Read carefully

Calculators may **NOT** be used in this paper.

Section A – Questions 1 – 20 (40 marks)

Instructions for completion of Section A are given on page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks).

1. Full credit will be given only where the solution contains appropriate working.

2. Answers obtained by readings from scale drawings will not receive any credit.
SECTION A

ALL questions should be attempted.

1. The midpoint of the line joining $G(-1, 3, 7)$ to $H(5, -1, p)$ is $M(q, 1, 4)$. What are the values of $p$ and $q$?

2. Given that $f(x) = \frac{1}{3x^5}$, find $f'(x)$.

3. If $x^2 + 12x + 7$ is written in the form $(x + a)^2 + r$, find the value of $r$.

4. A straight line passes through the points $(4, 3)$ and $(0, -1)$. What is the equation of the line?

5. Functions $f$ and $g$ are defined on the set of real numbers by
   
   $f(x) = x^2 + 1$ and $g(x) = 3x - 5$

   What is the value of $g(f(-1))$?

6. The vectors with components $\begin{pmatrix} 4 \\ 7 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} -5 \\ t \\ -2 \end{pmatrix}$ are perpendicular. What is the value of $t$?

7. The diagram shows a right-angled triangle with sides 1, 3 and $\sqrt{10}$.
   What is the value of $\cos 2x$?

8. Find $\int_{-2}^{0} 6x^2 \, dx$

9. For what value of $k$ does the equation $2x^2 - 4x + k = 0$ have equal roots?
10. $\overrightarrow{DE}$ and $\overrightarrow{EF}$ have components \[
\begin{pmatrix} 5 \\ 2 \\ 3 \\
\end{pmatrix} \text{ and } \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}
\] respectively.

Given that $D$ has coordinates $(-2, 0, -2)$, what are the coordinates of $F$?

11. What is the maximum value of $8 - 3 \sin \left( x - \frac{7\pi}{9} \right)$?

12. Find $\int (2x + 5)^3 \, dx$.

13. How many solutions does the equation $(\sqrt{7} \cos x + 3)(4 \tan x - 9) = 0$ have in the interval $0 \leq x < 2\pi$?

14. Given that $f(x) = 4 \sin 3x$, find $f'(\frac{\pi}{6})$.

15. The diagram shows the line $ST$ with equation $2x + y = 0$.

The angle between $ST$ and the positive direction of the $x$-axis is $\theta$.

Find an expression for $\theta$.

A $\theta = \tan^{-1} \frac{1}{2}$  
B $\theta = \pi - \tan^{-1} \frac{1}{2}$  
C $\theta = \tan^{-1} 2$  
D $\theta = \pi - \tan^{-1} 2$

16. What is the value of $\log_2 32 - \log_2 8$?

17. The diagram shows a sketch of the curve with equation $y = k(x + 2)(x - 2)(x + a)$

What are the values of $a$ and $k$?

18. Here are two statements about the function $f(x) = \sqrt{x^2 - 4}$.

(1) The largest possible domain is $-2 \leq x \leq 2$.
(2) The range is $f(x) \geq 0$.

Which of these statements is true?

19. Given that
\begin{align*}
f'(x) &= \begin{cases} 
> 0, & \text{for } x < 3 \\
= 0, & \text{for } x = 3 \\
> 0, & \text{for } x > 3 
\end{cases} 
\end{align*}
Sketch a curve to represent \( y = f(x) \)?

20. If \( 5^x = a^2 \), find an expression for \( x \).

End of Section A
SECTION B

ALL questions should be attempted.

21. A(−2, 4), B(10, 4) and C(4, 8) are the vertices of triangle ABC shown in the diagram.

\[ \text{(a) Write down the equation of the altitude from C.} \]
\[ \text{(b) Find the equation of the perpendicular bisector of BC.} \]
\[ \text{(c) Find the point of intersection of the lines found in (a) and (b).} \]

22. P is the point (4, 1, −2), Q is (5, 2, 0) and R is (7, 4, 4).

\[ \text{(a) Show that P, Q and R are collinear.} \]
\[ \text{(b) Find the ratio in which Q divides PR.} \]
23. Find the equation of the tangent to the curve with equation
\[ y = \frac{4}{x} \]
at the point where \( x = 2 \).

24. (a) Given that \( f'(x) = 3x^2 + 2x - 10 \) and \( (x - 2) \) is a factor of \( f(x) \), find a formula for \( f(x) \).

(b) Hence factorise \( f(x) \) fully.

(c) Solve \( f(x) = 0 \).

25. The graph illustrates the law \( y = ax^b \).

The straight line joins the points \( (0, 4) \) and \( (1, 0) \).

Find the values of \( a \) and \( b \).
ALL questions should be attempted.

1. A sequence is defined by recurrence relation \( u_{n+1} = ku_n - 6, u_0 = 0 \).

   (a) Given that \( u_2 = -8 \), find the value of \( k \).  

   (b) (i) Why does this sequence tend to a limit as \( n \to \infty \) ?

      (ii) Find the value of this limit.

2. \( f(x) = 2x^3 + px^2 + qx + 4 \).

   Given that \( (x - 2) \) is a factor of \( f(x) \), and the remainder when \( f(x) \) is divided by \( (x + 1) \) is 9, find the values of \( p \) and \( q \).

3. Security guards are watching a parked car, via two CCTV cameras, in a supermarket car park.

   With reference to a suitable set of axes, the car is at \( C(5, 3, 2) \) and the cameras are at positions \( A(-1, 6, 4) \) and \( B(7, 9, 5) \) as shown.

   Calculate the size of angle \( ACB \).
4. Part of the graphs of \( y = 3 - x - x^2 \) and \( y = 5 - 2x^2 \) are shown opposite.

The curves intersect at the points S and T.

(a) Find the coordinates of S and T.

(b) Find the shaded area enclosed between the two curves.

5. A circle with centre \( C_1 \) has equation \( x^2 + y^2 - 2x - 6y - 15 = 0 \).

(a) Write down the coordinates of the centre and calculate the length of the radius of this circle.

A second circle with centre \( C_2 \) has a diameter twice that of the circle with centre \( C_1 \).

\( C_1 \) lies on the circumference of this second circle.

The line joining \( C_1 \) and \( C_2 \) is parallel to the x-axis.

(b) Find the equation of the circle with centre \( C_2 \).
6. A manufacturer of executive desks estimates that the weekly cost, in £, of making $x$ desks is given by $C(x) = x^3 - 6x^2 + 560x + 800$.


(a) Show that the weekly profit made from making $x$ desks is given by

$$P(x) = -x^3 + 6x^2 + 1440x - 800$$

(b) (i) How many desks would the manufacturer have to make each week in order to maximise his profit?

(ii) What would his annual profit be?

7. The number of bacteria, $b$, in a culture after $t$ hours is given by $b = b_0 e^{kt}$ where $b_0$ is the original number of bacteria present.

(a) The number of bacteria in a culture increases from 800 to 2400 in 2 hours.

Find the value of $k$ correct to 3 significant figures.

(b) How many bacteria, to the nearest hundred, are present after a further 4 hours?

8. (a) Express $2\cos x - 5\sin x$ in the form $k\cos(x + a)$, where $k > 0$ and $0 < a < 90$.

(b) (i) Hence write $2\cos 2x - 5\sin 2x$ in the form $R\cos(2x + b)$, where $R > 0$ and $0 < b < 90$.

(ii) Solve $2\cos 2x - 5\sin 2x = 5$ in the interval $0 \leq x < 360$.

End of Question Paper