2011 Mathematics

Higher

Finalised Marking Instructions

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General Comments

These marking instructions are for use with the 2011 Higher Mathematics Examination.

For each question the marking instructions are in two sections, namely **Illustrative Scheme** and **Generic Scheme**. The **Illustrative Scheme** covers methods which you will commonly see throughout your marking. The **Generic Scheme** indicates the rationale for which each mark is awarded. In general you should use the **Illustrative Scheme** and only use the **Generic Scheme** where a candidate has used a method not covered in the **Illustrative Scheme**.

All markers should apply the following general marking principles throughout their marking:

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than deducted for what is wrong.

2. Award one mark for each •. There are no half marks.

3. Working subsequent to an error must be followed through, with possible full marks for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction of mark(s) should be made.

4. **Marking Symbols**
   - No comments, words or acronyms should be written on scripts. Please use the following and nothing else.
   - A tick should be used where a piece of working is correct and gains a mark. You are not expected to tick every line of working but you must check through the whole of a response.
   - Where a mark is lost, the error should be underlined in red at the point where it first occurs, and not at any subsequent stage of the working.
   - A cross-tick should be used to indicate “correct” working where a mark is awarded as a result of follow through from an error.
   - A double cross-tick should be used to indicate correct working which is irrelevant or insufficient to score any marks. This should also be used for working which has been eased.
   - A tilde should be used to indicate a minor transgression which is not being penalised, e.g. bad form.
   - This should be used where a candidate is given the benefit of the doubt.
   - A roof should be used to show that something is missing, such as a crucial step in the working or part of a solution.

These will help you to maintain consistency in your marking and will assist the examiners in the later stages of SQA procedures.
5 Regularly Occurring Responses (ROR) are shown on the marking scheme to help mark common solutions that are non-routine.

6 RORs may also be used as a guide in marking other non-routine candidate responses.

7 The mark for each part of a question should be entered in red in the outer right hand margin, opposite the end of the working concerned. The marks should correspond to those on the question paper and these marking instructions. Only the mark, as a single number, should be written.

8 Where a candidate has scored zero for any question, or part of a question, 0 should be written in the right hand margin beside their answer.

9 Every page of a candidate’s script should be checked for working. Unless blank, every page which is devoid of a marking symbol, should have a tick placed in the bottom right hand margin.

10 Where a solution is spread over several pages the marks should be recorded at the end of the solution. This should be indicated with a down arrow (\(\downarrow\)), in the margin, at the earlier stages.

The examples below illustrate the use of the marking symbols.

**Example 1**

\[ y = x^3 - 6x^2 \]

\[ dy \over dx = 3x^2 - 12 \checkmark \bullet 1 \checkmark \]

\[ 3x^2 - 12 = 0 \checkmark \bullet 1 \checkmark \]

\[ x = 2 \checkmark \bullet 4 \checkmark \]

\[ y = -16 \checkmark \bullet 5 \checkmark \]

**Example 2**

A(4,4,0), B(2,2,6), C(2,2,0)

\[ \overline{AB} = b + a = \begin{pmatrix} 6 \\ 6 \end{pmatrix} \checkmark \bullet 1 \]

\[ \overline{AC} = \begin{pmatrix} 6 \\ 6 \end{pmatrix} \checkmark \bullet 2 \checkmark \] (repeated error)

**Example 3**

\[ 3 \sin x - 5 \cos x \]

\[ k \sin x \cos a - \cos x \sin a \checkmark \bullet 1 \checkmark \]

\[ k \cos a = 3, k \sin a = 5 \checkmark \bullet 2 \checkmark \]

**Example 4**

Find intersection of \(x + 3y = 23\) and \(y = 3x - 9\)

\[ y - 3x = 9 \checkmark \bullet 1 \]

\[ 3y + x = 24 \checkmark \bullet 1 \checkmark \] (despite two errors)

\[ 3y - 9x = 27 \]

\[ x = -\frac{3}{10} \checkmark \bullet 2 \checkmark \]

The subsequent pd mark is lost (Note 12)
11 Where a transcription error (paper to script or within script) occurs, a mark is lost.  

e.g.

\[ x^2 + 5x + 7 = 9x + 4 \quad \checkmark \]
\[ x - 4x + 3 = 0 \quad \times \]
\[ x = 1 \quad \times \]

This is a transcription error and so mark is lost.

Eased as not solution of a quadratic equation.

Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt.

\[ x^2 + 5x + 7 = 9x + 4 \quad \checkmark \]
\[ x - 4x + 3 = 0 \quad \checkmark \]
\[ (x-3)(x-1) = 0 \quad \sim\sim \]
\[ x = 1 \text{ or } 3 \quad \checkmark \]

12 In general, as a consequence of an error perceived to be trivial, casual or insignificant, e.g. \( 6 \times 6 = 12 \), candidates lose the opportunity of gaining the appropriate ic or pd mark.

13 A processing error made at a strategy mark stage is penalised at the next pd or ic mark available within that part of the question. The strategy mark may still be awarded.

14 As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Throughout this paper, unless specifically mentioned in the marking scheme, a correct answer with no working receives no credit.

15 Unless specifically mentioned in the marking scheme, do not penalise:

- Working subsequent to a correct answer;
- Correct working in the wrong part of a question;
- Legitimate variations in numerical answers, e.g. angles in degrees rounded to nearest degree;
- Omission of units;
- Bad form.

16 No piece of working should be ignored without careful checking – even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme. Answers which are widely off-beam are unlikely to include anything of relevance, but candidates may still have the opportunity of gaining the odd mark or two, provided it satisfies the criteria for the marks.

17 In the exceptional circumstance where you are in doubt whether a mark should or should not be awarded, err on the generous side and award the mark.
18 Scored out or erased working which **has not been replaced** should be marked where still legible. However, if the scored out or erased working **has been replaced**, only the work which has not been scored out should be marked.

19 A valid approach, within Mathematical problem solving, is to try different strategies. Where this occurs, all working should be marked. The mark awarded to the candidate is from the *highest* scoring strategy. This is distinctly different from the candidate who gives two or more solutions to a question/part of a question, deliberately leaving all solutions, hoping to gain some benefit. All such contradictory responses should be marked and the *lowest* mark given.

20 It is of great importance that the utmost care should be exercised in adding up the marks. The recommended procedure is as follows:

   Step 1  Manually calculate the total from the candidate’s script.
   Step 2  Check this total using the grid issued with these marking instructions.
   Step 3  Input the scores and obtain confirmation of your total from the EMC screen. (This should highlight any discrepancies hitherto undiscovered.)

