Marking Instructions

These Marking Instructions have been provided to show how SQA would mark this Specimen Question Paper.

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General Marking Principles for Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the detailed marking instructions, which identify the key features required in candidate responses.

(a) Marks for each candidate response must always be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) Credit must be assigned in accordance with the specific assessment guidelines.

(d) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(e) Working subsequent to an error must be followed through, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working is easier, candidates lose the opportunity to gain credit.

(f) Where transcription errors occur, candidates would normally lose the opportunity to gain a processing mark.

(g) Scored out or erased working which has not been replaced should be marked where still legible. However, if the scored out or erased working has been replaced, only the work which has not been scored out should be judged.

(h) Unless specifically mentioned in the specific assessment guidelines, do not penalise:
   • Working subsequent to a correct answer
   • Correct working in the wrong part of a question
   • Legitimate variations in solutions
   • Repeated error within a question

Definitions of Mathematics-specific command words used in this Specimen Question Paper.

Determine: find a numerical value or values from the information given.

Expand: multiply out an algebraic expression by making use of the distributive law or a compound trigonometric expression by making use of one of the addition formulae for \( \sin(A \pm B) \) or \( \cos(A \pm B) \).

Show that: use mathematics to prove something, eg that a statement or given value is correct — all steps, including the required conclusion, must be shown.

Express: use given information to rewrite an expression in a specified form.

Hence: use the previous answer to proceed.

Hence, or otherwise: use the previous answer to proceed; however, another method may alternatively be used.

Justify: show good reason(s) for the conclusion(s) reached.
### Question 1 (a)

<table>
<thead>
<tr>
<th>Marking Instruction</th>
<th>Max Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. find coordinates of E</td>
<td>2</td>
</tr>
<tr>
<td>2. find coordinates of F</td>
<td></td>
</tr>
</tbody>
</table>

**Illustration of evidence for awarding a mark at each**

- E(45, 45, 40)
- F(60, 40, 0)

**Ans:**
- $-1750$
- $\overrightarrow{ED} = (-15, -15), \overrightarrow{EF} = (15, 40, -40)$
- $\overrightarrow{ED} \cdot \overrightarrow{EF} = -225 + 75 - 1600 = -1750$

### Question 1 (b)

<table>
<thead>
<tr>
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### Question 1 (c)

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**Illustration of evidence for awarding a mark at each**

- $-1750$
- $\overrightarrow{ED} = (-15, -15), \overrightarrow{EF} = (15, 40, -40)$
- $\overrightarrow{ED} \cdot \overrightarrow{EF} = -225 + 75 - 1600 = -1750$

**Ans:** $154^\circ$
- $\cos DEF = \frac{\overrightarrow{ED} \cdot \overrightarrow{EF}}{\left| \overrightarrow{ED} \right| \left| \overrightarrow{EF} \right|}$
- $\left| \overrightarrow{ED} \right| = \sqrt{2050}$
- $\left| \overrightarrow{EF} \right| = \sqrt{1850}$
- $\cos DEF = \frac{-1750}{\sqrt{2050}\sqrt{1850}}$
- $DEF = 153.977... = 154^\circ$

### Question 2 (a)

<table>
<thead>
<tr>
<th>Marking Instruction</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1. set up one equation</td>
<td>4</td>
</tr>
<tr>
<td>2. set up second equation</td>
<td></td>
</tr>
<tr>
<td>3. solve for one variable</td>
<td></td>
</tr>
<tr>
<td>4. solve for second variable</td>
<td></td>
</tr>
</tbody>
</table>

**Illustration of evidence for awarding a mark at each**

- $2500 = 2000a + b$
- $2980 = 2500a + b$
- $480 = 500a$ or $12500 = 10000a + 5b$
- $a = \frac{480}{500}$ or $11920 = 10000a + 4b$
- $580 = b$
- $a = 0.96$
- $b = 2500 - 2000 (0.96)
- $b = 2500 - 1920$
- $b = 580$
- $2000a = 2500 - 580$
- $a = \frac{1920}{2000}$
- $a = 0.96$

