Higher Mathematics

144 Exam Multiple Choice Questions
**FORMULAE LIST**

**Circle:**
The equation \( x^2 + y^2 + 2gx + 2fy + c = 0 \) represents a circle centre \((-g, -f)\) and radius \( \sqrt{g^2 + f^2 - c} \).
The equation \((x - a)^2 + (y - b)^2 = r^2\) represents a circle centre \((a, b)\) and radius \( r \).

**Scalar Product:** \( \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta \), where \( \theta \) is the angle between \( \mathbf{a} \) and \( \mathbf{b} \)

or \( \mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \) where \( \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \) and \( \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \).

**Trigonometric formulae:**
\[
\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A
\]
\[
= 2 \cos^2 A - 1
\]
\[
= 1 - 2 \sin^2 A
\]

**Table of standard derivatives:**

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>( f'(x) )</th>
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<tr>
<td>( \sin ax )</td>
<td>( a \cos ax )</td>
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<td>( \sin ax )</td>
<td>( -\frac{1}{a} \cos ax + c )</td>
</tr>
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<td>( \cos ax )</td>
<td>( \frac{1}{a} \sin ax + c )</td>
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</tbody>
</table>
1. For which real value of \( x \) is the function \( f \) given by \( f(x) = \frac{1}{\sqrt{1-x^2}} \) defined on the set of real numbers?

A  all \( x \) except 1 and -1  
B  \( x < 1 \) only  
C  \( x > 1, x < -1 \) only  
D  \(-1 < x < 1 \) only

2. Which of the graphs shown below is most likely to be the graph with equation \( y = 3x^2 - 2x + 4 \)?

A  
B  
C  
D  
3. The diagram shows part of the graph of a function with equation $y = f(x)$.

Which of the following diagrams shows the graph with equation $y = f(3-x)$?

A

B

C

D
4. \( f(x) = 2x^2 - 4 \) and \( g(x) = 1 - x \) define functions on the set of real numbers.

What is the value of \( f(g(2)) \)?

A 4  
B 3  
C 0  
D -2

5. When \( 2x^2 - 12x + 13 \) is written in the form \( 2(x + q)^2 + r \), what is the value of \( r \)?

A 13  
B 1  
C -5  
D -13

6. A function \( f \) is given by \( f(x) = (x - 2)^2 - 3 \).

The function \( g \) is given by \( g(x) = \frac{1}{f(x) + 10} \).

Which of the following statements about the stationary value of \( g \) is true?

A minimum value of \( g \) is 7  
B maximum value of \( g \) is 7  
C minimum value of \( g \) is \( \frac{1}{7} \)  
D maximum value of \( g \) is \( \frac{1}{7} \)
7. The diagram shows the graph of the function \( f \) where \( f(x) = p(x - q)^2 + r \). The line \( x = 0 \) is an axis of symmetry of the curve. Which of the following is true about \( p, q \) and \( r \)?

![Graph of \( f(x) \)](graph.png)

A. \( p > 0, q > 0, r > 0 \)
B. \( p > 0, q = 0, r < 0 \)
C. \( p < 0, q = 0, r > 0 \)
D. \( p < 0, q < 0, r = 0 \)

8. The population of hamsters in a breeding centre increases by 5% during each month. At the end of each month the breeder sells 30 hamsters. If \( u_n \) represents the hamster population at the beginning of a month, find an expression for \( u_{n+1} \).

\[
A. \quad u_{n+1} = 1.5u_n + 30 \\
B. \quad u_{n+1} = 5u_n - 30 \\
C. \quad u_{n+1} = 1.05u_n - 30 \\
D. \quad u_{n+1} = 0.95u_n + 30
\]

9. A sequence is defined by the recurrence relation \( u_{n+1} = au_n + b \) and \( u_0 = 4 \). Express \( u_2 \) in terms of \( a \) and \( b \).

\[
A. \quad u_2 = 4a^2 + ab + b \\
B. \quad u_2 = 4 + 2b \\
C. \quad u_2 = 4a^2 + a^2b \\
D. \quad u_2 = 2a + b
\]
10. A sequence is defined by the recurrence relation \( u_{n+1} = 0.5u_n + 2 \) and \( u_0 = 8 \). Here are two statements about this sequence:

1. A limit exists for this sequence.
2. No term in the sequence is greater than 8.

Which of the following is true?

A neither statement is correct
B only statement (1) is correct
C only statement (2) is correct
D both statements are correct

11. A sequence is defined by the recurrence relation \( u_{n+1} = \frac{1}{3}u_n - 7 \) and \( u_0 = -2 \). What is the limit of this sequence as \( n \to \infty \)?

A \( \frac{-21}{2} \)
B \( \frac{7}{3} \)
C \( \frac{-1}{18} \)
D \( \frac{-1}{24} \)

12. A parabola has equation \( y = x^2 + 6x - 8 \). At what value of \( x \) does the minimum point of the parabola occur?

A \(-8\)
B \(-3\)
C \(0\)
D \(3\)

13. Find the solution of \( x^2 + x - 12 < 0 \).

A \( x < -4 \) or \( x > 3 \)
B \( x < -3 \) or \( x > 4 \)
C \(-4 < x < 3\)
D \(-3 < x < 4\)
14. Here are two statements about the equation \((x-3)^2 = 17\):

(1) the roots of the equation are real
(2) the roots of the equation are equal

Which of the following is true?

A neither statement is correct
B only statement (1) is correct
C only statement (2) is correct
D both statements are correct

15. The equation \(x^2 + 2x + p = 0\) has no real roots.
What is the range of values of \(p\) ?

A \(p < -1\)
B \(p < 0\)
C \(p > 0\)
D \(p > 1\)

16. The roots of a quadratic equation are \(-1\) and \(p\).
Which of the following could be the quadratic equation?

A \(x^2 + (1-p)x - p = 0\)
B \(x^2 - (1+p)x + p = 0\)
C \(x^2 + (1+p)x + p = 0\)
D \(x^2 + (p-1)x - p = 0\)

17. If \(x-1\) is a factor of \(x^3 - 6x^2 + px - 6\), what is the value of \(p\) ?

A \(-6\)
B \(-1\)
C \(1\)
D \(11\)
18. If \( \log(x) = 2\log(y) - 3\log(z) \), find an expression for \( x \) in terms of \( y \) and \( z \).

