph &amp; ( vv )</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A22</td>
<td>sketch and annotate related functions</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A23</td>
<td>sketch/annotate graph given critical features</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A24</td>
<td>interpret equations and expressions</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A25</td>
<td>interpret functions(poly,exp,log)</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A26</td>
<td>sketch/annotate locus such as st.lines,para,poly,circle</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A27</td>
<td>determine nature of stationary points</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A28</td>
<td>determine range/domain</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A29</td>
<td>use the general equation of a parabola</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A30</td>
<td>solve quadratic inequality</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A31</td>
<td>find nature of roots of a quadratic</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A32</td>
<td>given nature of roots, find a condition on coeffs</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A33</td>
<td>complete the square</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A34</td>
<td>use the laws of logs to simplify/find equiv. expression</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A35</td>
<td>solve equations involving logarithms</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A36</td>
<td>use relationships of the form ( y = ax^2 ) or ( y = ab^2 )</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A37</td>
<td>apply A38-A33 to problems</td>
<td>calculate the length of a vector</td>
</tr>
</tbody>
</table>
Find the equation of the line through the point \((-1,4)\) which is parallel to the line with equation \(3x - y + 2 = 0\).

**Notes**
1. Accept any form of the answer (with or without working) for 3 marks

**Primary Method:** Give 1 mark for each:

1. \(y = 3x \ldots\) stated/implied by \(\star^2\)
2. gradient = 3 stated/implied by \(\star^3\)
3. \(y - 4 = 3(x - (-1))\) or
4. form is \(3x - y + c = 0\)
5. \(3 \times (-1) - 4 + c = 0\)
6. \(c = 7\)
1.02

Relative to a suitable coordinate system A and B are the points \((-2,1,-1)\) and \((1,3,2)\) respectively. A, B and C are collinear points and C is positioned such that \(BC = 2AB\).

Find the coordinates of C.

The primary marking scheme is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- **ss** introduces vectors
- **pd** completes
- **ic** interprets positions
- **ia** finds C

**Notes**

1. Treat \(C = \begin{pmatrix} 7 \\ 8 \end{pmatrix}\) as bad form

2. In Alt. method 2, without a diagram only **p2**, **p3** and **ia** are available.

**Primary Method**: Give 1 mark for each

\[ \overrightarrow{AB} = b - a \quad \text{stated or implied by **p2}} \]

\[ \overrightarrow{AB} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \]

\[ BC = \begin{pmatrix} 6 \\ 6 \end{pmatrix} \]

\[ C = (7,7,8) \]

**Alt. method 1**

\[ c - b = 2b - 2a \]

\[ c = 3b - 2a \]

\[ \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} \]

\[ C = (7,7,8) \]

**Alt. method 2**

\[ \text{Diagram} \rightarrow \]

\[ x = 7 \]

\[ y = 7 \]

\[ z = 8 \]
Functions \( f \) and \( g \), defined on suitable domains, are given by
\[
f(x) = x^2 + 1 \quad \text{and} \quad g(x) = 1 - 2x.
\]

Find
\[
\begin{align*}
(a) & \quad g\left(f(x)\right) \\
(b) & \quad g\left(g(x)\right)
\end{align*}
\]

The primary method m.s is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

\[\begin{array}{ll}
\text{Primary Method: Give 1 mark for each} & \\
\text{1} & \quad g\left(f(x)\right) = g(x^2 + 1) \quad \text{s/i by} \quad \bullet^2 \\
\text{2} & \quad 1 - 2(x^2 + 1) \\
\text{3} & \quad g\left(g(x)\right) = g(1 - 2x) \quad \text{s/i by} \quad \bullet^4 \\
\text{4} & \quad 1 - 2(1 - 2x)
\end{array}\]

**Notes**

1. in (a):
   - for finding \( f(g(x)) \):
     \[
     g(1 - 2x) \quad \text{no mark}
     \]
     \[
     (1 - 2x)^2 + 1 \quad \text{award} \quad \bullet^2
     \]
   - for finding \( f(f(x)) \): no marks

2. in (b):
   - for finding \( f(g(x)) \): no mark
   - for finding \( f(f(x)) \):
     \[
     f(x^2 + 1) \quad \text{no mark}
     \]
     \[
     (x^2 + 1)^2 + 1 \quad \text{award} \quad \bullet^4
     \]

3. There are no marks available for either \( g(x) \times f(x) \) or \( g(x) \times g(x) \).
Find the range of values of \( k \) such that the equation 
\[ kx^2 - x - 1 = 0 \]
has no real roots.