**Ans:** $a = 0.96, b = 580$
<table>
<thead>
<tr>
<th>Question</th>
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<th>Max mark</th>
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<tbody>
<tr>
<td>(b)</td>
<td>Ans: Yes. Stabilises at 14 500</td>
<td>3</td>
<td>•5 $u_{n+1} = 0.96u_n + 580, \quad -1 &lt; a &lt; 1$</td>
</tr>
<tr>
<td></td>
<td>•2 $L = \frac{b}{1-a}$</td>
<td></td>
<td>•6 $L = 14 500$</td>
</tr>
<tr>
<td></td>
<td>•3 $L = \frac{580}{1-0.96}$</td>
<td></td>
<td>•7 yes, conservation measures will end, since the predicted population stabilises at 14 500 and 14 500 &gt; 13 000</td>
</tr>
<tr>
<td>3</td>
<td>(a) Ans: $p = 1, q = 4$</td>
<td>1</td>
<td>•1 $p = 1, q = 4$</td>
</tr>
<tr>
<td></td>
<td>•1 state values of $p$ and $q$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>Ans: $y = 9(x - 1)$</td>
<td>4</td>
<td>•2 $f(x) = x^4 - 9x^3 + 24x^2 - 16x$</td>
</tr>
<tr>
<td></td>
<td>•2 expand brackets</td>
<td></td>
<td>•3 $f'(x) = 4x^3 - 27x^2 + 48x - 16$</td>
</tr>
<tr>
<td></td>
<td>•3 differentiate</td>
<td></td>
<td>•4 $f'(1) = 4 - 27 + 48 - 16 = 9$</td>
</tr>
<tr>
<td></td>
<td>•4 calculate gradient of tangent</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>•5 substitutes gradient and (1,0) into equation of line</td>
<td></td>
<td>•5 $y = 9(x - 1)$</td>
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</table>
| **4 (a)**| Ans: $y = \log_{4} x + \frac{1}{2}$  
• 1 using law of logarithms  
• 2 evaluating $\log_{4} 2$ | 2 |  
• 1 $\log_{4} 2x = \log_{4} 2 + \log_{4} x$  
• 2 $\log_{4} 2 = \frac{1}{2}$ |
| (b) | Ans: Graph of $y = \log_{4} x$ moved up by $\frac{1}{2}$  
or graph of $y = \log_{4} x$ compressed horizontally by a factor of 2.  
• 3 valid description of relationship | 1 |  
• 3 valid description — see answer |
| (c) | Ans: $x = \frac{1}{2}$  
• 4 setting $y = 0$  
• 5 solving for $x$ | 2 |  
• 4 $\log_{4} 2x = 0$  
• 5 $x = \frac{1}{2}$ |
| (d) | Ans:  
• 6 reflecting $y = \log_{4} 2x$ in the line $y = x$  
• 7 correct shape  
• 8 annotating (2 points) (or other valid method) | 3 |  
• 6 reflect in $y = x$  
• 7  
• 8 $\left(0, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, 1\right)$ |
<table>
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### Question 5

**(a)**

**Ans:**

\[ y - 1 = -2(x - 3) \]

- **1** calculate midpoint of AB
- **2** calculate gradient of AB
- **3** state gradient of perpendicular bisector
- **4** substitute into equation of line

**Max mark:** 4

- **1** \((3, 1)\)
- **2** \(\frac{1}{2}\)
- **3** -2
- **4** \(y - 1 = -2(x - 3)\)

### Question 5

**(b)**

**Ans:**

\[(x - 2)^2 + (y - 3)^2 = 25\]

- **5** knowing and using \(y = 3\)
- **6** solving for \(x\)
- **7** identifying the radius
- **8** obtain circle equation

**Max mark:** 4

- **5** \(3 = -2x + 7\)
- **6** \(x = 2\)
- **7** \(r = 5\)
- **8** \((x - 2)^2 + (y - 3)^2 = 25\)

### Question 6

**Ans:** \(x = -3\)

- **1** use perpendicular property
- **2** find \(\overrightarrow{CD}\)
- **3** find \(\overrightarrow{AB}\)
- **4** correct substitution into scalar product formula
- **5** calculates value of \(x\)