A \( x = 2y - 3z \)

B \( x = \frac{2y}{3z} \)

C \( x = \frac{y^2}{z^3} \)

D \( x = 2y + \frac{z}{3} \)

19. Given that \( \log_a(64) = \frac{3}{2} \), what is the value of \( a \)?

A 16

B \( 42 \frac{2}{3} \)

C 96

D 512

20. Given that \( \log_{10}(y) = 2\log_{10}(x) + \log_{10}(3) \), express \( y \) in terms of \( x \).

A \( y = 2x + 3 \)

B \( y = 6x \)

C \( y = 3x^2 \)

D \( y = 3 \times 2^x \)

21. The diagram shows the graph of \( \log_{10}(y) \) plotted against \( x \). The graph is a straight line through the origin with gradient 2.

What is the equation of this line?

A \( y = 2x \)

B \( y = 10^{2x} \)

C \( y = 10^{x^2} \)

D \( y = x^2 \)
22. If \( f(x) = 4x^3 + 5 \), what is the value of \( f(2) \)?

A 22  
B 26  
C 37  
D 48

23. If \( f(x) = 6x^3 - 2x^{-\frac{1}{2}} \) find \( f'(x) \).

A \( 18x^2 + x^{-\frac{3}{2}} \)  
B \( 2x^2 + 4x^{\frac{1}{2}} \)  
C \( 6x^2 - x^{-\frac{3}{2}} \)  
D \( 18x^2 + x^{\frac{1}{2}} \)

24. Given that \( f(x) = \frac{x^2 + 1}{x}, \ x \neq 0 \), find \( f'(x) \).

A \( 2x \)  
B \( 2x + 1 \)  
C \( 1 \)  
D \( 1 - \frac{1}{x^2} \)

25. The tangent to the curve with equation \( y = 2x^2 - 1 \) is drawn at the point where \( x = 0 \).

What is the gradient of this tangent?

A \(-1\)  
B \(0\)  
C \(1\)  
D \(2\)
26. The function \( f \) is defined by \( f(x) = 4x^3 - x^4 \), where \( x \) is a real number. What is the rate of change of \( f \) with respect to \( x \) at \( x = -1 \)?

A  -6  
B  -5  
C  5  
D  16

27. The graph of \( y = f(x) \) is shown with stationary points at \( x = 0.75, x = 1.5 \) and \( x = 3 \).

Here are two statements about \( f'(x) \):

(1) \( f(1) < 0 \)
(2) \( f(2) < 0 \)

Which of the following is true?

A  neither statement is correct  
B  only statement (1) is correct  
C  only statement (2) is correct  
D  both statements are correct

28. \( f(x) = ax^2 - 2x - 5 \) has a stationary value where \( x = 3 \). What is the value of \( a \)?

A  -1  
B  0  
C  \( \frac{1}{3} \)  
D  \( \frac{11}{9} \)
29. The diagram shows the graphs of two functions, \( f \) and \( g \). Here are two statements about the functions in the interval \( a \leq x \leq b \):

(1) Function \( f \) is differentiable for all values of \( x \)
(2) Function \( g \) is differentiable for all values of \( x \).

Which of the following is true?

A. neither statement is correct
B. only statement (1) is correct
C. only statement (2) is correct
D. both statements are correct

30. Find \( \int_{-1}^{1} x^4 \, dx \).

A. 0
B. \( \frac{1}{4} \)
C. \( \frac{2}{5} \)
D. 8
31. Find \( \int \left( 1 - x^{-\frac{1}{2}} \right) \, dx \)

A \( 2x^{\frac{1}{2}} + c \)
B \( x + 2x^{\frac{1}{2}} + c \)
C \( x - 2x^{\frac{1}{2}} + c \)
D \( x - 2x^{\frac{3}{2}} + c \)

32. Find \( \int \left( x^{4} + \frac{1}{x^{4}} \right) \, dx \)

A \( \frac{x^{5}}{5} - \frac{1}{3x^{3}} + c \)
B \( 4x^{3} - \frac{4}{x^{5}} + c \)
C \( \frac{x^{5}}{5} + \frac{1}{5x^{5}} + c \)
D \( \frac{x^{5}}{5} + \frac{1}{4x^{5}} + c \)

33. What is the value of \( \int_{-1}^{1} 3x^{2} \, dx \)?

A 20
B 24
C 28
D 32
34. Here are two statements about the numerical value of the shaded area shown in the diagram:

(1) Shaded area = \( 2 \int_{0}^{1} x \, dx \)

(2) Shaded area = \( \int_{-1}^{1} x \, dx \).

Which is of the following is true?

A neither statement is correct
B only statement (1) is correct
C only statement (2) is correct
D both statements are correct
35. The diagram shows the curves with equations \( y = x^2 \) and \( y = 4 - x^2 \).

Which of the following integrals gives the shaded area?

A \[\int_{0}^{4} \left(4 - 2x^2\right) dx.\]

B \[\int_{-2}^{2} \left(4 - 2x^2\right) dx.\]

C \[\int_{-\sqrt{2}}^{\sqrt{2}} \left(4 - 2x^2\right) dx.\]

D \[\int_{0}^{\sqrt{2}} \left(2x^2 - 4\right) dx.\]

36. If \( \frac{dy}{dx} = 2x + 1 \) and \( y = 3 \) when \( x = 1 \), express \( y \) in terms of \( x \).

A \( y = x^2 \)

B \( y = x^2 + x + 1 \)

C \( y = 2 \)

D \( y = x^2 + 2 \)
37. Given that $f(x) = \cos(3x^2 + 5)$, find $f'(x)$.

