Ex 10

1. \( y = 2x \)
   
   \( 2y + x = 5 \)
   
   but \( y = 2x \)
   
   \( y = 2.1 \)
   
   \( y = 2 \).
   
   \[ \text{Sub } y = 2x \text{ into } \]
   
   \[ 2y + x = 5 \]
   
   \[ 2(2x) + x = 5 \]
   
   \[ 4x + x = 5 \]
   
   \[ 5x = 5 \]
   
   \[ x = 1. \]

   Vertex has coords \((1, 2)\)

2. (a) Eq of \( PQ \) is \( 2x - 5y - 1 = 0 \)
   
   \( 5y = 2x - 1 \)
   
   \( y = \frac{2}{5}x - \frac{1}{5} \)
   
   \[ m_{PQ} = \frac{2}{5} \]

   \[ \Rightarrow m_{\text{perpendicular}} = -\frac{5}{2} \]

   \( A(1, 6) \)

   \[ y - 6 = m(x - a) \]

   \[ y - 6 = -\frac{5}{2}(x - 1) \]

   \[ 2y - 12 = -5x + 5 \]

   (b) Solve

   Eq of line

   \[ 5x + 2y - 17 = 0 \]

   Eq of line

   \[ 5x + 2y - 17 = 0 \]

   \[ 2x - 5y - 1 = 0 \]

   \[ 25x + 10y - 85 = 0 \]

   \[ 4x - 10y - 2 = 0 \]

   \[ 29x = 87 \]

   \[ x = 3 \]

   Sub \( x = 3 \) into \( 2x - 5y - 1 = 0 \)

   \[ 2(3) - 5y - 1 = 0 \]

   \[ 6 - 5y - 1 = 0 \]

   \[ 5 - 5y = 0 \]

   \[ y = 1 \]

   Line meets \( PQ \) at \((3, 1)\)
3. (a) \[ AB = \sqrt{(4-1)^2 + (8-2)^2} \]
\[ = \sqrt{3^2 + 6^2} \]
\[ = \sqrt{9 + 36} \]
\[ = \sqrt{45} \]

\[ BC = \sqrt{(1-7)^2 + (2-2)^2} \]
\[ = \sqrt{(-6)^2 + 0^2} \]
\[ = \sqrt{36} \]

\[ AC = \sqrt{(4-7)^2 + (8-2)^2} \]
\[ = \sqrt{(-3)^2 + 6^2} \]
\[ = \sqrt{9 + 36} \]

Since \( AB \neq BC \)

\( \triangle ABC \) is **isosceles**

(b)(i) \( D \) lies on \( BC \)

Since \( \triangle ABC \) is isosceles, \( D \) will be \((4, 2)\)

\( AD \) has equation \( x = 4 \)

\( E \) lies on \( CA \)

\[ m_{AC} = \frac{2-8}{7-4} = \frac{-6}{3} = -2 \]

\[ \Rightarrow m_{BE} = \frac{1}{2} \]

Eq of \( BE \)

\( B (1, 2) \)

To find \# solve

\[ x = 4 \]
\[ x - 2y + 3 = 0 \]

Sub \( x = 4 \) into \( x - 2y + 3 = 0 \)

\[ 4 - 2y + 3 = 0 \]
\[ 7 - 2y = 0 \]
\[ 2y = 7 \]
\[ y = 3.5 \]

\( H \) is point \((4, 3.5)\)

(2) \( DA = 8 - 2 = 6 \)

\( A(4, 8) \)

\( DH = 3.5 - 2 = 1.5 \)

\( D(4, 2) \)

\( DA = 6 \)

\( DH = 1.5 \)

\( 4 \times DH = 4 \times 1.5 = 6 = DA \)

\( H(4, 3.5) \)

\( 4 \times DH = 4 \times 1.5 = 6 = DA \).
Ex10
4. (a) Eq of OC is \( y = 6x \)
\[ m_{OC} = 6 \]
\[ \Rightarrow m_{AB} = 6 \]
Eq of AB
\[ B(3, 7) \]
\( y - 7 = m(x - 3) \)
\[ y = 6x - 18 \]
\[ y = 6x - 18 + 7 \]
\[ 6x - y - 11 = 0 \]
Eq of AO is \( y = -5x \)
\[ m_{AO} = -5 \]
\[ \Rightarrow m_{BC} = -5 \]
\[ B'(3, 7) \]
Equation of BC \[ 5x + y - 15 = 0 \]
(b) To find co-ords of A, solve
\[ y = -5x \]
Eq of AB \[ 6x - y - 11 = 0 \]
Sub \( y = -5x \) into \( 6x - y - 11 = 0 \)
\[ 6x - (-5x) - 11 = 0 \]
\[ 11x - 11 = 0 \]
\[ x = 1 \]
but \( y = -5x \)
\[ y = -5(1) \]
\[ y = -5 \]
A is \((1, -5)\)
Ex. 10.

4 (b) To find the coordinates of C

Solve \( y = 6x \)

eq. of BC \( 5x + y - 22 = 0 \)

Substitute \( y = 6x \) into \( 5x + y - 22 = 0 \)

\[
5x + 6x - 22 = 0
\]

\[
11x - 22 = 0
\]

\[
x = 2 \]

Substitute \( x = 2 \) into \( y = 6x \)

\[
y = 6 \cdot 2 = 12
\]

C is point \((2, 12)\).