**Max mark:** 5

- **1** If \(\overrightarrow{CD}\) is perpendicular to \(\overrightarrow{AB}\) then \(\overrightarrow{CD}.\overrightarrow{AB} = 0\)
- **2**
  \[
  \begin{pmatrix}
  x - 4 \\
  -3 \\
  -1 
  \end{pmatrix}
  \]
- **3**
  \[
  \begin{pmatrix}
  5 \\
  -10 \\
  -5 
  \end{pmatrix}
  \]
- **4** \(5(x - 4) + (-10)(-3) + (-5)(-1) = 0\)
- **5** \(x = -3\)
<table>
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<tr>
<td>7</td>
<td>Ans: A False and B True • 1 valid reason for statement A • 2 selecting true or false for statement A with valid reason • 3 setting $P(t) = 15$ • 4 taking log to base $e$ • 5 completing valid reason • 6 selecting true or false for statement B with valid reason</td>
<td>6</td>
<td>• 1 $P(0) = 30e^{-2} = 4.06$ • 2 false, since $P(0) \neq 30$ (do not award without valid reason) • 3 $15 = 30e^{-2}$ • 4 $\ln e^{-2} = \ln 0.5$ • 5 $t - 2 = \ln 0.5$ $t = \ln 0.5 + 2$ (1.3) • 6 true, since $t = 1.3$ to one decimal place and there is only one solution (do not award without valid reason)</td>
</tr>
</tbody>
</table>

Notes: Substituting $t = 1.3$ into $P(t) = 30e^{-t^2}$ is not sufficient to show that statement B is true, since it does not prove that $t = 1.3$ is the only solution.

8 (a) • 1 know to equate volume to 100 • 2 obtain an expression for $h$ • 3 complete area evaluation | 3        | • 1 $V = \pi r^2 h = 100$ • 2 $h = \frac{100}{\pi r^2}$ • 3 $A = \pi r^2 + 2\pi r^2 + 2\pi r \times \frac{100}{\pi r^2}$ |

8 (b) Ans: $r = 2.20$ m • 4 know to and start to differentiate • 5 complete differentiation • 6 set derivative to zero • 7 obtain $r$ • 8 justify nature of stationary point • 9 interpret result | 6        | • 4 $A'(r) = 6\pi r...$ • 5 $A'(r) = 6\pi r - \frac{200}{r^2}$ • 6 $6\pi r - \frac{200}{r^2} = 0$ • 7 $r = 2.20$ metres • 8 $A''(r) = 6\pi + \frac{400}{r^3} \Rightarrow A''(2.1974...) = 56.5...$ • 9 minimum (when $r = 2.20$ m) |

Notes: Candidates may use a nature table at • 8 to justify a minimum turning point when $r = 2.1974...$
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<tr>
<td>9</td>
<td>Ans: $\frac{5}{6}$</td>
<td>6</td>
<td>$\int (\frac{3}{4}x - \frac{3}{2} \pi)dx - \int (\cos(2x)dx$ $\frac{\pi}{2}$ $\frac{\pi}{2}$ $\frac{\pi}{2}$ $\int \left[-\frac{\pi}{4} \cos(\frac{\pi}{4}x - \frac{\pi}{2})\right] \ldots$ $\frac{\pi}{6}$ $(\frac{\pi}{6} - 0) - (\frac{\pi}{6} - 0) = \frac{\pi}{6}$ $([-\frac{\pi}{4} \cos(\frac{\pi}{4} \times \frac{\pi}{2} - \frac{\pi}{2})] - [-\frac{\pi}{4} \cos(0 - \frac{\pi}{2})]) - ([\frac{\pi}{4} \sin(2 \times \frac{\pi}{4})] - [\frac{\pi}{4} \sin(2 \times 0)])$</td>
</tr>
<tr>
<td>10 (a)</td>
<td>Ans: $k = 2, \alpha = \frac{\pi}{6}$ or equivalent</td>
<td>4</td>
<td>$\sin t + \sqrt{3} \cos t = k \cos (t - \alpha)$ or equivalent $k \cos \alpha \cos t + k \sin \alpha \sin t$ or equivalent $k \sin \alpha = 1, k \cos \alpha = \sqrt{3}$ or equivalent $k = 2, \alpha = \frac{\pi}{6}$ or equivalent</td>
</tr>
<tr>
<td></td>
<td>Ans: 5.9 $\cos \left(\frac{t - \pi}{6}\right) = 1.2$ or equivalent $\cos \left(\frac{t - \pi}{6}\right) = 0.6$ or equivalent $t \frac{\pi}{6} = 0.927 \ldots \ &amp; 5.355... $t \frac{\pi}{6} = 1.45... \ &amp; \frac{\pi}{6} 5.879...</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

[END OF SPECIMEN MARKING INSTRUCTIONS]