A $3\sin(3x^2 + 5)$
B $3\cos(3x^2 + 5)$
C $-\sin(6x)$
D $-6x\sin(3x^2 + 5)$

38. If $f(x) = (2x^2 - 1)^3$, find $f'(x)$.

A $\frac{1}{16x}(2x^2 - 1)^4$
B $12x(2x^2 - 1)^2$
C $48x^5$
D $48x^2$

39. Find $\int (4x - 1)^2 \, dx$.

A $\frac{1}{3}(2x^2 - x)^3 + c$
B $12(4x - 1)^3 + c$
C $\frac{1}{12}(4x - 1)^3 + c$
D $(2x^2 - x)^2 + c$
40. Find \( \int_{0}^{\frac{\pi}{2}} \cos 2x \, dx. \)

A \(-2\sqrt{2}\)
B \(\frac{1}{2}\)
C 0
D \(\sqrt{2}\)

41. What is the distance between the points \((-2, 5, 3)\) and \((4, -1, 1)\) ?

A 6
B 10
C \(2\sqrt{14}\)
D \(2\sqrt{19}\)

42. The line joining the points \((-2, -3)\) and \((6, k)\) has gradient \(\frac{2}{3}\). What is the value of \(k\) ?

A \(\frac{14}{3}\)
B \(\frac{7}{3}\)
C \(-\frac{1}{3}\)
D \(-9\)

43. A straight line passes through the points \(P(-5, -2)\) and \(Q(-2, -1)\). What is the equation of the straight line which passes through \(P\) and is perpendicular to \(PQ\) ?

A \(y + 2 = -3(x + 5)\)
B \(y - 2 = -\frac{1}{3}(x - 5)\)
C \(y - 1 = -\frac{1}{3}(x - 2)\)
D \(y - 1 = -\frac{1}{3}(x - 2)\)
44. The equation $ax + y + 4a = 0$ defines a set of straight lines for different values of $a$, where $a \neq 0$.

Here are two statements about this set of lines:

(1) All cut the $x$-axis at the same point

(2) They are parallel

Which of the following is true?

A neither statement is correct

B only statement (1) is correct

C only statement (2) is correct

D both statements are correct

45. P and Q are the points $(2, 3)$ and $(-1, 4)$.

What is the gradient of a line perpendicular to PQ?

A $\frac{8}{7}$

B 3

C 5

D 7

46. P is the point $(a, -2)$ and Q is $(0, b)$.

M(1, 2) is the midpoint of PQ.

What are the values of $a$ and $b$?

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<thead>
<tr>
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<th>$a$</th>
<th>$b$</th>
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<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>-6</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>-6</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

47. Triangle OPQ has vertices at $0(0, 0)$, $P(5, 3)$ and $Q(1, -7)$.

OS is a median. What are the coordinates of S?

A $(-5, -2)$

B $(3, -5)$

C $(3, -2)$

D $(2, 5)$
48. A circle has equation \( x^2 + y^2 = 4 - 4x + 2y \).
What is the radius of this circle?

A 2  
B 3  
C 4  
D 5  

49. PQ is a diameter of a circle.
P and Q have coordinates (3, 2) and (7, 2) respectively.
What is the equation of this circle?

A \( (x - 3)^2 + (y - 2)^2 = 16 \)  
B \( (x - 4)^2 + y^2 = 2 \)  
C \( (x + 5)^2 + (y + 2)^2 = 2 \)  
D \( (x - 5)^2 + (y - 2)^2 = 4 \)
50. The following diagrams each show a circle with centre Q(a, b) and radius 5 units, cutting the x and y axes in P and R respectively. In which diagram would the gradient of the tangent at P equal $\frac{-4}{3}$?

A

B

C

D
51. The line with equation \( y = k \) intersects the circle with equation \( x^2 + y^2 = 4 \) in at least one point.

What is the range of values of \( k \)?

A \(-2 \leq k \leq 2\)  
B \(-4 \leq k \leq 4\)  
C \(k \geq 2, \ k \leq -2\)  
D \(k \geq 4, \ k \leq -4\)

52. Given that \( u = \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix} \) and \( v = \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} \), what is the magnitude of \((u - v)\)?

A 1  
B \(\sqrt{20}\)  
C \(\sqrt{32}\)  
D \(\sqrt{34}\)

53. P, Q and R are points such that \( \overrightarrow{PQ} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \), \( \overrightarrow{PR} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \) and R is \((0, \ 2, \ 1)\).

What are the coordinates of Q ?

A \((−1, \ 3, \ 2)\)  
B \((−1, \ −1, \ 0)\)  
C \((1, \ 1, \ 0)\)  
D \((2, \ 0, \ 1)\)
54. The vector \( \mathbf{u} \) is given by \( \mathbf{u} = \frac{1}{4} i + pk \) where \( p > 0 \).

If \( \mathbf{u} \) is a unit vector, what is the value of \( p \)?

A  \( \frac{3}{4} \)
B  1
C  \( \frac{\sqrt{17}}{16} \)
D  \( \frac{\sqrt{15}}{4} \)

55. For what value of \( z \) are the vectors \( \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix} \) and \( \begin{pmatrix} 6 \\ -9 \\ z \end{pmatrix} \) parallel?

A  -18
B  -6
C  14
D  54

56. Given that \( \mathbf{p} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \), \( \mathbf{q} = \begin{pmatrix} 4 \\ -1 \\ -3 \end{pmatrix} \), and \( \mathbf{r} = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix} \), what are the components of \( \mathbf{p} - \mathbf{q} + 3\mathbf{r} \)?