21 Place the candidate’s script for Paper 2 inside the script for Paper 1 and write the candidate’s total score (i.e. Paper 1 Section B + Paper 2) in the space provided on the front cover of the script for Paper 1.

22 In cases of difficulty, covered neither in detail nor in principle in these instructions, contact your Team Leader (TL) in the first instance. A referral to the Principal Assessor (PA) should only be made in consultation with your TL. Please see the General Marking Instructions for PA Referrals.
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
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<tbody>
<tr>
<td>1</td>
<td>C</td>
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<td>2</td>
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<td>A</td>
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<td>C</td>
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<td>19</td>
<td>C</td>
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<td>20</td>
<td>D</td>
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**Summary**

<table>
<thead>
<tr>
<th>Grade</th>
<th>Count</th>
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<tbody>
<tr>
<td>A</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
</tr>
<tr>
<td>D</td>
<td>6</td>
</tr>
</tbody>
</table>
A quadrilateral has vertices $A(-1, 8)$, $B(7, 12)$, $C(8, 5)$ and $D(2, -3)$ as shown in the diagram.

(a) Find the equation of diagonal $BD$.

(b) The equation of diagonal $AC$ is $x + 3y = 23$.

Find the coordinates of $E$, the point of intersection of the diagonals.

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21 (a)

1. Find gradient of $BD$.
2. State equation of $BD$.

Notes

1. There is no need to simplify $m_{BD}$ for 1; however, it must be simplified before 2 can be awarded.
2. If $m_{BD}$ cannot be simplified, due to an error, then 2 is still available.
3. Candidates who determine the equation of $AC$ lose 1 but may still gain 2.
4. Candidates lose 1 and 2 for the equation of any side of the quadrilateral.

Regularly occurring responses

Response 1

Using $y = mx + c$

$y = 3x + c \checkmark$ 1

$12 = 3 \times 7 + c$ or $-3 = 3 \times 2 + c$

$c = -9 \checkmark$ 2

2 marks out of 2

Response 2

$m_{AC} = -\frac{1}{3}$

$m_{BD} = 3$

$y - (-3) = 3(x - 2)$ or $y - 12 = 3(x - 7)$

Candidate has assumed diagonals are perpendicular - without evidence.

1 mark out of 2

---

21 (b)

1. Start solution of simultaneous equations.
2. Solve for one variable.
3. Solve for second variable.

Notes

1. Candidates who find the equation of $AC$ in (a), correctly or incorrectly, lose 1, 2 and 3 in (b).
2. Any other incorrect answer from (a) may still gain 1, 2 and 3 as follow through.

Regularly occurring responses

Response 3

$3x - y = 3$ and $x + 3y = 23$ 1

$x = 3 \cdot 2$ 2

$y = 6 \cdot 6$ 3

Subsequent to gaining 3 an error was made in simplifying the equation in (a), but strategy mark still awarded in (b).

2 marks out of 3

Error going from (a) to (b) is penalised at first pd (or ic) mark.
21 (c) (i) Find the equation of the perpendicular bisector of AB.
(ii) Show that this line passes through E.

<table>
<thead>
<tr>
<th>Generic Scheme</th>
<th>Illustrative Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 ss know and find midpoint of AB</td>
<td>6 (3,10)</td>
</tr>
<tr>
<td>7 pd find gradient of AB</td>
<td>7 ( \frac{4}{8} ) or equivalent</td>
</tr>
<tr>
<td>8 ic interpret perpendicular gradient</td>
<td>8 ( \frac{-8}{4} ) or equivalent stated, or implied by 6</td>
</tr>
<tr>
<td>9 ic state equation of perp. bisector</td>
<td>9 ( y - 10 = -2(x - 3) ) but not ( y - 6 = -2(x - 5) )</td>
</tr>
<tr>
<td>10 ic justification of point on line</td>
<td>10 when ( x = 5 ), ( y = -2 \times 5 + 16 = 6 ) or ( 2 \times 5 + 6 - 16 = 0 )</td>
</tr>
</tbody>
</table>

**Notes**

7. Candidates who do not simplify the gradient in (a) and (c) should only be penalised once.
8. \( \star \) is only available as a consequence of using a midpoint and perpendicular gradient.
9. Candidates who use \( y - 6 = -2(x - 5) \) at \( \star \) stage, lose \( 8 \) and \( 10 \).
10. Candidates who show that the point of intersection of BD or AC and the perpendicular bisector is E gain \( 10 \).

**Regularly occurring responses**

**Response 4**

\( m_{\text{perp bisector}} = -2 \)
\( m_{\text{ME}} = \ldots = -2 \)

So perpendicular bisector goes through E \( \neq 10 \)

There must be reference to the midpoint being a common point to gain this mark.

**Response 5**

From (i) equation of perpendicular bisector is \( y = -2x + 16 \), using (3, 10).

Then in (ii) using \( m = -2 \) and E(5, 6) leads to \( y = -2x + 16 \). Same equation so E lies on line. \( \checkmark 10 \)

**Response 6**

From (b) E(3·2, 6·6)

\( x = 3 \cdot 2, \ y = \ldots = 9 \cdot 6, \) so line does not pass through E. \( \times 10 \)

Comment must be consistent with E from (b).
22 A function \( f \) is defined on the set of real numbers by \( f(x) = (x - 2)(x^2 + 1) \).

(a) Find where the graph of \( y = f(x) \) cuts:
(i) the \( x \)-axis; (ii) the \( y \)-axis.  

(b) Find the coordinates of the stationary points on the curve with equation \( y = f(x) \) and determine their nature.  

**Generic Scheme**

<table>
<thead>
<tr>
<th></th>
<th>ic interpret ( x ) intercept</th>
<th>ic interpret ( y ) intercept</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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</table>

**Illustrative Scheme**

<table>
<thead>
<tr>
<th></th>
<th>ic interpret ( x ) intercept</th>
<th>ic interpret ( y ) intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(2, 0) (minimum response &quot;(i) 2&quot;)</td>
<td>(0, -2) (minimum response &quot;(ii) -2&quot;)</td>
</tr>
</tbody>
</table>

Notes

1. Candidates who obtain extra \( x \)-axis intercepts lose 1.
2. Candidates who obtain extra \( y \)-axis intercepts lose 2.
3. Candidates who interchange intercepts can gain at most one mark.