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide
but only where a candidate does not use the primary method or any
alternative method shown in detail in the marking scheme.

- ss know to use discriminant < 0
- ic interpret the values of \( a, b \) and \( c \)
- pd solve an inequation

Notes
1. The " < 0 " has to appear at least once
   at the \( \bullet^1 \) stage or the \( \bullet^3 \) stage for \( \bullet^1 \) to
   be awarded
2. If an \( x \) appears at \( \bullet^2 \) stage, none of \( \bullet^2, \bullet^3 \) or
   \( \bullet^4 \) are available
3. Some candidates may start with the quadratic
   formula. Apply the marking scheme to the part
   underneath the square root sign
4. The use of any expression masquerading as the
   discriminant can only gain \( \bullet^2 \) at most

Primary Method: Give 1 mark for each:

- \( \bullet^1 b^2 - 4ac < 0 \)
- \( \bullet^2 a = k, b = -1, c = -1 \) s/1 by \( \bullet^3 \)
- \( \bullet^3 1 + 4k \)
- \( \bullet^4 k < -\frac{1}{4} \)

Common Error 1

- \( \bullet^1 X b^2 - 4ac \)
- \( \bullet^2 \sqrt[3]{1 + 4k} \)
- \( k = -\frac{1}{4} \)
- \( \bullet^4 X k < -\frac{1}{4} \)
The large circle has equation $x^2 + y^2 - 14x - 16y + 77 = 0$.

Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the x-axis.

This pattern is continued, as shown in the diagram.

Find the equation of the circle with centre D.

**Primary Method:** Give 1 mark for each.

1. $B = (7,8)$
2. $r_{\text{avg}} = \sqrt{7^2 + 8^2 - 77} = 6$
3. $r_{\text{small}} = \frac{6}{3}$
4. $D = (15,8)$
5. $(x - 10)^2 + (y - 8)^2 = 2^2$

**Note**

1. If $D = (31,8)$ then $\bullet^4$ is not available; however either of
   
   $(x - 31)^2 + (y - 8)^2 = 2^2$
   
   or $(x - 31)^2 + (y - 8)^2 = 6^2$

   may be awarded $\bullet^5$

2. $\bullet^5$ is only awarded for substituting numerical values for the centre and the radius.
Solve the equation \( \sin(2x°) = 6 \cos(x°) \) for \( 0 \leq x \leq 360° \).

The primary method m.s. is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- **1** ss know and use double angle formula
- **2** pd write in st. form and factorise
- **3** pd start to solve
- **4** ic know and use exact values

Primary Method : Give 1 mark for each •

1. \( 2 \sin(x°) \cos(x°) \)
2. \( \cos(x°)(2 \sin(x°) - 6) = 0 \)
3. \( \cos(x°) = 0 \) and \( x = 90, 270° \)
4. \( \sin(x°) = 3 \) and no solution

or
5. \( \cos(x°) = 0 \) and \( \sin(x°) = 3 \)
6. \( x = 90, 270° \) and no solution

Alt. method : Division by \( \cos(x°) \)

1. \( 2 \sin(x°) \cos(x°) \)
2. either \( \cos(x°) = 0 \) or \( \cos(x°) = 0 \) stated explicitly
3. \( \cos(x°) = 0 \) \( \Rightarrow \) \( x = 90 \) or 270°
4. \( 2 \sin(x°) = 6 \Rightarrow \) no solution

Notes
1. **1** is NOT available for \( 2 \sin A \cos A \) with no further working
2. The " has to appear at least once at the **1** stage or the **2** stage
3. The inclusion of extra answers which would have been correct but are outside the given interval should be treated as bad form (i.e. not penalised)
4. In following through from an error, **4** is only available for solving an equation with no solution
5. The phrase "no solution" does not always appear after \( \sin(x) = 3 \). The minimum indication that no solution exists might simply be a line drawn through or underneath the equation.
A sequence is defined by the recurrence relation
\[ u_{n+1} = \frac{1}{4} u_n + 16, \quad u_0 = 0. \]

(a) Calculate the values of \( u_1, u_2 \) and \( u_3 \).

Four terms of this sequence, \( u_1, u_2, u_3 \) and \( u_4 \) are plotted as shown in the graph.
As \( n \to \infty \), the points on the graph approach the line \( u_n = k \), where \( k \) is the limit of this sequence.

(b) (i) Give a reason why this sequence has a limit.
(ii) Find the exact value of \( k \).

