X100/301

NATIONAL QUALIFICATIONS 2005

FRIDAY, 20 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 1
(Non-calculator)

Read Carefully

1 **Calculators may <u>NOT</u> be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

SCOTTISH QUALIFICATIONS AUTHORITY

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos \theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

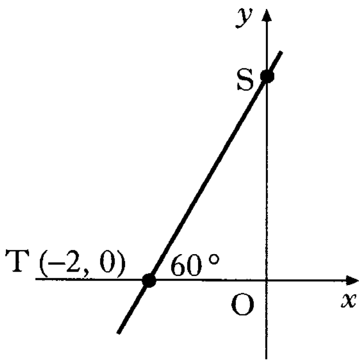
Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted.

Marks

1. Find the equation of the line ST, where T is the point (–2, 0) and angle STO is 60°. **3**



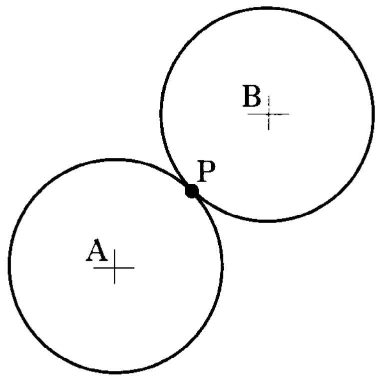
2. Two congruent circles, with centres A and B, touch at P.

Relative to suitable axes, their equations are

$x^2 + y^2 + 6x + 4y - 12 = 0$ and
$x^2 + y^2 - 6x - 12y + 20 = 0$.

(*a*) Find the coordinates of P. **3**

(*b*) Find the length of AB. **2**

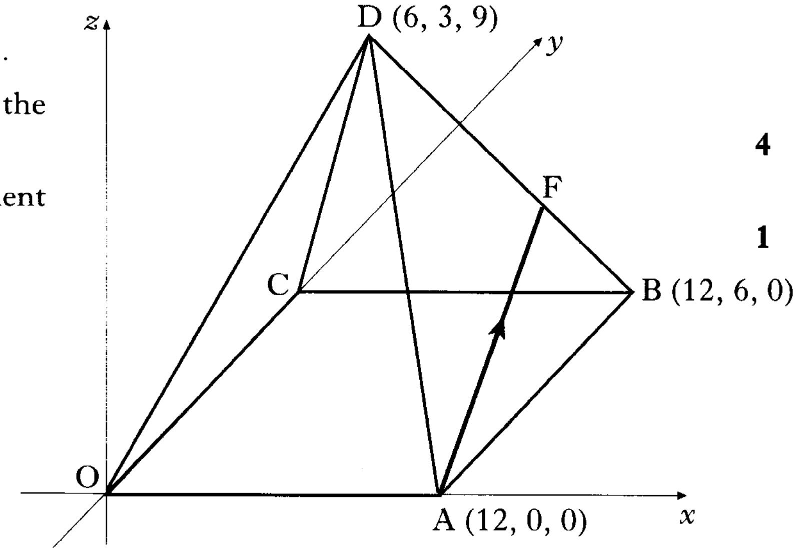


3. D,OABC is a pyramid. A is the point (12, 0, 0), B is (12, 6, 0) and D is (6, 3, 9).

F divides DB in the ratio 2:1.

(*a*) Find the coordinates of the point F. **4**

(*b*) Express $\overrightarrow{AF}$ in component form. **1**



[Turn over

4. Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$ are defined on the set of real numbers.

(*a*) Find $h(x)$ where $h(x) = g(f(x))$. **2**

(*b*) (i) Write down the coordinates of the minimum turning point of $y = h(x)$.

(ii) Hence state the range of the function h. **2**

5. Differentiate $(1 + 2\sin x)^4$ with respect to x. **2**

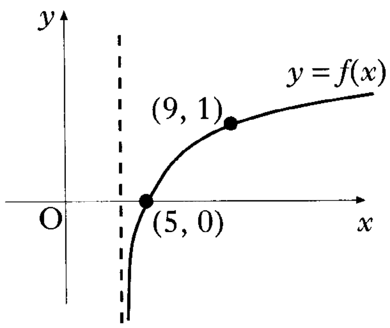
6. (*a*) The terms of a sequence satisfy $u_{n+1} = ku_n + 5$. Find the value of k which produces a sequence with a limit of 4. **2**

(*b*) A sequence satisfies the recurrence relation $u_{n+1} = mu_n + 5$, $u_0 = 3$.