22 (b)

<table>
<thead>
<tr>
<th></th>
<th>ic write in differentiable form</th>
<th>ss know to and start to differentiate</th>
<th>pd complete derivative and equate to 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
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<td>4</td>
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<table>
<thead>
<tr>
<th></th>
<th>pd factorise derivative</th>
<th>pd process for ( x )</th>
<th>pd evaluate ( y )-coordinates</th>
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<tbody>
<tr>
<td>6</td>
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<table>
<thead>
<tr>
<th></th>
<th>ic justify nature of stationary points</th>
<th>ic interpret and state conclusions</th>
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<tbody>
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<td>9</td>
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<td>10</td>
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Notes

4. **5** is only available if " 0 " appears at or before **6** stage.
5. **3**, **4** and **5** are the only marks available to candidates who solve \( 3x^2 - 4x = -1 \).
6. At **5** the nature can be determined using the second derivative.
7. **9** is only available if the nature table is consistent with the candidate’s derivative.
8. **10** is awarded for correct interpretation of the candidate’s nature table in words.

Regularly occurring responses

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This question may be marked vertically. The dotted rectangle shows what is required for **10**.
22 (c) On separate diagrams sketch the graphs:
   (i) \( y = f(x) \);
   (ii) \( y = -f(x) \).

\begin{itemize}
  \item \textit{ic} curve showing points from (a) and (b) without annotation
  \item \textit{ic} \textbf{cubic} curve showing all intercepts and stationary points annotated
  \item \textit{ic} curve from (i) reflected in \( x \)-axis
\end{itemize}

Notes

9. \textit{•} \textsuperscript{11} is for any curve consistent with all points found in (a) and (b). Ignore any extra critical points.

10. In (c)(ii), the minimum requirement is the curve from (c)(i) reflected in \( x \)-axis showing \textbf{at least one} \( x \)-intercept unchanged and \textbf{at least one} stationary point correctly annotated.

Regularly occurring responses

Follow through from candidate’s work in (a) and (b).

Response 2

\begin{itemize}
  \item \textit{•} \textsuperscript{11} \textbf{cubic} curve showing all points found in (a) and (b) without annotation
  \item \textit{•} \textsuperscript{12} \textbf{cubic} curve showing \textbf{all} intercepts and stationary points annotated
  \item \textit{•} \textsuperscript{13} curve from (i) reflected in \( x \)-axis
\end{itemize}

Response 3

\begin{itemize}
  \item \textit{•} \textsuperscript{11} \textit{cubic} curve showing \textbf{all} intercepts and stationary points annotated
  \item \textit{•} \textsuperscript{12} \textit{cubic} curve showing \textbf{all} intercepts and stationary points annotated
  \item \textit{•} \textsuperscript{13} reflected sketch
\end{itemize}

Response 4

\begin{itemize}
  \item \textit{•} \textsuperscript{11} \textbf{cubic} curve showing \textbf{all} intercepts and stationary points annotated
  \item \textit{•} \textsuperscript{12} \textbf{cubic} curve showing \textbf{all} intercepts and stationary points annotated
  \item \textit{•} \textsuperscript{13} reflected sketch
\end{itemize}

Response 5

\begin{itemize}
  \item \textit{•} \textsuperscript{11} \textbf{cubic} curve showing \textbf{all} intercepts and stationary points annotated
  \item \textit{•} \textsuperscript{12} \textbf{cubic} curve showing \textbf{all} intercepts and stationary points annotated
  \item \textit{•} \textsuperscript{13} reflected sketch
\end{itemize}

No marks available for a quadratic.

Not a cubic due to apparent change of concavity.
23 (a) Solve \( \cos 2x - 3 \cos x + 2 = 0 \) for \( 0 \leq x < 360 \).

### Generic Scheme

- **1** ss know to use double angle formula
- **2** ic express as a quadratic in \( \cos x \)
- **3** ss start to solve

### Illustrative Scheme

**Method 1**: Using factorisation
- **1** \( 2 \cos^2 x - 1 \ldots \) stated, or implied by **2**
- **3** \( 2 \cos^2 x - 3 \cos x + 1 \) \( = 0 \) must appear at either
- **1** \( (2 \cos x - 1)(\cos x - 1) \) of these lines to gain **3**.

**Method 2**: Using quadratic formula
- **1** \( 2 \cos^2 x - 1 \ldots \)
- **3** \( 2 \cos^2 x - 3 \cos x + 1 = 0 \) stated explicitly
- **3** \( x \) \( = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 2 \times 1}}{2 \times 2} \)

In both methods:
- **4** \( \cos x = \frac{1}{2} \) and \( \cos x = 1 \) Candidates who include 360 lose **5**
- **4** 0, 60 and 300 or
- **4** \( \cos x = 1 \) and \( x = 0 \) Candidates who include 360 lose **4**
- **4** \( \cos x = \frac{1}{2} \) and \( x = 60 \) or 300

### Notes

1. **1** is not available for simply stating that \( \cos 2A = 2 \cos^2 A - 1 \) with no further working.
2. In the event of \( \cos^2 x - \sin^2 x \) or \( 1 - 2 \sin^2 x \) being substituted for \( \cos 2x \), **1** cannot be awarded until the equation reduces to a quadratic in \( \cos x \).
3. Substituting \( \cos 2A = 2 \cos^2 A - 1 \) or \( \cos 2a = 2 \cos^2 a - 1 \) etc. should be treated as bad form throughout.
4. Candidates may express the quadratic equation obtained at the **2** stage in the form \( 2x^2 - 3x + 1 \) or \( 2x^2 - 3x + 1 \) etc. For candidates who do not solve a trigonometric quadratic equation at **1**, \( \cos x \) must appear explicitly to gain **2**.
5. **4** and **5** are only available as a consequence of solving a quadratic equation.
6. Any attempt to solve \( ax^2 + bx = c \) loses **4**, **1** and **2**.
7. **5** is not available to candidates who work in radian measure and do not convert their answers into degree measure.

