2019 P1 Q9

9. Vectors \( \mathbf{u} \) and \( \mathbf{v} \) have components \[
\begin{pmatrix}
  p \\
  -2 \\
  4
\end{pmatrix}
\quad \text{and} \quad
\begin{pmatrix}
  2p+16 \\
  -3 \\
  6
\end{pmatrix},
p \in \mathbb{R}.
\]

(a) (i) Find an expression for \( \mathbf{u} \cdot \mathbf{v} \).

(ii) Determine the values of \( p \) for which \( \mathbf{u} \) and \( \mathbf{v} \) are perpendicular.

(b) Determine the value of \( p \) for which \( \mathbf{u} \) and \( \mathbf{v} \) are parallel.

---

**Answers**

<table>
<thead>
<tr>
<th>Question</th>
<th>Generic scheme</th>
<th>Illustrative scheme</th>
<th>Max mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>9. (a) (i)</td>
<td>• form an expression</td>
<td>•(^1) ( p(2p+16) + (-2)(-3) + (4)(6) )</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>• equate scalar product to 0</td>
<td>•(^2) ( p(2p+16) + (-2)(-3) + (4)(6) = 0 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• factorise</td>
<td>•(^3) ( 2(p+5)(p+3) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• state values of ( p )</td>
<td>•(^4) (-5 \text{ and } -3 )</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>• interpret relationship</td>
<td>•(^5) ( 3(p) = 2(2p+16) ) or ( 3\mathbf{u} = 2\mathbf{v} ) or equivalent</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>• determine value of ( p )</td>
<td>•(^6) (-32 )</td>
<td></td>
</tr>
</tbody>
</table>

---