(i) Express u_1 and u_2 in terms of m.

(ii) Given that $u_2 = 7$, find the value of m which produces a sequence with no limit. **5**

7. The function f is of the form $f(x) = \log_b(x - a)$. The graph of $y = f(x)$ is shown in the diagram.



(*a*) Write down the values of a and b. **2**

(*b*) State the domain of f. **1**

8. A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.

(*a*) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully. **5**

(*b*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. **2**

(*c*) Find the greatest and least values of f in the interval $-2 \leq x \leq 2$. **5**

9. If $\cos 2x = \frac{7}{25}$ and $0 < x < \frac{\pi}{2}$, find the exact values of $\cos x$ and $\sin x$. **4**

Marks

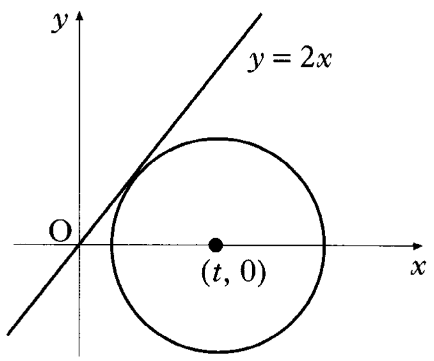
10. (*a*) Express $\sin x - \sqrt{3}\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \leq a \leq 2\pi$. **4**

(*b*) Hence, or otherwise, sketch the curve with equation $y = 3 + \sin x - \sqrt{3}\cos x$ in the interval $0 \leq x \leq 2\pi$. **5**

11. (*a*) A circle has centre $(t, 0)$, $t > 0$, and radius 2 units.

Write down the equation of the circle. **1**

(*b*) Find the exact value of t such that the line $y = 2x$ is a tangent to the circle. **5**



[*END OF QUESTION PAPER*]

X100/303

NATIONAL QUALIFICATIONS 2005

FRIDAY, 20 MAY
10.30 AM – 12.00 NOON

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 2

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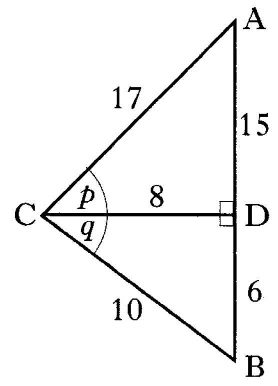
Marks

1. Find $\int \frac{4x^3 - 1}{x^2}\,dx, \quad x \neq 0.$ **4**

2. Triangles ACD and BCD are right-angled at D with angles p and q and lengths as shown in the diagram.



(*a*) Show that the exact value of $\sin(p + q)$ is $\frac{84}{85}$. **4**

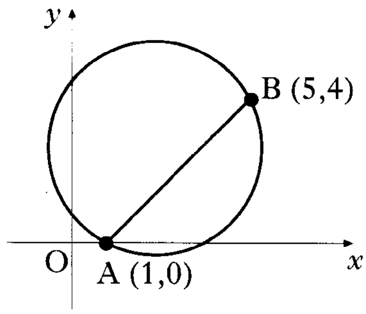
(*b*) Calculate the exact values of:

(i) $\cos(p + q)$;

(ii) $\tan(p + q)$. **3**

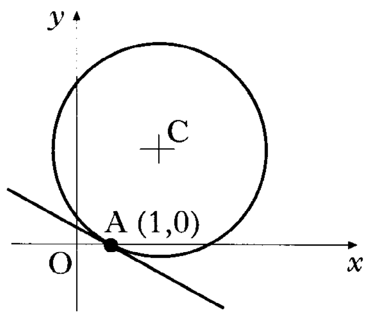
3. (*a*) A chord joins the points A(1,0) and B(5,4) on the circle as shown in the diagram.

Show that the equation of the perpendicular bisector of chord AB is $x + y = 5$. **4**



(*b*) The point C is the centre of this circle. The tangent at the point A on the circle has equation $x + 3y = 1$.

Find the equation of the radius CA. **4**



(*c*) (i) Determine the coordinates of the point C.

