1. The diagram shows a cuboid OPQR, STUV relative to the coordinate axes.

P is the point (4,0,0), Q is (4,2,0) and U is (4,2,3).

M is the midpoint of OR.

N is the point on UQ such that \( UN = \frac{1}{3} UQ \).

(a) State the coordinates of M and N.  
(b) Express \( VM \) and \( VN \) in component form.  
(c) Calculate the size of angle MVN.

2. Find \( \int \frac{4x^3-1}{x^2} \, dx, \ x \neq 0. \)

3. The value \( V \) (in £ million) of a cruise ship \( t \) years after launch is given by the formula \( V = 252e^{-0.06335t} \).  
(a) What was the value when it was launched?  
(b) The owners decide to sell the ship once its value falls below £20 million.  
After how many years will it be sold?
4. (a) (i) Show that the line with the equation \( y = 3 - x \) is a tangent to the circle with equation \( x^2 + y^2 + 14x + 4y - 19 = 0 \).

(ii) Find the coordinates of the point of contact P.

(b) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (a) and a second smaller circle with centre C.

The line \( y = 3 - x \) is a common tangent at the point P.

The radius of the larger circle is three times the radius of the smaller circle.

Find the equation of the smaller circle.

5. Two functions, \( f \) and \( g \), are defined by

\[ f(x) = k \sin 2x \quad \text{and} \quad g(x) = \sin x \]

where \( k > 1 \).

The diagram shows the graphs of \( y = f(x) \) and \( y = g(x) \) intersecting at O, A, B, C and D.

Show that, at A and C, \( \cos x = \frac{1}{2k} \).
6. The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.

A rectangle PQRS is placed between the two parabolas as shown, so that:

- Q and R lie on the upper parabola;
- RQ and SP are parallel to the $x$-axis;
- T, the turning point of the lower parabola lies on SP.

(a) (i) If $TP = x$ units, find an expression for the length of PQ.

(ii) Hence show that the area $A$, of rectangle PQRS is given by $A(x) = 12x - 2x^3$.

(b) Find the maximum area of this rectangle.

7. (a) Given that $\log_4 x = P$, show that $\log_{16} x = \frac{1}{2}P$.

(b) Solve $\log_3 x + \log_9 x = 12$. 
8. (a) A curve has equation \( y = (2x - 9)^{\frac{1}{3}} \).
   Show that the equation of the tangent to this curve at the point where \( x = 9 \) is
   \( y = \frac{1}{3}x \).

   (b) Diagram 1 shows part of the curve and the tangent.
   The curve cuts the \( x \)-axis at the point \( A \).

   \[ \text{Diagram 1} \]

   Find the coordinates of point \( A \).

   (c) Calculate the shaded area shown in diagram 2.

   \[ \text{Diagram 2} \]

9. (a) Show that \( x = 1 \) is a solution of the cubic equation \( x^3 + px^2 + px + 1 = 0 \)
   
   (b) Hence find the range of values of \( p \) for which all the roots of the cubic equation are real.