Higher Mathematics
SQA Multiple Choice Questions
2015
FORMULAE LIST

Circle:
The equation \( x^2 + y^2 + 2gx + 2fy + c = 0 \) represents a circle centre \((-g, -f)\) and radius \( \sqrt{g^2 + f^2 - c} \).
The equation \((x - a)^2 + (y - b)^2 = r^2\) represents a circle centre \((a, b)\) and radius \(r\).

Scalar Product: \( \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta \), where \(\theta\) is the angle between \(\mathbf{a}\) and \(\mathbf{b}\)

or \( \mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 \) where \( \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \) and \( \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \).

Trigonometric formulae:
\[
\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\sin 2A = 2\sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A
\]
\[
= 2\cos^2 A - 1
\]
\[
= 1 - 2\sin^2 A
\]

Table of standard derivatives:

<table>
<thead>
<tr>
<th>(f(x))</th>
<th>(f'(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sin ax)</td>
<td>(a \cos ax)</td>
</tr>
<tr>
<td>(\cos ax)</td>
<td>(-a \sin ax)</td>
</tr>
</tbody>
</table>

Table of standard integrals:

<table>
<thead>
<tr>
<th>(f(x))</th>
<th>(\int f(x)dx)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sin ax)</td>
<td>(-\frac{1}{a} \cos ax + c)</td>
</tr>
<tr>
<td>(\cos ax)</td>
<td>(\frac{1}{a} \sin ax + c)</td>
</tr>
</tbody>
</table>

[Turn over]
SECTION A

ALL questions should be attempted.

1. Given \( f(x) = 2x^3 - 7 \) find the value of \( f'(2) \).
   
   A \( -6 \)
   
   B \( 9 \)
   
   C \( 24 \)
   
   D \( 137 \)

2. The line with equation \( 2y = 3x + 5 \) is perpendicular to the line with equation \( y = kx \).
   What is the value of \( k \)?
   
   A \( \frac{-3}{2} \)
   
   B \( \frac{-2}{3} \)
   
   C \( \frac{2}{3} \)
   
   D \( \frac{3}{2} \)

3. If \( 2x^3 + x^2 - 4x + 1 \) is divided by \( (x - 2) \), what is the remainder?
   
   A \( -11 \)
   
   B \( 0 \)
   
   C \( 1 \)
   
   D \( 13 \)
4. The diagram shows the graph with equation of the form \( y = a \cos bx \) for \( 0 \leq x \leq 2\pi \).

What is the equation of this graph?

A \( y = -3\cos 2x \)

B \( y = -3\cos 3x \)

C \( y = 3\cos 2x \)

D \( y = 3\cos 3x \)

5. A sequence is defined by the recurrence relation \( u_{n+1} = 0.2u_n + 9 \), \( u_5 = 11 \).

What is the value of \( u_3 \)?

A \( 11.24 \)

B \( 9.4 \)

C \( 5 \)

D \( 4 \)

6. The points P, Q and R are collinear.

\( \overrightarrow{QR} = \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix} \).

P is the point \((-1, 6, 4)\), Q is the point \((2, 0, 13)\) and \( QR \).

Calculate the ratio in which Q divides PR.

A \( 2 : 3 \)

B \( 3 : 2 \)

C \( 3 : 5 \)

D \( 5 : 2 \)
7. What is \( \int (x + 4)(x - 4) \, dx \)?

A  \( 2x + c \)

B  \( \frac{1}{3}x^3 + c \)

C  \( \frac{1}{3}x^3 - 16x + c \)

D  \( \left( \frac{1}{2}x^2 + 4x \right) \left( \frac{1}{2}x^2 - 4x \right) + c \)

8. A straight line makes an angle of 60° with the x-axis as shown in the diagram.

What is the gradient of this line?

A  \( \frac{1}{2} \)

B  \( \frac{1}{\sqrt{3}} \)

C  \( \frac{\sqrt{3}}{2} \)

D  \( \sqrt{3} \)
9. Find the minimum value of $3\sin 2x + 5$ and the value of $x$ where this occurs in the interval $0 \leq x < \pi$.