A  \( \begin{pmatrix} -3 \\ 0 \\ -2 \end{pmatrix} \)
B  \( \begin{pmatrix} 5 \\ 0 \\ -8 \end{pmatrix} \)
C  \( \begin{pmatrix} 0 \\ 0 \\ 54 \end{pmatrix} \)
D  \( \begin{pmatrix} -3 \\ -2 \\ 10 \end{pmatrix} \)
57. The diagram shows a square PQRS where $\overrightarrow{SP} = u$ and $\overrightarrow{SR} = v$.

Express $\overrightarrow{ST}$ in terms of $u$ and $v$.

A $\overrightarrow{ST} = u + \frac{1}{2}v$
B $\overrightarrow{ST} = \frac{1}{2}u + \frac{1}{2}v$
C $\overrightarrow{ST} = u - \frac{1}{2}v$
D $\overrightarrow{ST} = \frac{1}{2}u - \frac{1}{2}v$

58. PQRS, KLMN is a cuboid as shown in the diagram. $\overrightarrow{SN} = u$, $\overrightarrow{SR} = v$ and $\overrightarrow{SP} = w$.

$T$ is the midpoint of KR.

Express $\overrightarrow{KT}$ in terms of $u$, $v$ and $w$.

A $\overrightarrow{KT} = -\frac{1}{2}u + \frac{1}{2}v - \frac{1}{2}w$
B $\overrightarrow{KT} = -u + v - w$
C $\overrightarrow{KT} = \frac{1}{2}u + \frac{1}{2}v + \frac{1}{2}w$
D $\overrightarrow{KT} = u - v + w$
59. The points A(1, 4, 2), B(3, 2, z) and C(7, y, -1) are collinear. What are the values of y and z?

<table>
<thead>
<tr>
<th></th>
<th>y</th>
<th>z</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>-3</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>-2</td>
<td>-3</td>
</tr>
<tr>
<td>D</td>
<td>-2</td>
<td>1</td>
</tr>
</tbody>
</table>

60. The point N divides the line LM in the ratio 3 : 1.

L has coordinates (-1, 1, 0) and \(\overrightarrow{LM} = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix}\).

What are the coordinates of N?

A \(\left( \frac{3}{2}, 2, 1 \right)\)
B \((2, 4, 3)\)
C \(\left( \frac{5}{2}, 4, 3 \right)\)
D \((5, 3, 4)\)

61. The components of vectors \(\mathbf{u}\) and \(\mathbf{v}\) are given by \(\mathbf{u} = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}\) and \(\mathbf{v} = \begin{pmatrix} 3 \\ -1 \\ -5 \end{pmatrix}\).

What is the value of \(\mathbf{u} \cdot \mathbf{v}\)?

A -10
B -3
C 3
D 5
62. The vectors \( \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \) and \( \begin{pmatrix} -5 \\ 2 \\ z \end{pmatrix} \) are perpendicular. What is the value of \( z \)?

A \( -1 \)  
B \( 0 \)  
C \( \frac{1}{4} \)  
D \( 4 \)

63. What is the angle between the vectors \( \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \) and \( \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \)?

A \( \frac{\pi}{6} \)  
B \( \frac{\pi}{4} \)  
C \( \frac{\pi}{3} \)  
D \( \frac{\pi}{2} \)

64. What is the value of \((i + 2j)(j + 2k)\)?

A \( 0 \)  
B \( 2 \)  
C \( 5 \)  
D \( 9 \)
65. Here are two statements about a stationary value for the function \( f(x) = 4\sin x - 2 \):

(1) \( f \) has a stationary value when \( x = \frac{\pi}{3} \)

(2) \( f \) has a stationary value when \( x = \frac{\pi}{2} \)

Which of the following is true?

A neither statement is correct
B only statement (1) is correct
C only statement (2) is correct
D both statements are correct

66. What is the exact value of \( \sin \frac{2\pi}{3} + \sin \frac{7\pi}{3} \)?

A 0
B 1
C \( \sqrt{3} \)
D 3

67. The diagram shows the graph of a trigonometric function.

Which of the following could be the equation of the graph?

A \( y = 1 + \sin x^\circ \)
B \( y = 1 - \sin x^\circ \)
C \( y = 2 - \cos x^\circ \)
D \( y = 2 \cos x^\circ - 1 \)
68. What is the minimum value of \(4\cos\left(x - \frac{\pi}{3}\right) + 6\)?

A 10  
B 9  
C 5  
D 2

69. Given that \(3\cos x^\circ + 4\sin x^\circ = 5\cos(x - 53.1)^\circ\), which of the following equations has a solution when \(x\) is a real number?

(1) \(3\cos x^\circ + 4\sin x^\circ = 2\)
(2) \(3\cos x^\circ + 4\sin x^\circ = 8\).

A neither equation has a solution  
B only equation (1) has a solution  
C only equation (2) has a solution  
D both equations have a solution

70. If \(\sin x^\circ = \frac{4}{5}\) and \(0 < x < 90\), what is the exact value of \(\sin 2x^\circ\)?

A \(\frac{17}{25}\)  
B \(\frac{8}{10}\)  
C \(\frac{24}{25}\)  
D \(\frac{6}{5}\)
71. The diagram shows an isosceles triangle with lengths as shown.

Express \(\sin 2t^\circ\) in terms of \(p, q\) and \(r\).

A \(\sin 2t^\circ = \frac{2q^2}{r^2}\)
B \(\sin 2t^\circ = \frac{2q}{r}\)
C \(\sin 2t^\circ = \frac{2p}{r}\)
D \(\sin 2t^\circ = \frac{2pq}{r^2}\)

72. If \(\sqrt{3} \cos x + \sin x = k \cos x \cos p + k \sin x \sin p\), where \(k > 0\), what is the value of \(k\) ?

A 1
B 2
C 3
D 4

73. A function \(f\) is defined by \(f(x) = 5 + 2\cos 3x\), where \(x\) is a real number. What is the range of \(f\) ?

A \(3 \leq f(x) \leq 7\)
B \(5 \leq f(x) \leq 7\)
C \(5 \leq f(x) \leq 11\)
D \(-1 \leq f(x) \leq 11\)
74. The graph with equation \( y = (x - 4)^2 + k \) passes through the point (3, 9). What are the coordinates of the stationary point of the graph?