### Regularly occurring responses

**Response 1**
(Reading \( \cos 2x \) as \( \cos 2x \))

\[
\begin{align*}
\cos^2 x' - 3 \cos x' + 2 &= 0 \times \times \times \\
(\cos x' - 2)(\cos x' - 1) &= 0 \times \times \times \\
\cos x' &= 2 \quad \text{or} \quad \cos x' &= 1 \times \times \times \\
\text{no solution} \quad x &= 0 \times \times \times \\
\end{align*}
\]

2 marks out of 5

**Response 2A**
(See note 6 above)

\[
\begin{align*}
2 \cos^2 x' - 1 - 3 \cos x' + 2 &= 0 \checkmark \times \times \\
2 \cos^2 x' - 3 \cos x' &= -1 \times \times \times \\
\cos x' (2 \cos x' - 3) &= -1 \times \times \times \\
\cos x' &= -1 \quad \text{or} \quad \cos x' &= 1 \checkmark \times \times \\
x &= 180 \quad x = 0 \times \times \times \\
\end{align*}
\]

1 mark out of 5

**Response 2B**
(See note 6 above)

\[
\begin{align*}
2 \cos^2 x' - 1 - 3 \cos x' + 2 &= 0 \checkmark \times \times \\
2 \cos^2 x' - 3 \cos x' + 1 &= 0 \checkmark \times \\
2 \cos^2 x' - 3 \cos x' &= -1 \checkmark \times \\
\cos x' (2 \cos x' - 3) &= -1 \checkmark \times \\
\cos x' &= -1 \quad \text{or} \quad \cos x' &= 1 \checkmark \times \\
x &= 180 \quad x = 0 \times \times \times \\
\end{align*}
\]

2 marks out of 5
Hence solve $\cos 4x - 3\cos 2x + 2 = 0$ for $0 \leq x < 360$.  

**Generic Scheme**

- $\bullet^6$ ic interpret relationship with (a)
- $\bullet^7$ ic interpret periodicity

**Illustrative Scheme**

- $\bullet^6$ $2x = 0$ and $60$ and $300$
- $\bullet^7$ $0, 30, 150, 180, 210$ and $330$

**Notes**

8. Do not penalise the inclusion of $360$ in (b).
9. Ignore extra answers, correct or incorrect, outside the given interval, but penalise incorrect answers within the interval.
10. Do not penalise candidates who use radians in (b) if they have already been penalised in (a).
11. Candidates who go back to ‘first principles’ for (b) can only gain $\bullet^6$ and $\bullet^7$ for a correct method leading to valid solutions as stated in the Illustrative Scheme.

**Regularly occurring responses**

**Response 3A**

From (a) $x = 0, 60, 300$
(b) $\cos 4x - 3\cos 2x + 2 = 0$

- $2(\cos 2x - 3\cos x + 1) = 0 \times \bullet^6$
- $x = 0, 30, 150, 180, 210, 330 \times \bullet^7$

1 mark out of 2

**Response 3B**

From (a) $x = 0, 60, 300$
(b) $\cdots \times \bullet^6$

- $x = 0, 30, 150, 180, 210, 330 \times \bullet^7$

1 mark out of 2

**Response 4A**

From (a) $x = 0, 60, 300$
(b) $x^2 = 0, 30, 150 \times \bullet^6 \times \bullet^7$

0 marks out of 2

**Response 4B**

From (a) $x = 0, 60, 300$
(b) $x^2 = 0, 30, 150, 180, 210, 330 \times \bullet^6 \times \bullet^7$

1 mark out of 2

**Response 5**

From (a) $x = 0, 60, 300$
(b) $\cos(2.2x) - 3\cos 2x + 2 = 0 \checkmark \bullet^6$

- $x = 0, 30, 150, 180, 210, 330 \checkmark \bullet^7$

2 marks out of 2

**Response 6**

From (a) $x = 0, 60, 300$
(b) period $\div 2 \checkmark \bullet^6$

- so $x = 0, 30, 150, 180, 210, 330, 360, 570 \checkmark \bullet^7$

2 marks out of 2

**Response 7**

From (a) $x = 0, 60, 300$
(b) $2x$ repeats every $180 \checkmark \bullet^6$

- $x = 0, 60, 300, 0 + 180, 60 + 180$
- $= 0, 60, 180, 240, 300 \times \bullet^6$

0 marks out of 2

**Response 8** (Wrong angles from (a))

From (a) $x = 0, 30, 330$
(b) $2x = 0, 30, 330 \times \bullet^6$

- $x = 0, 15, 165, 180, 195, 345 \checkmark \bullet^7$

2 marks out of 2
1 D,OABC is a square based pyramid as shown in the diagram below.

O is the origin, D is the point (2, 2, 6) and OA = 4 units.

M is the mid-point of OA.

(a) State the coordinates of B.

(b) Express $DB$ and $DM$ in component form.

Throughout this question, coordinates written as components and vice versa are treated as bad form.

### Generic Scheme

1 (a)

| ic state coordinates of B | (4, 4, 0) |

1 (b)

| pd state components of $DB$ | $\begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$ |
| ic state coordinates of M | (2, 0, 0) |
| pd state components of $DM$ | $\begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$ |

### Illustrative Scheme

**Response 1A** (Transcription error for D)  
$DB = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix} \times \begin{pmatrix} 2 \\ -2 \\ -6 \end{pmatrix}$

**Response 1B** (Transcription error for D)  
$DB = \begin{pmatrix} 2 \\ -2 \\ -6 \end{pmatrix}$ and $DM = \begin{pmatrix} 0 \\ -6 \\ -6 \end{pmatrix}$ with no working.

**Response 2A**  
$DB = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix} \times \begin{pmatrix} 2 \\ -2 \\ -6 \end{pmatrix}$

**Response 2B**  
$DB = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$ and $DM = \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix}$ with no working.

| 2 marks out of 3 | 0 marks out of 3 |
1 (c) Find the size of angle BDM.

**Generic Scheme**

- **5** ss know to use scalar product
- **6** pd find scalar product
- **7** pd find magnitude of a vector
- **8** pd find magnitude of a vector
- **9** pd evaluate angle BDM

**Illustrative Scheme**

- **3** \( \cos \angle BDM = \frac{\overrightarrow{DB} \cdot \overrightarrow{DM}}{|\overrightarrow{DB}| |\overrightarrow{DM}|} \)
- **6** \( \overrightarrow{DB} \cdot \overrightarrow{DM} = 32 \)
- **7** \( |\overrightarrow{DB}| = \sqrt{44} \)
- **8** \( |\overrightarrow{DM}| = \sqrt{40} \)
- **9** \( 40 \cdot 3 \text{° or } 0.703 \text{ radians} \)

**Notes**

1. **5** is not available to candidates who evaluate the wrong angle.
2. If candidates do not attempt **9**, then **5** is only available if the formula quoted relates to the labelling in the question.
3. **9** should be awarded to any answer which rounds to 40° or 0.7 radians.
4. In the event that both magnitudes are equal or there is only one non-zero component, **8** is not available.