5. (a) ODC has equation \( y = \frac{1}{2}x \)

So \( m_{DC} = \frac{1}{2} \)

\( m_{AD} = -2 \)

A \((3, 4)\)

\[
y - 4 = m(x - 3)
\]

\[
y - 4 = -2(x - 3)
\]

\[
y - 4 = -2x + 6
\]

eqn of AD \( 2x + y - 10 = 0 \)

(b) Solve \( y = \frac{1}{2}x \)

\[
2x + y - 10 = 0
\]

Subs. \( y = \frac{1}{2}x \) into \( 2x + y - 10 = 0 \)

\[
2x + \frac{1}{2}x - 10 = 0
\]

\[
\frac{5}{2}x = 10
\]

\[
x = 20
\]

but \( y = \frac{1}{2}x \)

\[
y = \frac{1}{2} \cdot 2 = 2
\]

\( D(4, 2) \)
Ex \(10\)

5 (c) \(A (3,4)\)
\(D (4,2)\)

\[ AD = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \]
\[ = \sqrt{(4-3)^2 + (2-4)^2} \]
\[ = \sqrt{1^2 + (-2)^2} \]
\[ = \sqrt{1 + 4} \]
\[ = \sqrt{5} \]

Area = \(\sqrt{5} \times \sqrt{5}\)
= 5 sq. units

6. (a) Solve \(\begin{cases} y = 7 \\ 2y = 3x + 8 \end{cases}\)

eqn of QS \(\text{eqn of PR (diagonals)}\)

Sub \(y = 7\) into
\(2y = 3x + 8\)
\(14 = 3x + 8\)
\(3x = 6\)
\(x = 2\)

(b) \(CS = 4\) units
\(QC = 4\) units (on line \(y = 7\))

\(Q (-2,7)\)

P to C 2 along 3 up
C to R 2 along 3 up

R(4,10)

7. Midpoint of BC is \(\left(\frac{3+6}{2}, \frac{3+2}{2}\right)\)
\(\left(\frac{9}{2}, \frac{5}{2}\right)\)

\[ M_{BC} = \frac{x_2-x_1}{y_2-y_1} = \frac{-1}{3} \]

\[ \Rightarrow M_{y_{AC}} = 3 \]
10. (a) \( m_{BC} = \frac{-3-9}{10-4} = \frac{-12}{6} = -2 \)

\[ \Rightarrow m_{AH} = \frac{1}{2} \]

A (2, 3)

\[ y - 3 = m(x - 2) \]
\[ y - 3 = \frac{1}{2}(x - 2) \]
\[ 2y - 6 = x - 2 \]

Equation of AH

\[ x - 2y + 4 = 0 \]

\( \phi \)

\( d = \sqrt{(10-4)^2 + (-3-9)^2} \)
\[ = \sqrt{6^2 + (-12)^2} \]
\[ = \sqrt{36 + 144} \]
\[ = \sqrt{180} \]

AH

\[ = \sqrt{(6-2)^2 + (5-3)^2} \]
\[ = \sqrt{4^2 + 2^2} \]
\[ = \sqrt{16 + 4} \]
\[ = \sqrt{20} \]

Area

\[ = \frac{1}{2} \sqrt{180} \sqrt{20} \]
\[ = \frac{1}{2} \sqrt{3600} \]
\[ = \frac{1}{2} 60 \]
\[ = 30 \text{ sq units} \]

11. (a) \( m_{RT} = \frac{-4-(-2)}{6-(4)} = \frac{-2}{10} = \frac{-1}{5} \)

T (6, 4)

\[ y - 4 = m(x - 6) \]
\[ y - 4 = \frac{-1}{5}(x - 6) \]
\[ 5y + 20 = -x + 12 \]

Equation of RT

\[ 3x + 5y + 2 = 0 \]

\[ m_{SU} = \frac{1}{3} \]

M (1, -3)

\[ y - (-3) = \frac{1}{3}(x - 1) \]
\[ y + 3 = \frac{1}{3}x + \frac{1}{3} \]
\[ 3y + 9 = \frac{1}{3}x + 3 \]
\[ 9x + 3y + 27 = 0 \]

Equation of SU

\[ 5x - y - 8 = 0 \]
Eqn of Rs

Eqn of Su

\[ 3y - 2x + 2 = 0 \]  \[ \quad \text{(1)} \]

\[ 5x + 8y - 10 = 0 \]  \[ \quad \text{(2)} \]

Add

\[ 7x + 6 = 0 \]
\[ -13x + 26 = 0 \]
\[ 13x = 26 \]
\[ x = 2 \]

Substitute \( x = 2 \) in

\[ 3y = 2x + 2 \]
\[ 3y = 2(2) + 2 \]
\[ 3y = 6 \]
\[ y = 2 \]

S(2,2) \( \rightarrow \) M(1,-3) \( \rightarrow \) S(1,5)

S(2,2) \( \rightarrow \) U(3,2,10) \( \rightarrow \) U(0,-8)