Notes 1
1 In (a) only numerical values for \( u_1, u_2 \) and \( u_3 \) are acceptable
2 For (b)(i) accept
   \[ \left| \frac{1}{4} \right| < 1 \]
   \[ 0 < \frac{1}{4} < 1 \]
   \[ \frac{1}{4} \text{ lies between } -1 \text{ and } 1 \]
   \[ \frac{1}{4} \text{ is a proper fraction} \]
3 For (b)(i) do NOT accept
   \[ -1 \leq \frac{1}{4} \leq 1 \]
   \[ \frac{1}{4} < 1 \]
   \[ -1 < a < 1 \text{ unless } a \text{ is clearly identified/replaced by } a \frac{1}{4} \text{ anywhere in} \]
   the answer
The diagram shows a sketch of the graph of \( y = x^3 - 4x^2 + x + 6 \).

(a) Show that the graph cuts the \( x \)-axis at (3,0)

(b) Hence or otherwise find the coordinates of \( A \).

(c) Find the shaded area.

The primary method m.s. is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only if a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- \( \bullet \) 1: ss know to evaluate, and evaluate at \( x = 3 \)
- \( \bullet \) 2: ss strategy for finding other factors
- \( \bullet \) 3: ic quadratic factor
- \( \bullet \) 4: pd find \( +ve \) root and identify
- \( \bullet \) 5: ss know to integrate
- \( \bullet \) 6: ic identify limits
- \( \bullet \) 7: pd integrate
- \( \bullet \) 8: ic substitute limits
- \( \bullet \) 9: pd process limits

Primary Method: Give 1 mark for each:

- \( \bullet \) 1 \( f(3) = 27 - 36 + 3 + 6 = 0 \)
- \( \bullet \) 2 \( (x - 3)(x^2... \ldots) \)
- \( \bullet \) 3 \( (x - 3)(x^2 - x - 2) \)
- \( \bullet \) 4 \( (x - 3)(x - 2)(x + 1) \) so \( A = (2, 0) \)
- \( \bullet \) 5 \( \int (x^3 - 4x^2 + x + 6) \, dx \)
- \( \bullet \) 6 \( \int_0^2 x^3 - 4x^2 + x + 6 \, dx \)
- \( \bullet \) 7 \( \frac{1}{4}x^4 - \frac{4}{3}x^3 + \frac{1}{2}x^2 + 6x \)
- \( \bullet \) 8 \( \frac{1}{4}x^4 - \frac{4}{3}x^3 + \frac{1}{2}x^2 + 6x \)
- \( \bullet \) 9 \( 22 \) 

Alt. Method 1 for \( \bullet \) 1 to \( \bullet \) 4

\[
\begin{array}{cccccc}
3 & 1 & -4 & 1 & 6 \\
3 & -3 & -6 \\
1 & -1 & -2 & 0 & 2
\end{array}
\]

- \( \bullet \) 1 \( x^2 - x - 2 \)
- \( \bullet \) 2 \( x = 2, x = -1 \) \textit{AND} \( x_A = 2 \)

Alt. Method 2 for \( \bullet \) 1 to \( \bullet \) 4

- \( \bullet \) 1 \( f(3) = \ldots \ldots = 0 \)
- \( \bullet \) 2 try \( f(n) = \ldots \ldots \) where \( n > 0 \)
- \( \bullet \) 3 \( f(2) = \ldots \ldots = 0 \)
- \( \bullet \) 4 \( x_A = 2 \)

Notes

1 The working & evidence for (a) may appear in part (b) and vice versa

2 In Alternative Method 1, \( \bullet \) 1, candidates must show some acknowledgement of the resulting "zero". Although a statement with respect to the "zero" is preferable, accept something as simple as an underlining of the zero

3 In (c) the appearance of \( \int_0^2 \) may NOT be used as evidence for \( \bullet \) 4

4 Since the area is totally above the \( x \)-axis, \( \bullet \) 9 is not available for a negative answer irrespective of whether or not the candidate tries to deal with it

5 For information:

\[
\int_0^1 = \frac{27}{4} \quad \int_0^1 = \frac{65}{12} \quad \int_0^4 = \frac{32}{3} \quad \int_0^6 = 90
\]

6 For candidates who differentiate, or fail to even try to integrate, \( \bullet \) 7, \( \bullet \) 8 and \( \bullet \) 9 are not available
A function \( f \) is defined by the formula \( f(x) = 3x - x^3 \).

(a) Find the exact values where the graph of \( y = f(x) \) meets the \( x \)- and \( y \)-axes.
(b) Find the coordinates of the stationary points of the function and determine their nature.
(c) Sketch the graph of \( y = f(x) \).

