H

National
Qualifications
SPECIMEN ONLY

SQ30/H/01

Mathematics
Paper 1
(Non-Calculator)

Date — Not applicable

Duration — 1 hour and 10 minutes

Total marks — 60

Attempt ALL questions.

You may NOT use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the answer booklet provided. In the answer booklet you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not you may lose all the marks for this paper.

SQ30H01

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$, where θ is the angle between **a** and **b**

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

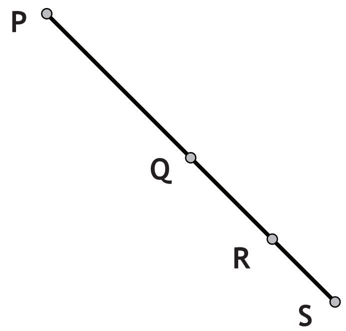
Attempt ALL questions **MARKS**

Total marks — 60

1. Find $\int \frac{3x^3+1}{2x^2}\, dx,\ x \neq 0$. **4**

2. Find the coordinates of the points of intersection of the curve $y = x^3 - 2x^2 + x + 4$ and the line $y = 4x + 4$. **5**

3. In the diagram, P has coordinates $(-6, 3, 9)$,



$\overrightarrow{PQ} = 6\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}$ and $\overrightarrow{PQ} = 2\overrightarrow{QR} = 3\overrightarrow{RS}$.

Find the coordinates of S. **5**

4. Given that $2x^2 + px + p + 6 = 0$ has no real roots, find the range of values for p, where $p \in \mathbb{R}$. **4**

5. Line l_1 has equation $\sqrt{3}y - x = 0$.

(a) Line l_2 is perpendicular to l_1. Find the gradient of l_2. **2**

(b) Calculate the angle l_2 makes with the positive direction of the x-axis. **2**

6. (a) Find an equivalent expression for $\sin(x + 60)^\circ$. **1**

(b) Hence, or otherwise, determine the exact value of $\sin 105^\circ$. **3**

7. (a) Show that $(x + 1)$ is a factor of $x^3 - 13x - 12$. **3**

(b) Factorise $x^3 - 13x - 12$ fully. **2**

MARKS

8. $f(x)$ and $g(x)$ are functions, defined on the set of real numbers, such that

$f(x) = 1 - \frac{1}{2}x$ and $g(x) = 8x^2 - 3$.

(a) Given that $h(x) = g(f(x))$, show that $h(x) = 2x^2 - 8x + 5$. **3**

(b) Express $h(x)$ in the form $a(x+p)^2 + q$. **3**

(c) Hence, or otherwise, state the coordinates of the turning point on the graph of $y = h(x)$. **1**

(d) Sketch the graph of $y = h(x) + 3$, showing clearly the coordinates of the turning point and the y-axis intercept. **2**

9. (a) AB is a line parallel to the line with equation $y + 3x = 25$.

A has coordinates $(-1, 10)$.

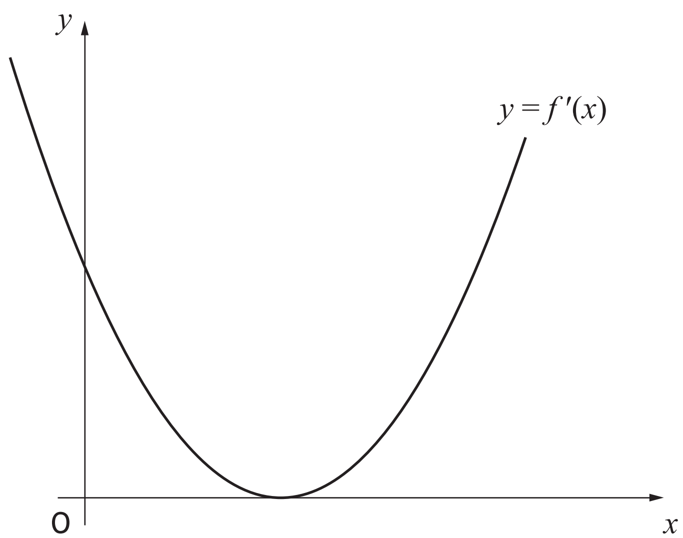
Find the equation of AB. **1**

(b) $3y = x + 11$ is the perpendicular bisector of AB.

Determine the coordinates of B. **5**

10. Find the rate of change of the function $f(x) = 4\sin^3 x$ when $x = \frac{5\pi}{6}$. **3**

11. The diagram shows the graph of $y = f'(x)$. The x-axis is a tangent to this graph.



(a) Explain why the function $f(x)$ is never decreasing. **1**

(b) On a graph of $y = f(x)$, the y-coordinate of the stationary point is negative. Sketch a possible graph for $y = f(x)$. **2**

MARKS

12. The voltage, $V(t)$, produced by a generator is described by the function $V(t) = 120\sin 100\pi t,\ t > 0$, where t is the time in seconds.

(a) Determine the period of $V(t)$. **2**

(b) Find the first three times for which $V(t) = -60$. **6**

[END OF SPECIMEN QUESTION PAPER]