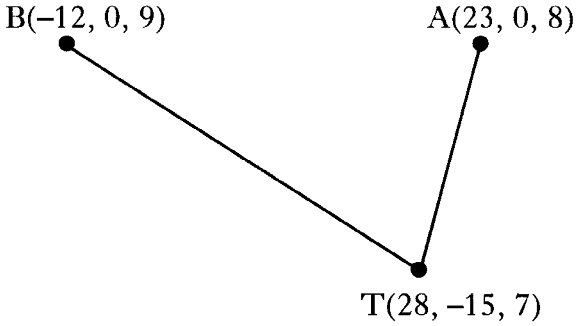
(ii) Find the equation of the circle. **4**

[Turn over

4. The sketch shows the positions of Andrew(A), Bob(B) and Tracy(T) on three hill-tops.

Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(−12, 0, 9) and T(28, −15, 7).

In the dark, Andrew and Bob locate Tracy using heat-seeking beams.

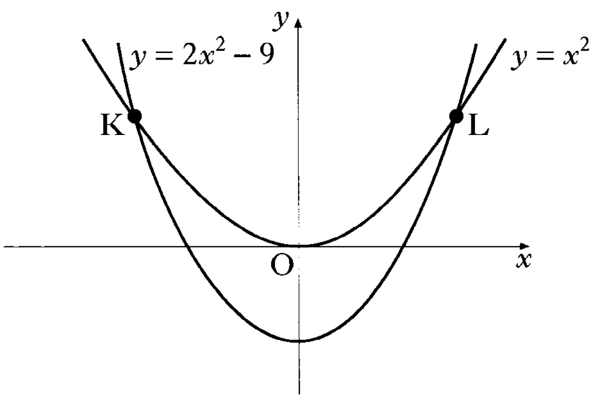


(*a*) Express the vectors $\overrightarrow{TA}$ and $\overrightarrow{TB}$ in component form. **2**

(*b*) Calculate the angle between these two beams. **5**

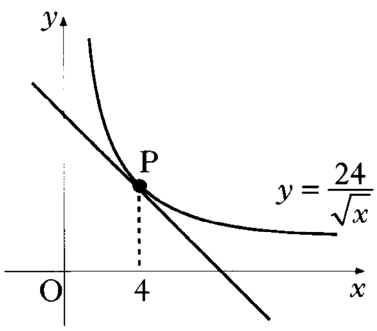
5. The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown.

Calculate the area enclosed between the curves. **8**



6. The diagram shows the graph of $y = \dfrac{24}{\sqrt{x}}$, $x > 0$.

Find the equation of the tangent at P, where $x = 4$. **6**



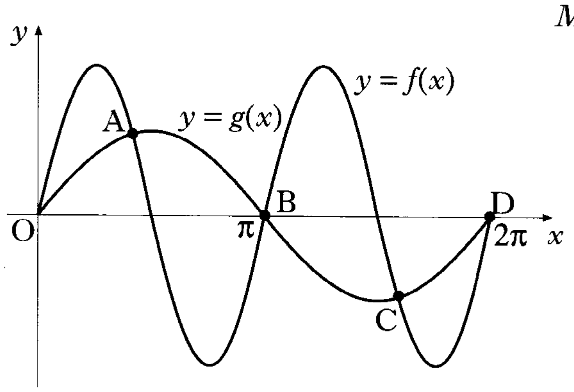
7. Solve the equation $\log_4(5 - x) - \log_4(3 - x) = 2$, $x < 3$. **4**

Marks

8. Two functions, f and g, are defined by $f(x) = k\sin 2x$ and $g(x) = \sin x$ where $k > 1$.

The diagram shows the graphs of $y = f(x)$ and $y = g(x)$ intersecting at O, A, B, C and D.

Show that, at A and C, $\cos x = \frac{1}{2k}$. **5**



9. The value V (in £ million) of a cruise ship t years after launch is given by the formula $V = 252e^{-0\cdot06335t}$.

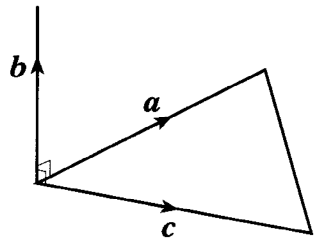
(a) What was its value when launched? **1**

(b) The owners decide to sell the ship once its value falls below £20 million. After how many years will it be sold? **4**

10. Vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ are represented by two sides of an equilateral triangle with sides of length 3 units, as shown in the diagram.

Vector $\boldsymbol{b}$ is 2 units long and $\boldsymbol{b}$ is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{c}$.

Evaluate the scalar product $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$. **4**



11. (a) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$. **1**

(b) Hence find the range of values of p for which all the roots of the cubic equation are real. **7**

[*END OF QUESTION PAPER*]