### Notes 1
1. \( \bullet^2 \) is only available if \( \bullet^4 \) has been awarded
2. The "\( n = 0 \)" shown at \( \bullet^5 \) must appear at least once somewhere in the working between \( \bullet^3 \) and \( \bullet^6 \)
3. \( \bullet^6 \) is only available as a consequence of solving \( f'(x) = 0 \)
4. An unsimplified \( \sqrt{1} \) should be penalised at the first occurrence
5. The evidence for \( \bullet^7 \) and \( \bullet^9 \) may not appear until the sketch
6. The nature table must reflect previous working from \( \bullet^4 \) and \( \bullet^6 \)
7. The minimum requirement for the sketch is a cubic passing through the origin and with turning points annotated
Given that \( y = \sqrt{3x^2 + 2} \), find \( \frac{dy}{dx} \).

The primary method mark scheme is based on the following generic marks:

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

1. **ss** expresses in standard form
2. **pd** differentiate a binomial to fractional power
3. **ss** know and use chain rule

### Primary Method: Give 1 mark for each

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( (3x^2 + 2)^{\frac{1}{2}} )</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( \frac{1}{2}(3x^2 + 2)^{-\frac{1}{2}} )</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( \times 6x )</td>
<td></td>
</tr>
</tbody>
</table>

### Common Errors

1. \( y = (3x^2 + 2)^{-1} \)
   - \( x = 1, y = 2 \)
   - \( x = -1, y = -2 \)

2. \( y = (3x^2 + 2)^{\frac{1}{2}} \)
   - \( \frac{dy}{dx} = -\frac{1}{2}(3x^2 + 2)^{\frac{3}{2}} \)
   - \( \times 6x \)

### Marking in series or parallel

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( x = 1 )</td>
<td>( x = -1 )</td>
</tr>
<tr>
<td>2</td>
<td>( y = 2 )</td>
<td>( y = -2 )</td>
</tr>
</tbody>
</table>

### Example of a minimum requirement nature table

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( f' )</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>minimum</td>
<td>maximum</td>
</tr>
</tbody>
</table>

### Example of a preferred nature table

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( x \rightarrow -1 \rightarrow 1 \rightarrow )</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( f' \rightarrow 0 \rightarrow 0 \rightarrow )</td>
<td></td>
</tr>
</tbody>
</table>

min at \( x = -1 \)
max at \( x = 1 \)
(a) Express \( f(x) = \sqrt{3} \cos(x) + \sin(x) \) in the form \( k \cos(x - a) \), where \( k > 0 \) and \( 0 < a < \frac{\pi}{2} \).

(b) Hence or otherwise sketch the graph of \( y = f(x) \) in the interval \( 0 \leq x \leq 2\pi \).

Notes 1

1. In the whole question, do not penalise more than once for not using radians.
2. Table showing marks lost for using degrees:

<table>
<thead>
<tr>
<th>Value</th>
<th>-1</th>
<th>-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>graph in degrees</td>
<td>(30^\circ)</td>
<td>(60^\circ)</td>
</tr>
<tr>
<td>graph in radians</td>
<td>(-\frac{\pi}{6})</td>
<td>(-\frac{\pi}{3})</td>
</tr>
</tbody>
</table>

In (a)

2. \( k(\cos x \cos a + \sin x \sin a) \) is acceptable for •
3. \( k = \sqrt{3} \) does NOT earn •
4. \( 2(\cos x \cos a + \sin x \sin a) \) etc is acceptable for • & •
5. Candidates may use any form of the wave equation as long as their final answer is in the form \( k \cos(x - a) \). If not then • is NOT available
6. Treat \( k \cos x \cos a + \sin x \sin a \) as bad form ONLY if • is gained.

Notes 2

7. Do not penalise graphs which go beyond \( 0 \leq x \leq 2\pi \)
8. A maximum of 3 marks are available for candidates who attempt to sketch graphs of \( k \cos(x + a) \), \( k \sin(x + a) \) or \( k \sin(x - a) \). No other graphs can earn any credit

9. Alternative marking for 2 marks for candidates who do not make a sketch

max\( \left(\frac{\pi}{6}, ..., \frac{4\pi}{3}\right) \), \( \min\left(\frac{\pi}{6}, ..., -2\right) \), \( \left(\frac{2\pi}{3}, 0\right), \left(\frac{4\pi}{3}, 0\right) \) and \( \left(0, \sqrt{3}\right) \)

• any two from the above list

• another two from the above list