1. Triangle ABC has vertices A(4,0), B(-4,16) and C(18,20) as shown in the diagram opposite.

Medians AP and CR intersect at the point T(6, 12).

(a) Find the equation of median BQ
(b) Verify that T lies on BQ.
(c) Find the ratio in which T divides BQ.

2. Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$ are defined on the set of real numbers.

(a) Find $h(x)$ where $h(x) = g(f(x))$.
(b) (i) Write down the coordinates of the minimum turning point of $y = h(x)$.
(ii) Hence state the range of function $h$.

3. Two congruent circles, with centres A and B, touch at P. Relative to suitable axes, their equations are

$x^2 + y^2 + 6x + 4y - 12 = 0$ and

$x^2 + y^2 - 6x - 12y + 20 = 0$.

(a) Find the coordinates of P.
(b) Find the length of AB.
4. Differentiate \((1 + 2 \sin x)^4\) with respect to \(x\).

5. (a) The terms of a sequence satisfy \(u_{n+1} = ku_n + 5\). Find the value of \(k\) which produces a sequence with a limit of 4.

(b) A sequence satisfies the recurrence relation \(u_{n+1} = mu_n + 5, \ u_1 = 3\).

(i) Express \(u_1\) and \(u_2\) in terms of \(m\).

(ii) Given that \(u_2 = 7\), find the value of \(m\) which produces a sequence with no limit.

6. The function \(f\) is of the form \(f(x) = \log_b(x-a)\).

The graph of \(y = f(x)\) is shown in the diagram.

(a) Write down the values of \(a\) and \(b\).

(b) State the domain of \(f\).

7. A function \(f\) is defined by the formula \(f(x) = 2x^3 - 7x^2 + 9\) where \(x\) is a real number.

(a) Show that \((x-3)\) is a factor of \(f(x)\), and hence factorise \(f(x)\) fully.

(b) Find the co-ordinates of the points where the curve with equation \(y = f(x)\) crosses the x- and y-axes.

(c) Find the greatest and least values of \(f\) in the interval \(-2 \leq x \leq 2\).

8. (a) Express \(\sin x - \sqrt{3} \cos x\) in the form \(k \sin(x-a)\) where \(k > 0\) and \(0 \leq a \leq 2\pi\).

(b) Hence, or otherwise, sketch the curve with equation \(y = 3 + \sin x - \sqrt{3} \cos x\) in the interval \(0 \leq x \leq 2\pi\).
9. (a) Diagram 1 shows a right angled triangle, where the line OA has equation $3x - 2y = 0$.

(i) Show that $\tan a = \frac{3}{2}$.

(ii) Find the value of $\sin a$.

(b) A second right angled triangle is added as shown in Diagram 2.

The line OB has equation $3x - 4y = 0$.

Find the values of $\sin b$ and $\cos b$.

(i) Find the value of $\sin(a - b)$.

(ii) State the value of $\sin(b - a)$.

[END OF QUESTION PAPER]
The diagram shows a cuboid OPQR, STUV relative to the coordinate axes.

P is the point (4,0,0), Q is (4,2,0) and U is (4,2,3).

M is the midpoint of OR.
N is the point on UQ such that \( UN = \frac{1}{3} UQ \).

(a) State the coordinates of M and N.

(b) Express \( \vec{VM} \) and \( \vec{VN} \) in component form.

(c) Calculate the size of angle MVN.

2. Find \( \int 4x^3 - 1 \, dx, \; x \neq 0 \).

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3. The value \( V \) (in £ million) of a cruise ship \( t \) years after launch is given by the formula \( V = 252e^{-0.06335t} \).

(a) What was the value when it was launched?

(b) The owners decide to sell the ship once its value falls below £20 million.

After how many years will it be sold?

1 4
4. (a) (i) Show that the line with the equation \( y = 3 - x \) is a tangent to the circle with equation \( x^2 + y^2 + 14x + 4y - 19 = 0 \).

(ii) Find the coordinates of the point of contact \( P \).

(b) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (a) and a second smaller circle with centre \( C \).

![Diagram of circles and point P]

The line \( y = 3 - x \) is a common tangent at the point \( P \).

The radius of the larger circle is three times the radius of the smaller circle.

Find the equation of the smaller circle.

5. Two functions, \( f \) and \( g \), are defined by

\[
 f(x) = k \sin 2x \quad \text{and} \quad g(x) = \sin x \quad \text{where} \quad k > 1.
\]

The diagram shows the graphs of \( y = f(x) \) and \( y = g(x) \) intersecting at \( O, A, B, C \) and \( D \).

Show that, at \( A \) and \( C \), \( \cos x = \frac{1}{2k} \).
6. The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.

A rectangle PQRS is placed between the two parabolas as shown, so that:

- Q and R lie on the upper parabola;
- RQ and SP are parallel to the $x$-axis;
- T, the turning point of the lower parabola lies on SP.

(a) (i) If $TP = x$ units, find an expression for the length of PQ.

(ii) Hence show that the area $A$, of rectangle PQRS is given by

$$A(x) = 12x - 2x^3.$$ 

(b) Find the maximum area of this rectangle.

7. (a) Given that $\log_4 x = P$, show that $\log_{16} x = \frac{1}{2}P$.

(b) Solve $\log_3 x + \log_9 x = 12$. 

8. (a) A curve has equation \( y = (2x - 9)^{\frac{1}{2}} \).
Show that the equation of the tangent to this curve at the point where \( x = 9 \) is
\( y = \frac{1}{3}x \).

(b) Diagram 1 shows part of the curve and the tangent.
The curve cuts the \( x \)-axis at the point A.

![Diagram 1](image1)

Find the coordinates of point A.

(c) Calculate the shaded area shown in diagram 2.

![Diagram 2](image2)

9. (a) Show that \( x = 1 \) is a solution of the cubic equation \( x^3 + px^2 + px + 1 = 0 \).

(b) Hence find the range of values of \( p \) for which all the roots of the cubic equation are real.