FORMULAE LIST

Circle:
The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.
The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre $(a, b)$ and radius $r$.

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where $\theta$ is the angle between $\mathbf{a}$ and $\mathbf{b}$
or $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:
- $\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$
- $\sin 2A = 2\sin A \cos A$
- $\cos 2A = \cos^2 A - \sin^2 A$
- $\cos 2A = 2\cos^2 A - 1$
- $\cos 2A = 1 - 2\sin^2 A$

Table of standard derivatives:

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>$f'(x)$</th>
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<tbody>
<tr>
<td>$\sin ax$</td>
<td>$a \cos ax$</td>
</tr>
<tr>
<td>$\cos ax$</td>
<td>$-a \sin ax$</td>
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Table of standard integrals:

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>$\int f(x) , dx$</th>
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<tbody>
<tr>
<td>$\sin ax$</td>
<td>$\frac{-1}{a} \cos ax + C$</td>
</tr>
<tr>
<td>$\cos ax$</td>
<td>$\frac{1}{a} \sin ax + C$</td>
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SECTION A

ALL questions should be attempted.

1. Given that \( p = \begin{pmatrix} 2 \\ 5 \\ -7 \end{pmatrix}, \ q = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \) and \( r = \begin{pmatrix} -4 \\ 2 \\ 0 \end{pmatrix} \), express \( 2p - q - \frac{1}{2}r \) in component form.

\[
\begin{pmatrix} A \\ B \\ C \end{pmatrix}
\]

\[
\begin{pmatrix} 1 \\ 9 \\ -15 \end{pmatrix}
\]

\[
\begin{pmatrix} 5 \\ 9 \\ -13 \end{pmatrix}
\]

\[
\begin{pmatrix} 5 \\ 11 \\ -15 \end{pmatrix}
\]

2. A line \( l \) has equation \( 3y + 2x = 6 \).

What is the gradient of any line parallel to \( l \)?

\[
\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix}
\]

\[
-2
\]

\[
\frac{2}{3}
\]

\[
\frac{3}{2}
\]

\[
2
\]
3. The diagram shows the graph of \( y = f(x) \).

Which of the following shows the graph of \( y = f(x + 2) - 1 \)?

A

\[ (-4, 3) \]

\[ (-1, -2) \]

B

\[ (3, 1) \]

\[ (3, -2) \]

C

\[ (-1, 1) \]

\[ (-4, -4) \]

D

\[ (1, 2) \]

\[ (-2, -3) \]
4. A tangent to the curve with equation \( y = x^3 - 2x \) is drawn at the point \((2, 4)\). What is the gradient of this tangent?

A 2  
B 3  
C 4  
D 10

5. If \( x^2 - 8x + 7 \) is written in the form \((x - p)^2 + q\), what is the value of \(q\)?

A \(-9\)  
B \(-1\)  
C 7  
D 23

6. The point \(P(2, -3)\) lies on the circle with centre \(C\) as shown.

The gradient of \(CP\) is \(-2\).

What is the equation of the tangent at \(P\)?

A \(y + 3 = -2(x - 2)\)  
B \(y - 3 = -2(x + 2)\)  
C \(y + 3 = \frac{1}{2}(x - 2)\)  
D \(y - 3 = \frac{1}{2}(x + 2)\)

7. A function \(f\) is defined on the set of real numbers by \(f(x) = x^3 - x^2 + x + 3\).

What is the remainder when \(f(x)\) is divided by \((x - 1)\)?

A 0  
B 2  
C 3  
D 4
8. A line makes an angle of 30° with the positive direction of the x-axis as shown.

What is the gradient of the line?

A $\frac{1}{\sqrt{3}}$
B $\frac{1}{\sqrt{2}}$
C $\frac{1}{2}$
D $\frac{\sqrt{3}}{2}$

9. The discriminant of a quadratic equation is 23.
Here are two statements about this quadratic equation:
(1) the roots are real;
(2) the roots are rational.