**Regularly occurring responses**

<table>
<thead>
<tr>
<th>Response 3A</th>
<th>Response 3B</th>
<th>Response 3C</th>
<th>Response 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \cos \angle BDM = \frac{\overrightarrow{OB} \cdot \overrightarrow{DM}}{</td>
<td>\overrightarrow{OB}</td>
<td></td>
<td>\overrightarrow{DM}</td>
</tr>
<tr>
<td>( \overrightarrow{OB} \cdot \overrightarrow{DM} = 8 \times 9 )</td>
<td>( \overrightarrow{OB} \cdot \overrightarrow{OD} = 16 \times 10 )</td>
<td>( \overrightarrow{BD} \cdot \overrightarrow{BM} = 12 \times 11 )</td>
<td>( \overrightarrow{BD} \cdot \overrightarrow{DM} = -32 \times 12 )</td>
</tr>
<tr>
<td>(</td>
<td>\overrightarrow{OB}</td>
<td>= \sqrt{32} \times 13 )</td>
<td>(</td>
</tr>
<tr>
<td>(</td>
<td>\overrightarrow{DM}</td>
<td>= 2 \times 17 )</td>
<td>(</td>
</tr>
<tr>
<td>45° ( \times \times 21 )</td>
<td>64.8° ( \times \times 22 )</td>
<td>66.1° ( \times \times 23 )</td>
<td>139.7° ( \times \times 24 )</td>
</tr>
<tr>
<td>3 marks out of 5</td>
<td>4 marks out of 5</td>
<td>4 marks out of 5</td>
<td>4 marks out of 5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Response 5A</th>
<th>Response 5B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Scalar Product is 0)</td>
<td></td>
</tr>
<tr>
<td>In 1(b) ( \overrightarrow{DB} = \begin{pmatrix} 2 \ -2 \ -6 \end{pmatrix} ), ( \overrightarrow{DM} = \begin{pmatrix} 0 \ 6 \ -2 \end{pmatrix} )</td>
<td>In 1(b) ( \overrightarrow{DB} = \begin{pmatrix} 2 \ -2 \ -6 \end{pmatrix} ), ( \overrightarrow{DM} = \begin{pmatrix} 0 \ 6 \ -2 \end{pmatrix} )</td>
</tr>
<tr>
<td>( \cos \angle BDM = \frac{\overrightarrow{DB} \cdot \overrightarrow{DM}}{</td>
<td>\overrightarrow{DB}</td>
</tr>
<tr>
<td>( \overrightarrow{DB} \cdot \overrightarrow{DM} = 0 \times 27 )</td>
<td>( \overrightarrow{DB} \cdot \overrightarrow{DM} = 0 \times 28 )</td>
</tr>
<tr>
<td>(</td>
<td>\overrightarrow{DB}</td>
</tr>
<tr>
<td>(</td>
<td>\overrightarrow{DM}</td>
</tr>
<tr>
<td>90° ( \times \times 33 )</td>
<td>( \times \times 34 )</td>
</tr>
<tr>
<td>5 marks out of 5</td>
<td>3 marks out of 5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Response 6 (Cosine rule)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \cos \angle BDM = \frac{DB^2 + DM^2 - BM^2}{2 \times DB \times DM} \times 35 )</td>
</tr>
<tr>
<td>( DB = \sqrt{44} \times 36 )</td>
</tr>
<tr>
<td>( DM = \sqrt{40} \times 37 )</td>
</tr>
<tr>
<td>( BM = \sqrt{20} \times 38 )</td>
</tr>
<tr>
<td>( 40 \cdot 3 \text{°} \times \times 39 )</td>
</tr>
<tr>
<td>5 marks out of 5</td>
</tr>
</tbody>
</table>
Functions \( f, g \) and \( h \) are defined on the set of real numbers by

\[
\begin{align*}
\bullet & \quad f(x) = x^3 - 1 \\
\bullet & \quad g(x) = 3x + 1 \\
\bullet & \quad h(x) = 4x - 5
\end{align*}
\]

(a) Find \( g(f(x)) \).

(b) Show that \( g(f(x)) + xh(x) = 3x^2 + 4x^2 - 5x - 2 \).

### Notes

1. \( 3x^3 - 2 \) without working gains only 1 mark.
2. \( f(g(x)) \) loses \( \bullet^1 \) but will gain \( \bullet^2 \) for \( (3x+1)^3 - 1 \).
3. \( f(x) \times g(x) \) loses both \( \bullet^1 \) and \( \bullet^2 \).

### Regularly occurring responses

CAVE: Watch out for erroneous working leading to the required cubic.

| Response 1 | \( 3x^3 - 2 + x(4x+5) = 3x^3 + 4x^2 - 5x - 2 \) | X \( \bullet^3 \) |
| Response 2 | \( 3x^3 - 4 + x(4x-5) = 3x^3 + 4x^2 - 5x - 2 \) | X \( \bullet^3 \) |
| Response 3 | From (a) \( (3x+1)^3 - 1 \)  
In (b) \( 3x^3 + 3 - 1 + x(4x-5) = 3x^3 + 2 + 4x^2 - 5x \) | X \( \bullet^3 \) |
| Response 4A | From (a) \( g(f(x)) = 3x^3 - 2 \)  
In (b) \( xh(x) = 4x^2 - 5x \) | \( 3x^3 + 4x^2 - 5x - 2 \) |
| Response 4B | From (a) \( g(f(x)) = 3x^3 - 2 \)  
In (b) \( 3x^2 - 2 + 4x^2 - 5x \) | X \( \bullet^3 \) |

Note: \( \bullet^3 \) is not available to candidates who leave their answer as \( 3x^3 - 2 + 4x^2 - 5x \).
2  (c)  (i) Show that \((x - 1)\) is a factor of \(3x^3 + 4x^2 - 5x - 2\).

(ii) Factorise \(3x^3 + 4x^2 - 5x - 2\) fully.

(d) Hence solve \(g(f(x)) + x h(x) = 0\).