A (4, 8)  
B (4, 9)  
C (4, 10)  
D (4, 11)

75. The diagram shows sketches of \( y = f(x) \) and \( y = kf(x) + c \).

What are the values of \( k \) and \( c \)?

<table>
<thead>
<tr>
<th></th>
<th>( k )</th>
<th>( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>-2</td>
</tr>
</tbody>
</table>
76. \( f(x) = 2x - 1 \) and \( g(x) = 2x + 1 \) are functions defined on the set of real numbers. Find an expression for \( f(g(x)) \).

A \( f(g(x)) = 4x^2 - 1 \)
B \( f(g(x)) = 4x^2 \)
C \( f(g(x)) = 4x \)
D \( f(g(x)) = 4x + 1 \)

77. When \( x^2 + 8x + 5 \) is expressed in the form \( (x + a)^2 + b \), what is the value of \( b \)?

A \(-59\)
B \(-11\)
C \(0\)
D \(5\)

78. A function \( f \) is given by \( f(x) = 4 - 2 \cos 3x \) on a suitable domain. What is the minimum value of \( f \)?

A \(1\)
B \(2\)
C \(6\)
D \(7\)

79. The diagram shows part of the graph of a cubic function.

What is the equation of this graph?

A \( y = 3(x + 2)^2(x - 1) \)
B \( y = (x + 2)(x - 1)^2 \)
C \( y = 3(x + 2)(x - 1)^2 \)
D \( y = (x + 2)(x - 1)(x + 1) \)
80. A fish farm starts with a stock of 5000 fish. Each Friday 30% of the fish are removed for sale and it is then restocked with 400 new fish. Let \( u_n \) represent the number of fish after restocking \( n \) times.

What is the recurrence relation that describes the situation after restocking?

A \[ u_{n+1} = 0 \cdot 3u_n + 400 \] and \( u_0 = 5000 \)
B \[ u_{n+1} = 0 \cdot 7u_n + 400 \] and \( u_0 = 5000 \)
C \[ u_{n+1} = 3(u_n + 400) \] and \( u_0 = 5000 \)
D \[ u_{n+1} = 7(u_n + 400) \] and \( u_0 = 5000 \)

81. A sequence is defined by the recurrence relation
\[ u_{n+1} = 3u_n - 7 \] and \( u_0 = 1 \).

What is the value of \( u_2 \)?
A \(-19\)
B \(-11\)
C \(-4\)
D \(-1\)

82. A sequence is generated by the recurrence relation
\[ 2u_{n+1} = ku_n + 7 . \]

What is the largest range of \( k \) for which the sequence has a limit?
A \(-0.5 < k < 0.5\)
B \(-1 < k < 1\)
C \(-2 < k < 2\)
D \(0 < k < 3\)

83. A sequence is defined by the recurrence relation
\[ u_{n+1} = 0 \cdot 6u_n + k \] and \( u_0 = 3 \).

As \( n \to \infty \), the limit of this sequence is 5.

What is the value of \( k \)?
A \(0\)
B \(0.88\)
C \(2\)
D \(8\)
84. The diagram shows the graph of a parabola.

What is the equation of this graph?

A \( y = \frac{1}{7}x^2 + \frac{1}{7}x - 6 \)
B \( y = x^2 + x - 12 \)
C \( y = \frac{1}{7}x^2 - \frac{1}{7}x - 6 \)
D \( y = 6x^2 + 6x - 72 \)

85. What is the solution of \( 2(x-3)(x+5) > 0 \) ?

A \( 2 < x < 5 \)
B \( x < -5, x > 3 \)
C \( -5 < x < 3 \)
D \( x < -3, x > 5 \)

86. The function \( g \) is given by \( g(x) = 4x^2 - 12x + 9 \).

Which condition describes the nature of the roots of \( g(x) = 0 \) ?

A Equal roots
B Exactly three distinct roots
C Exactly two distinct roots
D No real roots
87. The diagram shows part of the graph of a parabola with equation \( y = px^2 + qx + r \).
The \( x \)-axis is a tangent to the parabola.

What is the relationship between \( p \), \( q \) and \( r \)?

A \( q^2 = 4pr \)
B \( q^2 > 4pr \)
C \( q^2 < 4pr \)
D \( q^2 = -4pr \)

88. The diagram shows part of the graph of a cubic function.

What is the equation of this graph?

A \( y = 2(x - 2)(x - 1)(x + 3) \)
B \( y = 12(x - 2)(x - 1)(x + 3) \)
C \( y = -2(x - 3)(x + 1)(x + 2) \)
D \( y = 12(x - 3)(x + 1)(x + 2) \)
89. What is the remainder on dividing the polynomial $5x^3 - 4x + 8$ by $x - 2$?

   A $-24$
   B $0$
   C $8$
   D $40$

90. What is the value of $\frac{\log_3(8)}{\log_3(2)}$?

   A $\log_3(4)$
   B $\log_3(6)$
   C $4$
   D $3$

91. If $\log_y(x) = \frac{1}{4}$, what is the value of $x$?

   A $\sqrt{3}$
   B $\frac{9}{4}$
   C $\left(\frac{1}{4}\right)^9$
   D $\frac{3}{2}$

92. Given that $\log_{10}(x) = y \log_{10}(3) + 1$, express $x$ in terms of $y$.

   A $x = 10 \times 3^y$
   B $x = 30^{10y}$
   C $x = 3y + 10$
   D $x = y^3 + 10$
93. Given that \( y = kn^x \) where \( k \) and \( n \) are constants, what would you plot in order to get a straight line graph?