<table>
<thead>
<tr>
<th>min value</th>
<th>$x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 2</td>
<td>$\frac{3\pi}{2}$</td>
</tr>
<tr>
<td>B 2</td>
<td>$\frac{3\pi}{4}$</td>
</tr>
<tr>
<td>C 4</td>
<td>$\frac{3\pi}{2}$</td>
</tr>
<tr>
<td>D 4</td>
<td>$\frac{3\pi}{4}$</td>
</tr>
</tbody>
</table>

10. Solve $2\cos x + 1 = 0$ for $x$, where $\pi \leq x \leq \frac{3\pi}{2}$.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A $\frac{5\pi}{6}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B $\frac{7\pi}{6}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C $\frac{5\pi}{4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D $\frac{4\pi}{3}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

11. The curve $y = f(x)$ is such that $f'(x) = 4x - 1$.

The curve passes through the point (2, 9).

What is the equation of the curve?

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A $y = 2x^2 - x - 5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B $y = 2x^2 + 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C $y = 2x^2 - x + 3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D $y = 2x^2 - x$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
12. Given that the point R is (3, –1, 2), \( \overrightarrow{RS} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} \) and \( \overrightarrow{RT} = 3\overrightarrow{RS} \), find the coordinates of T.

A. (3, 2, –11)
B. (3, 4, –11)
C. (9, 2, –7)
D. (9, 4, –7)

13. The diagram shows a curve with equation of the form \( y = ax^2 + bx + c \).

Here are two statements about \( a, b \) and \( c \):

(1) \( a > 0 \)
(2) \( b^2 - 4ac > 0 \)

Which of the following is true?

A. Neither statement is correct.
B. Only statement (1) is correct.
C. Only statement (2) is correct.
D. Both statements are correct.
14. If $\cos x = -\frac{2}{5}$, what is the value of $\cos 2x$?

A $\frac{33}{25}$

B $\frac{17}{25}$

C $-\frac{4}{5}$

D $-\frac{17}{25}$

15. The graph of a cubic function, $y = f(x)$, is shown below.
It passes through the points $(-2, 0)$, $(-1, 0)$, $(3, 0)$ and $(0, -3)$.

What is the equation of this curve?

A $y = \frac{1}{2}(x - 3)(x + 1)(x + 2)$

B $y = 2(x - 3)(x + 1)(x + 2)$

C $y = -\frac{1}{2}(x + 3)(x - 1)(x - 2)$

D $y = -2(x + 3)(x - 1)(x - 2)$
16. If $e^{4t} = 6$, find an expression for $t$.

A $t = \log_e \frac{3}{2}$

B $t = \frac{\log_e 6}{4}$

C $t = \frac{6}{\log_e 4}$

D $t = \frac{\log_e 6}{\log_e 4}$

17. Vectors $\mathbf{u}$ and $\mathbf{v}$ have components $\begin{pmatrix} 3 \\ 5 \\ t \end{pmatrix}$ and $\begin{pmatrix} -6 \\ 0 \\ -10 \end{pmatrix}$ respectively.

Here are two statements about $\mathbf{u}$ and $\mathbf{v}$:

(1) when $t = \frac{4}{5}$, $\mathbf{u}$ is a unit vector

(2) when $t = 1$, $\mathbf{u}$ and $\mathbf{v}$ are parallel

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

18. The circle with equation $x^2 + y^2 - 12x - 10y + k = 0$ meets the coordinate axes at exactly three points.

What is the value of $k$?

A 5

B 6

C 25

D 36
19. The diagram shows part of the graph of \( y = a \cos bx \).

The shaded area is \( \frac{1}{2} \) unit\(^2\).

![Graph of y = a \cos bx]

What is the value of \( \int_0^{3\pi/4} (a \cos bx) \, dx \)?

A. \(-1\)

B. \(-\frac{1}{2}\)

C. \(\frac{1}{2}\)

D. \(1\frac{1}{2}\)

20. The only stationary point on the graph of \( y = f(x) \) is the point \((a, b)\).

What are the coordinates of the only stationary point on the graph of \( y = -f(2x)\)?

A. \((\frac{1}{2}a, -b)\)

B. \((2a, -b)\)

C. \((-\frac{1}{2}a, b)\)

D. \((-2a, b)\)

[END OF SECTION A]