<table>
<thead>
<tr>
<th>Generic Scheme</th>
<th>Illustrative Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\cdot^4) ss know to use (x = 1)</td>
<td>(\cdot^1) 1 3 4 -5 -2</td>
</tr>
<tr>
<td>(\cdot^5) pd complete evaluation</td>
<td>(\cdot^3) 1 3 4 -5 -2</td>
</tr>
<tr>
<td>(\cdot^6) ic state conclusion</td>
<td>3 7 2 0</td>
</tr>
<tr>
<td>(\cdot^7) ic find quadratic factor</td>
<td>If only the word 'factor' appears, it must be linked to the 0 in the table. The link could be as little as 'so', ':', '→', '⇒' or 'hence'. The word 'factor' only, with no link, does not gain (\cdot^6).</td>
</tr>
<tr>
<td>(\cdot^8) pd factorise completely</td>
<td>3 7 2 0</td>
</tr>
</tbody>
</table>

Method 1 : Using synthetic division

\[
\begin{array}{c|ccccc}
& 1 & 3 & 4 & -5 & -2 \\
\hline
3 & 3 & 15 & 3 \hline
& 3 & 7 & 2 & 0 \\
\end{array}
\]

\(\cdot^7\) stated or implied by \(\cdot^3\)

\(\cdot^6\) “remainder is zero so \((x - 1)\) is a factor”, accept “\((x - 1)\) is a factor” stated explicitly

Method 2 : Using substitution and inspection

\(\cdot^1\) know to use \(x = 1\)

\(\cdot^5\) \(3 + 4 - 5 - 2 = 0\)

\(\cdot^6\) \((x - 1)\) is a factor

\(\cdot^7\) \((x - 1)(3x^2 + 7x + 2)\) stated or implied by \(\cdot^3\)

\(\cdot^8\) \((x - 1)(3x + 1)(x + 2)\) stated explicitly

Notes

4. \(\cdot^6\) is only available as a consequence of the evidence for \(\cdot^4\) and \(\cdot^5\).

5. Communication at \(\cdot^6\) must be consistent with working at \(\cdot^5\). i.e. candidate’s working must arrive legitimately at zero before \(\cdot^6\) is awarded.

If the remainder is not 0 then an appropriate statement would be ‘\((x - 1)\) is not a factor’.

6. Unacceptable statements: \(x = 1\) is a factor, \((x + 1)\) is a factor, \(x = 1\) is a root, \((x - 1)\) is a root etc.

7. \(\cdot^9\) cannot be awarded for solving \(3x^3 + 4x^2 - 5x - 2 = 0\) in (c).

2 (d)

\(\cdot^9\) ic interpret and solve equation in (d)

\(\cdot^5\) \(-2, -\frac{1}{3}\) and 1

These must appear explicitly here at (d).

Notes

8. From (c) \((x - 1)(3x + 1)(x + 2)\) leading to \(x = 1, x = -\frac{1}{3}\) and \(x = -2\) then \((1, 0), (\frac{1}{3}, 0)\) and \((-2, 0)\) gains \(\cdot^9\).

However, \((x - 1)(3x + 1)(x + 2)\) leading to \((1, 0), (\frac{1}{3}, 0)\) and \((-2, 0)\) only does not gain \(\cdot^9\).

9. From (c) \((3x + 1)(x + 2)\) only, leading to \(x = -\frac{1}{3}, x = -2\) does not gain \(\cdot^9\) as equation solved is not a cubic.
3 (a) A sequence is defined by \( u_{n+1} = -\frac{1}{2} u_n \) with \( u_0 = -16 \).

Write down the values of \( u_1 \) and \( u_2 \).

(b) A second sequence is given by 4, 5, 7, 11, . . .

It is generated by the recurrence relation \( v_{n+1} = pv_n + q \) with \( v_1 = 4 \).

Find the values of \( p \) and \( q \).

### Generic Scheme vs. Illustrative Scheme

<table>
<thead>
<tr>
<th>3 (a)</th>
<th>3 (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>•1 pd</td>
<td>•1 s</td>
</tr>
<tr>
<td></td>
<td>•2 ic</td>
</tr>
<tr>
<td></td>
<td>•3 ss</td>
</tr>
<tr>
<td></td>
<td>•4 pd</td>
</tr>
</tbody>
</table>

| 1 mark out of 3 | 2 marks out of 3 |

### Notes
1. Candidates may use \( 7p + q = 11 \) as one of their equations at •2.
2. Treat equations like \( p4 + q = 5 \) or \( p(4) + q = 5 \) as bad form.
3. Candidates should not be penalised for using \( u_{n+1} = pu_n + q \).

### Regularly occurring responses

<table>
<thead>
<tr>
<th>Response 1A (No working)</th>
<th>Response 1B (Only one equation)</th>
<th>Response 1C (By verification)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p = 2 ) and ( q = -3 )</td>
<td>( 4p + q = 5 )</td>
<td>( p = 2 ) and ( q = -3 ) (ex nihilo)</td>
</tr>
<tr>
<td>or ( v_{n+1} = 2v_n - 3 )</td>
<td>( p = 2 ) and ( q = -3 )</td>
<td>( v_2 = 8 - 3 = 5 ) and ( v_3 = 10 - 3 = 7 )</td>
</tr>
</tbody>
</table>
Detailed Marking Instructions: Higher Mathematics 2011 Final

3 (c) Either the sequence in (a) or the sequence in (b) has a limit.
   (i) Calculate this limit.
   (ii) Why does the other sequence not have a limit?

**Notes**
4. Just stating that \( l = a + b \) or \( l = \frac{b}{1-a} \) is not sufficient for \( \bullet^5 \).
5. Any calculations based on formulae masquerading as a limit rules cannot gain \( \bullet^5 \) and \( \bullet^6 \).
6. For candidates who use ‘\( b = 0 \)’, \( \bullet^6 \) is only available to those who simplify \( \frac{0}{x} \) to 0.
7. Accept \( 2 > 1 \) or \( p > 1 \) for \( \bullet^7 \). This may be expressed in words.
8. Candidates who use a without reference to \( p \) or 2 cannot gain \( \bullet^7 \).

**Regularly occurring responses**

<table>
<thead>
<tr>
<th>Response 2</th>
<th>Response 3A</th>
<th>Response 3B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l = \frac{0}{\frac{1}{2}} \times \bullet^5 )</td>
<td>From (b) ( p = \frac{3}{4} ) and ( q = 2 )</td>
<td>From (b) ( p = \frac{3}{4} ) and ( q = 2 )</td>
</tr>
<tr>
<td>( l = 0 \times \bullet^6 )</td>
<td>In (c) ( l = \frac{3}{4} + 2 ) and ( l = -\frac{1}{2} ) ( \checkmark \ \bullet^5 ) ( l = 8 )</td>
<td>In (c) ( l = \frac{3}{4} + 2 ) and ( l = -\frac{1}{2} ) ( \checkmark \ \bullet^5 ) ( l = 8 )</td>
</tr>
<tr>
<td>See note 5</td>
<td>Both limits exist as ( -1 &lt; \frac{3}{4} &lt; 1 ) and ( -1 &lt; -\frac{1}{2} &lt; 1 ) ( \checkmark \ \bullet^7 )</td>
<td>Impossible ( \times \ \bullet^7 )</td>
</tr>
</tbody>
</table>