A. \( x \) against \( y \)
B. \( x \) against \( \log (y) \)
C. \( \log (x) \) against \( y \)
D. \( \log (x) \) against \( \log (y) \)

94. Given that \( f(x) = 2x^3 - 8x \), what is the value of \( f(-1) \)?

A. \(-2\)
B. \(0\)
C. \(\frac{4}{2} \)
D. \(6\)

95. If \( f(x) = 4x^{\frac{1}{2}} \), what is the value of \( f(4) \)?

A. \(-\frac{1}{4}\)
B. \(\frac{1}{4}\)
C. \(2\)
D. \(4\)

96. If \( f(x) = 3x^2(2x^3 + 4x - 1) \), find \( f(x) \).

A. \(30x^4 + 36x^2 - 6x\)
B. \(36x^3 + 24x\)
C. \(30x^4 + 12x^3 - 3x^2\)
D. \(x^6 - 3x^4 - x^3\)

97. At a point \( P \) on the curve \( y = 6 - 3x^2 \), the gradient is 6. What is the \( x \)-coordinate of \( P \)?

A. \(-102\)
B. \(-3\)
C. \(-1\)
D. \(6\)
98. A function $f$ is defined by $f(x) = (x - 2)^3$.
What is the rate of change of $f$ with respect to $x$ at $x = 3$?

A 0
B 1
C 3
D 19

99. Which of the functions shown satisfies the conditions $f'(x) < 0$ for $x < 0$ and $f'(x) > 0$ for $x > 0$, where $x$ is a real number and $x \neq 0$?
100. A function \( f \) is given by \( f(x) = (x - 1)(x + 5) \).

\( f \) has a stationary value when \( x = a \).

What is the value of \( a \) ?

A \( -5 \)  
B \( -2 \)  
C \( 0 \)  
D \( 1 \)

101. Which of the following could represent a function \( f \) such that \( f(0) = 0, f(1) = 0, f'(0) = 1 \) and \( f'(1) = 0 \)?
102. The graph of a function \( f \) passes through the point \((1, 5)\).

If \( f(x) = \int 3x^2 \, dx \), find an explanation for \( f(x) \).

A \( f(x) = x^3 - 1 \)
B \( f(x) = 6x + 5 \)
C \( f(x) = x^3 + 5 \)
D \( f(x) = x^3 + 4 \)

103. If \( f(x) = \frac{1}{\sqrt[4]{x^3}} \), what is \( f(x) \)?

A \( f(x) = \frac{1}{4} x^{-\frac{1}{3}} + c \)
B \( f(x) = 4x^{\frac{1}{4}} + c \)
C \( f(x) = -\frac{4}{7} x^{-\frac{7}{4}} + c \)
D \( f(x) = \frac{3}{4} x^{-\frac{7}{4}} + c \)

104. Find \( \int \frac{1}{5\sqrt{x}} \, dx \).

A \( \frac{2}{5} x^{\frac{1}{2}} + c \)
B \( \frac{5}{2} x^{\frac{1}{2}} + c \)
C \( -\frac{1}{10} x^{\frac{3}{2}} + c \)
D \( \frac{1}{10} x^{\frac{3}{2}} + c \)
105. What is the value of \( \int_0^3 (3x^2 + 4x) \, dx \)?

A 22  
B 31  
C 39  
D 45

106. In the diagram area P = 5 sq. units and area Q = 3 sq. units. Here are two statements relating to this diagram:

(1) \( \int_0^3 f(x) \, dx = 8 \)

(2) \( \int_2^3 f(x) \, dx = 3 \)

Which of the following is true?

A neither statement is correct  
B only statement (1) is correct  
C only statement (2) is correct  
D both statements are correct
107. The graphs of functions $f$ and $g$ are shown in the diagram.

Which of the following gives the area of the shaded section?

A \[ \int_{1}^{12} (g(x) - f(x)) \, dx \]

B \[ \int_{1}^{12} (f(x) - g(x)) \, dx \]

C \[ \int_{2}^{7} (g(x) - f(x)) \, dx \]

D \[ \int_{2}^{7} (f(x) - g(x)) \, dx \]
108. A curve passes through the point (2, 3). At every point on the curve \( \frac{dy}{dx} = 6x^2 \). What is the equation of the curve?

A \( y = 18x^3 - 141 \)
B \( y = 2x^3 - 13 \)
C \( y = 2x^3 \)
D \( y = 12x - 21 \)

109. If \( y = \sin 3x - \cos x \), what is \( \frac{dy}{dx} \)?

A \( -3 \cos 3x - \sin x \)
B \( 3 \cos 3x + \sin x \)
C \( \cos 3x - \sin x \)
D \( 3 \cos 2x + \sin x \)

110. If \( f(x) = (x^3 + 7)^2 \), find \( f'(x) \).

A \( \frac{1}{3} (x^3 + 7)^3 \)
B \( 6x^2 (x^3 + 7) \)
C \( 2(3x^2 + 7) \)
D \( 6x^2 \)

111. Find \( \int (4x+1)^{-\frac{1}{3}} \, dx \)

A \( 2 \left( 2x^2 + 1 \right)^{\frac{1}{3}} + c \)
B \( \frac{1}{2} (4x+1)^{\frac{1}{3}} + c \)
C \( \frac{1}{4} (4x+1)^{\frac{1}{3}} + c \)
D \( -\frac{8}{3} (4x+1)^{-\frac{2}{3}} + c \)
112. Find \( \int_{0}^{\pi} (1 + \cos x) \, dx \).

A. 1  
B. \( \pi - 2 \)  
C. 2  
D. \( \pi \)

113. The point \( P(7, 6) \) lies on a circle with centre \((-5, 1)\) as shown in the diagram.

What is the length of the diameter?