Which of the following is true?
A Neither statement is correct.
B Only statement (1) is correct.
C Only statement (2) is correct.
D Both statements are correct.
10. Solve $2 \cos x = \sqrt{3}$ for $x$, where $0 \leq x < 2\pi$.

A $\dfrac{\pi}{3}$ and $\dfrac{5\pi}{3}$

B $\dfrac{\pi}{3}$ and $\dfrac{2\pi}{3}$

C $\dfrac{\pi}{6}$ and $\dfrac{5\pi}{6}$

D $\dfrac{\pi}{6}$ and $\dfrac{11\pi}{6}$

11. Find $\int \left(4x^\frac{1}{3} + x^{-3}\right) dx$, where $x > 0$.

A $2x^{-\frac{1}{2}} - 3x^{-4} + c$

B $2x^{-\frac{1}{2}} - \dfrac{1}{2}x^{-2} + c$

C $\dfrac{8}{3}x^{\frac{3}{2}} - 3x^{-4} + c$

D $\dfrac{8}{3}x^{\frac{3}{2}} - \dfrac{1}{2}x^{-2} + c$
12. The diagram shows two right-angled triangles with sides and angles as given.

What is the value of \( \sin(p + q) \)?

A. \( \frac{2}{\sqrt{5}} + \frac{2}{3} \)

B. \( \frac{2}{\sqrt{5}} + \frac{\sqrt{5}}{3} \)

C. \( \frac{2}{3} + \frac{2}{3\sqrt{5}} \)

D. \( \frac{4}{3\sqrt{5}} + \frac{1}{3} \)

13. Given that \( f(x) = 4 \sin 3x \), find \( f'(0) \).

A. 0

B. 1

C. 12

D. 36
14. An equilateral triangle of side 3 units is shown. The vectors \( \mathbf{p} \) and \( \mathbf{q} \) are as represented in the diagram. What is the value of \( \mathbf{p} \cdot \mathbf{q} \)?

A 9  
B \( \frac{9}{2} \)  
C \( \frac{9}{\sqrt{2}} \)  
D 0

15. Given that the points S(–4, 5, 1), T(–16, –4, 16) and U(–24, –10, 26) are collinear, calculate the ratio in which T divides SU.

A 2 : 3  
B 3 : 2  
C 2 : 5  
D 3 : 5

16. Find \( \int \frac{1}{3x^4} \, dx \), where \( x \neq 0 \).

A \(-\frac{1}{9x^3} + c\)  
B \(-\frac{1}{x^3} + c\)  
C \(\frac{1}{x^3} + c\)  
D \(\frac{1}{12x^3} + c\)
17. The diagram shows the graph of a cubic.

What is the equation of this cubic?
A $y = -x(x + 1)(x - 2)$
B $y = -x(x - 1)(x + 2)$
C $y = x(x + 1)(x - 2)$
D $y = x(x - 1)(x + 2)$

18. If $f(x) = (x - 3)(x + 5)$, for what values of $x$ is the graph of $y = f(x)$ above the $x$-axis?
A $-5 < x < 3$
B $-3 < x < 5$
C $x < -5, x > 3$
D $x < -3, x > 5$
19. Which of the following diagrams represents the graph with equation $\log_3 y = x$?
20. On a suitable domain, D, a function \( g \) is defined by \( g(x) = \sin^2 \sqrt{x - 2} \).

Which of the following gives the real values of \( x \) in D and the corresponding values of \( g(x) \)?

A  \( x \geq 0 \) and \( -1 \leq g(x) \leq 1 \)
B  \( x \geq 0 \) and \( 0 \leq g(x) \leq 1 \)
C  \( x \geq 2 \) and \( -1 \leq g(x) \leq 1 \)
D  \( x \geq 2 \) and \( 0 \leq g(x) \leq 1 \)

[END OF SECTION A]
## 2011 Answers

<table>
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<tr>
<th>Question</th>
<th>Answer</th>
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**Summary**

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