| 0 marks out of 3 | 3 marks out of 3 | 2 marks out of 3 |

<table>
<thead>
<tr>
<th>Response 4A</th>
<th>Response 4B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l = \frac{0}{1-\left(\frac{1}{2}\right)} ) and ( l = -\frac{3}{1-2} ) ( \checkmark \ \bullet^5 ) ( l = 0 ) ( l = 3 \times \bullet^6 )</td>
<td>( l = \frac{0}{1-\left(\frac{1}{2}\right)} ) and ( l = -\frac{3}{1-2} ) ( \checkmark \ \bullet^5 ) ( l = 0 ) ( l = 3 \times \bullet^6 )</td>
</tr>
<tr>
<td>First has limit because ( -1 &lt; -\frac{1}{2} &lt; 1 ) ( \checkmark \ \bullet^7 )</td>
<td>Second sequence has no limit as ( -1 &lt; 2 &lt; 1 ) not true ( \checkmark \ \bullet^7 )</td>
</tr>
</tbody>
</table>

| 1 mark out of 3 | 2 marks out of 3 |

<table>
<thead>
<tr>
<th>Response 5A</th>
<th>Response 5B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l = -\frac{1}{2} ) and ( l = 2 - 3 ) ( \checkmark \ \bullet^5 ) ( l = 0 ) ( l = 3 \times \bullet^6 )</td>
<td>( l = -\frac{1}{2} ) and ( l = 2 - 3 ) ( \checkmark \ \bullet^5 ) ( l = 0 ) ( l = 3 \times \bullet^6 )</td>
</tr>
<tr>
<td>1st has limit because ( -1 &lt; 0 &lt; 1 ) ( \checkmark \ \bullet^7 )</td>
<td>Second sequence has no limit because 2 is not between ( \checkmark \ \bullet^7 ) ( -1 ) and 1</td>
</tr>
</tbody>
</table>

| 2 marks out of 3 | 3 marks out of 3 |
The diagram shows the curve with equation \( y = x^3 - x^2 - 4x + 4 \) and the line with equation \( y = 2x + 4 \).

The curve and the line intersect at the points \((-2, 0), (0, 4)\) and \((3, 10)\).

Calculate the total shaded area.

### Generic Scheme

1. **ss** know to integrate
2. **ic** know to deal with areas on each side of \( y \)-axis
3. **ic** interpret limits of one area
4. **ic** use "upper – lower"
5. **pd** integrate
6. **ic** substitute in limits
7. **pd** evaluate the area on one side
8. **ss** interpret integrand with limits of the other area
9. **pd** evaluate the area on the other side
10. **ic** state total area

### Illustrative Scheme

\[ \int_{-2}^{0} \left( x^3 - x^2 - 4x + 4 \right) - \left( 2x + 4 \right) \, dx \]
\[ = \left[ \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2 + \frac{1}{4}x^4 \right]_{-2}^{0} \]
\[ = \frac{3}{4}x^3 - \frac{1}{3}x^3 - 3(-2)^2 \]
\[ = \frac{3}{4}(3)^3 - \frac{1}{3}(3)^3 - 3(-2)^2 \]
\[ = 3 \left( \frac{27}{4} \right) - \frac{3}{4} \left( \frac{27}{3} \right) - 3(4) \]
\[ = \frac{81}{4} - \frac{27}{4} - 12 \]
\[ = \frac{81 - 27 - 48}{4} \]
\[ = \frac{24}{4} \]
\[ = 6 \]

\[ \int_{-2}^{3} \left( x^3 - x^2 - 4x + 4 \right) \, dx - \int_{0}^{3} \left( 2x + 4 \right) \, dx \]
\[ = \left[ \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2 + \frac{1}{4}x^4 \right]_{-2}^{3} \]
\[ = \frac{3}{4}x^3 - \frac{1}{3}x^3 - 3(-2)^2 \]
\[ = \frac{3}{4}(27) - \frac{1}{3}(27) - 3(4) \]
\[ = \frac{81}{4} - \frac{27}{4} - 12 \]
\[ = \frac{81 - 27 - 48}{4} \]
\[ = \frac{16}{4} \]
\[ = 4 \]

### Notes

1. The evidence for **2** may not appear until **6** stage.
2. The evidence for **2** may appear in a diagram e.g.
3. Where a candidate differentiates at **5**, then **6**, **7** and **9** are not available.
4. Candidates who substitute at **6**, without integrating at **5** lose **5**, **6** and **7**. However **8**, **9** and **10** are still available.
### Regularly occurring responses

#### General comment to markers

In this question you should scan the entire response before starting to mark. Where errors occur in the integration/evaluation, use • to • to mark the better solution and • and • to mark the poorer solution.

A tabular approach to allocating marks is particularly useful in questions like this, where a candidate’s response is spread over several pages, or contains working which appears randomly set out. Response 1 indicates the approach to take here.

### Response 1

\[
\int_{-2}^{3} (x^3 - x^2 - 6x) \, dx
\]

\[
= \left[ \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2 \right]_{-2}^{3}
\]

\[
= \left( \frac{27}{4} - 9 - 27 \right) - \left( \frac{16}{4} - \frac{8}{3} - 12 \right)
\]

\[
= \frac{-125}{12}
\]

\[\checkmark \quad \bullet^7 \quad \bullet^8 \]

---

### Response 2

\[
\int_{-2}^{3} x^3 - x^2 - 4x + 4 \, dx
\]

\[
= \left[ \frac{1}{4}x^4 - \frac{1}{3}x^3 - 2x^2 + 4x \right]_{-2}^{3}
\]

\[
= \left( \frac{27}{4} - \frac{9}{3} - 18 + 12 \right) - \left( \frac{16}{4} - \frac{8}{3} + 8 \right)
\]

\[
= \frac{-7}{12} \checkmark
\]

\[\checkmark \quad \bullet^9 \]

---

### Response 3

\[
\int_{-2}^{3} x^3 - x^2 - 4x + 4 \, dx
\]

\[
= \left[ \frac{1}{4}x^4 - \frac{1}{3}x^3 - 2x^2 + 4x \right]_{-2}^{3}
\]

\[
= \left( \frac{27}{4} - \frac{9}{3} - 18 + 12 \right) - \left( \frac{16}{4} - \frac{8}{3} + 8 \right)
\]

\[
= \frac{-7}{12} \checkmark
\]

---

### N.B.

If due to an error the evaluation is negative it must be dealt with correctly. The responses below illustrate what is required under this circumstance. If both integrals lead to negative values only • or • is lost.