A. \( 2\sqrt{53} \) units  
B. \( 2\sqrt{111} \) units  
C. \( 2\sqrt{157} \) units  
D. 26 units

114. What is the exact value of \( \tan \frac{7\pi}{6} \)?

A. \( -\sqrt{3} \)  
B. \( -\frac{\sqrt{3}}{2} \)  
C. \( \frac{1}{\sqrt{3}} \)  
D. \( \sqrt{3} \)
115. A line L is parallel to the line with equation $4x + 2y = 6$ and passes through the point $(-3, 1)$. What is the equation of L?

A $y - 1 = -2(x - 3)$  
B $y - 1 = 4(x - 3)$  
C $y - 1 = -2(x + 3)$  
D $y + 3 = -2(x - 1)$

116. The lines with the equations $ax - 2y + 5 = 0$ and $3x + y - 4 = 0$ are parallel. What is the value of $a$?

A $-6$  
B $-2$  
C $-\frac{1}{3}$  
D $3$

117. A line L has equation $x + 3y + 7 = 0$. What is the gradient of a line perpendicular to L?

A $-\frac{4}{3}$  
B $-1$  
C $1$  
D $3$
118. A straight line passes through the points G, M and H where G = (−2, 5) and M = (4, −3). M is the midpoint of GH. What are the coordinates of H?

A (6, −8)  
B (6, 1)  
C (−6, 1)  
D (10, −11)

119. P and Q have coordinates (4, −7) and (−2, 5) respectively. The perpendicular bisector of PQ has a gradient of \( \frac{1}{2} \).

What is the equation of the perpendicular bisector of PQ?

A \( 2y = x − 3 \)  
B \( y = −2x + 1 \)  
C \( y = 2x + 3 \)  
D \( 2y = −x − 1 \)
120. Q is the centre of the circle with equation \( x^2 + y^2 + 2x - 4y - 15 = 0 \) and R(3, 4) lies on the circumference. What is the gradient of QR?

A \( \frac{1}{8} \)
B \( \frac{1}{2} \)
C 1
D \( \frac{8}{5} \)

121. The diagram shows a circle with the y-axis as a tangent. M and N have coordinates (0, −8) and (10, −8) and angle MKN equals 90°.

What is the equation of the circle passing through M, K and N?

A \( (x + 5)^2 + (y - 8)^2 = 100 \)
B \( (x - 10)^2 + (y + 8)^2 = 100 \)
C \( (x + 5)^2 + (y - 8)^2 = 5 \)
D \( (x - 5)^2 + (y + 8)^2 = 25 \)
122. The point $P(-3, 4)$ lies on the circle $x^2 + y^2 = 25$ as shown in the diagram.

What is the gradient of the tangent at $P$?

A $-\frac{4}{3}$
B $-\frac{1}{5}$
C $\frac{3}{4}$
D $\frac{5}{3}$

123. The line with the equation $y = 2x$ intersects the circle with equation $x^2 + y^2 = 1$ at the point $T$.
What is the $x$-coordinate of $T$?

A $\frac{1}{3}$
B $\frac{1}{\sqrt{6}}$
C $\frac{1}{\sqrt{5}}$
D $\frac{1}{2}$
124. What is the magnitude of the vector \( \mathbf{v} = -2\mathbf{i} + 5\mathbf{j} + \mathbf{k} \) ?

A  \(3\)  
B  \(4\)  
C  \(\sqrt{21}\)  
D  \(\sqrt{30}\)

125. P is the point \((1,2,3)\), \(\overrightarrow{PR}\) represents the vector \(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\) and \(\overrightarrow{RQ}\) represents the vector \(\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}\).

What are the coordinates of Q?

A  \((4, 3, 5)\)  
B  \((5, 4, 6)\)  
C  \((-2, 0, -1)\)  
D  \((3, 2, 4)\)

126. Vector \(\mathbf{p}\) has components \(\begin{pmatrix} 2 \\ \frac{\sqrt{5}}{5} \\ \frac{a}{5} \end{pmatrix}\), where \(a > 0\).

If \(\mathbf{p}\) is a unit vector, what is possible value of \(a\)?

A  \(\frac{3 - \sqrt{5}}{5}\)  
B  \(\frac{9}{25}\)  
C  \(\frac{3}{5}\)  
D  \(\frac{4}{5}\)
127. A vector $\mathbf{u}$ has components \[
\begin{pmatrix}
2 \\
-3 \\
6
\end{pmatrix}.
\]

What are the components of a unit vector parallel to $\mathbf{u}$?

A \[
\begin{pmatrix}
\frac{5}{2} \\
-\frac{5}{3} \\
\frac{5}{6}
\end{pmatrix}
\]

B \[
\begin{pmatrix}
\frac{2}{7} \\
-\frac{3}{7} \\
\frac{6}{7}
\end{pmatrix}
\]

C \[
\begin{pmatrix}
\frac{2}{11} \\
-\frac{3}{11} \\
\frac{6}{11}
\end{pmatrix}
\]

D \[
\begin{pmatrix}
\frac{4}{12} \\
-\frac{6}{12}
\end{pmatrix}
\]
128. Vector \( u \) and \( v \) are given by \( u = 2i + k \) and \( v = i - 3j + 4k \).

What are the components of vector \( 2u - v \)?

A \[
\begin{pmatrix}
6 \\
8 \\
-8
\end{pmatrix}
\]

B \[
\begin{pmatrix}
-1 \\
1 \\
-2
\end{pmatrix}
\]

C \[
\begin{pmatrix}
3 \\
3 \\
-2
\end{pmatrix}
\]

D \[
\begin{pmatrix}
4 \\
6 \\
-6
\end{pmatrix}
\]

129. The diagram shows a trapezium PQRS. PS is parallel to QR and \(|PS| = 3|QR|\).

\( \overrightarrow{PQ} \) and \( \overrightarrow{PS} \) represent vectors \( a \) and \( b \) respectively.

Express \( \overrightarrow{SR} \) in terms of \( a \) and \( b \).

A \( \overrightarrow{SR} = a \)

B \( \overrightarrow{SR} = a - \frac{2}{3}b \)

C \( \overrightarrow{SR} = -a + \frac{4}{3}b \)

D \( \overrightarrow{SR} = a - 4b \)
130. OABC,DEFG is a cuboid where $A$ is the point $(5, 0, 0)$ and $F$ is $(5, 3, 4)$, as shown in the diagram.

![Diagram of a cuboid](image)

What are the components of $\overrightarrow{AG}$?

- **A** \[
\begin{pmatrix}
-5 \\
3 \\
4
\end{pmatrix}
\]

- **B** \[
\begin{pmatrix}
3 \\
4 \\
0
\end{pmatrix}
\]

- **C** \[
\begin{pmatrix}
4 \\
-5 \\
-3
\end{pmatrix}
\]

- **D** \[
\begin{pmatrix}
5 \\
4 \\
3
\end{pmatrix}
\]
131. The diagram shows three collinear points P, Q and R where $3\overrightarrow{PQ} = 2\overrightarrow{PR}$.

What is the ratio in which Q divides PR?

A 2 : 1  
B 3 : 1  
C 3 : 2  
D 5 : 3

132. A is the point (1, 4, −2) and $\overrightarrow{AB} = \begin{pmatrix} -1 \\ -5 \\ 7 \end{pmatrix}$.

If $\overrightarrow{AC} = 3\overrightarrow{AB}$, what are the coordinates of C?

A (1, 1, 13)  
B (−3, −15, 21)  
C (−2, −11, 19)  
D (3, 15, −21)

133. Vectors $u$ and $v$ are defined by $u = i + 2j - 4k$ and $v = 3i + 2k$.

What is the value of $u \cdot v$?

A −5  
B −1  
C 0  
D 3

134. Vectors $u$ and $v$ are given $u = 2i - j + 5k$ and $v = 3i + pj - k$.

If $u$ and $v$ are perpendicular, what is the value of $p$?

A 1  
B 4  
C 7  
D 8
135. Vectors \( \mathbf{a} \) and \( \mathbf{b} \) are inclined at an angle of \( t \) radians to each other, as shown in the diagram.

![Diagram of vectors a and b inclined at angle t]

If \( \mathbf{a} \cdot \mathbf{b} = 2 \) and \( |\mathbf{a}| = |\mathbf{b}| = \sqrt{3} \) units, what is the value of \( \cos t \) ?

A  \(-1\)  
B  \(\frac{2}{3}\)  
C  \(\frac{2}{\sqrt{3}}\)  
D  \(\frac{3}{2}\)

136. Two vectors, \( \mathbf{a} \) and \( \mathbf{b} \), are perpendicular and \( |\mathbf{a}| = 2 \) units, \( |\mathbf{b}| = 3 \) units. What is the value of \( \mathbf{a} \cdot (\mathbf{a} + \mathbf{b}) \)?

A  0  
B  4  
C  7  
D  10
137. Which of the four graphs is most likely to show the graph of \( y = \cos 2x^\circ \) for \( 0 \leq x \leq 360^\circ \)?

A

B

C

D

138. If \( f(x) = 1 + \cos x \), what is the value of \( f\left(\frac{2\pi}{3}\right) \)?

A \( -\frac{\sqrt{3}}{2} \)

B \( -\frac{1}{2} \)

C \( \frac{1}{2} \)

D \( \frac{1}{\sqrt{3}} \)
139. The diagram shows part of the graph whose equation is of the form $y = a \sin bx$.

![Graph of $y = a \sin bx$.]

What is the equation of this graph?

A $y = -3 \sin \frac{1}{2}x$
B $y = 3 \sin \frac{1}{3}x$
C $y = -3 \sin 2x$
D $y = 3 \sin 2x$

140. The maximum value of $\int_0^{2\pi} \cos \left( x - \frac{\pi}{6} \right) dx$ occurs when $x = t$.

What is the value of $t$?

A 0
B $\frac{\pi}{6}$
C $\frac{2\pi}{3}$
D $\frac{7\pi}{6}$
141. What is the solution of the equation \( \sqrt{3} \sin x = -\cos x \) where \( 0 \leq x \leq \frac{3\pi}{2} \)?

A \( \frac{2\pi}{3} \)
B \( \frac{5\pi}{6} \)
C \( \frac{7\pi}{6} \)
D \( \frac{4\pi}{3} \)

142. Expand \( \cos \left( x + \frac{\pi}{4} \right) \).

A \( \cos \left( x + \frac{\pi}{4} \right) = \frac{\sqrt{2}}{2} \cos x - \frac{\sqrt{2}}{2} \sin x \)
B \( \cos \left( x + \frac{\pi}{4} \right) = \cos x + \frac{1}{\sqrt{2}} \)
C \( \cos \left( x + \frac{\pi}{4} \right) = \cos x - \frac{1}{\sqrt{2}} \)
D \( \cos \left( x + \frac{\pi}{4} \right) = \frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x \)

143. The diagram shows a right-angled triangle with side lengths of 2, \( \sqrt{21} \) and 5.

What is the exact value of \( \sin 2a \)?

A \( \frac{4}{5} \)
B \( \frac{17}{25} \)
C \( \frac{4\sqrt{21}}{25} \)
D \( \frac{2\sqrt{21}}{5} \)
144. \( k \) and \( a \) are given by
\[
\begin{align*}
\sin a & = 1 \\
\cos a & = 1 \\
\end{align*}
\]
where \( k > 0 \) and \( 0 \leq a \leq \frac{\pi}{2} \).

What are the values of \( k \) and \( a \)?

\[
\begin{array}{c|c|c}
\hline
k & a \\
\hline
\sqrt{2} & 0 & A \\
\sqrt{2} & \frac{\pi}{4} & B \\
2 & 0 & C \\
2 & \frac{\pi}{4} & D \\
\hline
\end{array}
\]