### Response 4A

\[
\int_{0}^{3} \frac{63}{4} \, dx
\]

\[
= \left[ \frac{63}{4}x \right]_{0}^{3}
\]

\[
= \frac{63}{4} \times 3 - \frac{63}{4} \times 0
\]

\[
= \frac{189}{4}
\]

Area = \frac{125}{12} •

\[\bullet^9 \]

---

### Response 4B

\[
\int_{-2}^{3} x^3 - x^2 - 4x + 4 \, dx
\]

\[
= \left[ \frac{1}{4}x^4 - \frac{1}{3}x^3 - 2x^2 + 4x \right]_{-2}^{3}
\]

\[
= \left( \frac{27}{4} - \frac{9}{3} - 18 + 12 \right) - \left( \frac{16}{4} - \frac{8}{3} + 8 \right)
\]

\[
= \frac{-7}{12} \checkmark
\]

Area = \frac{253}{12} \checkmark

\[\bullet^9 \]

---

### Response 4C

\[
\int_{-2}^{3} x^3 - x^2 - 4x + 4 \, dx
\]

\[
= \left[ \frac{1}{4}x^4 - \frac{1}{3}x^3 - 2x^2 + 4x \right]_{-2}^{3}
\]

\[
= \left( \frac{27}{4} - \frac{9}{3} - 18 + 12 \right) - \left( \frac{16}{4} - \frac{8}{3} + 8 \right)
\]

\[
= \frac{-7}{12} \checkmark
\]

Area = \frac{253}{12} \checkmark

\[\bullet^9 \]
Variables \(x\) and \(y\) are related by the equation \(y = kx^n\).

The graph of \(\log_2 y\) against \(\log_2 x\) is a straight line through the points \((0, 5)\) and \((4, 7)\), as shown in the diagram.

Find the values of \(k\) and \(n\).

**Method 1**

1. **ss** introduce logarithms to \(y = kx^n\)
2. **ic** use laws of logarithms
3. **ic** interpret intercept
4. **ic** solve for \(k\)
5. **ic** interpret gradient

**Method 2**

1. **ss** state linear equation
2. **ic** introduce logarithms
3. **ic** use laws of logarithms
4. **ic** use laws of logarithms
5. **ic** interpret result

**Method 3**

1. **ic** interpret point on log. graph
2. **ic** convert from log. to exp. form
3. **ic** interpret point and convert
4. **ss** know to substitute points
5. **ic** interpret result

---

**Notes**

1. Omission of base 2 is treated as bad form at the \(\bullet^1\) and \(\bullet^2\) stage.
2. Gradient \((m) = \frac{1}{2}\) is not sufficient for \(\bullet^3\).
3. Throughout this question accept 32 in lieu of \(2^5\).
4. Markers should not pick and choose within methods. Use the method which gives the candidate the highest mark.
<table>
<thead>
<tr>
<th>Response 1A</th>
<th>Response 1B</th>
</tr>
</thead>
<tbody>
<tr>
<td>With no working</td>
<td>With no working</td>
</tr>
<tr>
<td>$k = 32$ ✓ 1</td>
<td>$k = \frac{1}{2}$ X 3</td>
</tr>
<tr>
<td>$n = \frac{1}{2}$ ✓ 5</td>
<td>$n = 32$ X 5</td>
</tr>
</tbody>
</table>

2 marks out of 5                                   0 marks out of 5

Response 2 (Method 1)

$$\log_2 k = 5$$ ✓ 3

$$k = 32$$ ✓ 4

$$n = \frac{1}{2}$$ ✓ 5

3 marks out of 5

Response 3 (Variation of Method 2 and Response 1A)

$$\log_2 y = \frac{1}{2} \log_1 x + 5$$ ✓ 1

$$\log_2 y = \log_2 \sqrt{x} + 5$$ ✓ 2

$$y = \sqrt{x} + 5$$

$$k = 1$$

$$n = \frac{1}{2}$$ ✓ 5

3 marks out of 5

Response 4 (Variation of Method 2 and Response 1A)

$$y = \frac{1}{2} x + 5$$

$$\log_2 y = \frac{1}{2} \log_2 x + 5$$ ✓ 1

$$\log_2 y - \log_2 x^2 = 5$$ ✓ 2

$$\frac{y}{\sqrt{x}} = 5$$ X

$$y = 5\sqrt{x}$$

$$k = 5$$ X

$$n = \frac{1}{2}$$ ✓ 5

3 marks out of 5
The expression $3 \sin x - 5 \cos x$ can be written in the form $R \sin(x + a)$ where $R > 0$ and $0 \leq a < 2\pi$.

Calculate the values of $R$ and $a$.

**Generic Scheme**

- $\text{ss}$ use compound angle formula
- $\text{ic}$ compare coefficients
- $\text{pd}$ process $R$
- $\text{pd}$ process $a$

**Illustrative Scheme**

- $R \sin x \cos a + R \cos x \sin a$ stated explicitly
- $R \cos a = 3$ and $R \sin a = -5$ stated explicitly
- $\sqrt{34}$ (Accept $5.8$) with or without working
- $5.253$ (Accept $5.3$) must be consistent with

**Notes**

1. Treat as bad form the use of $k$ instead of $R$.
2. Treat $R \sin x \cos a + \cos x \sin a$ as bad form only if the equations at the $\text{pd}$ stage both contain $R$.
3. $\sqrt{34} \sin x \cos a + \sqrt{34} \cos x \sin a$ or $\sqrt{34} (\sin x \cos a + \cos x \sin a)$ is acceptable for $\text{ic}$ and $\text{ss}$.
4. $\text{pd}$ is not available for $R \cos x = 3$ and $R \sin x = 5$, however, $\text{pd}$ is still available.
5. $\text{pd}$ is only available for a single value of $a$.
6. Candidates who work in degrees and don’t convert to radian measure lose $\text{pd}$: Do not accept $\frac{301\pi}{180}$ or $\frac{5\pi}{3}$.
7. Candidates may use any form of the wave equation for $\text{ss}$, $\text{ic}$ and $\text{ss}$, however, $\text{pd}$ is only available if the value of $a$ is interpreted for the form $R \sin(x + a)$.

**Regularly occurring responses**

**Response 1A**

- $R \cos a = 3$  $R \sin a = 5 \times \text{ss}$
- $\tan a = \frac{5}{3}$
- $a = 1.03 \times 4$

**Response 1B**

- $R \cos a = 3$  $R \sin a = 5 \times \text{ss}$
- $\tan a = \frac{3}{5}$
- $a = 0.54 \times \text{pd}$

**Response 1C**

- $R \cos a = 3$  $R \sin a = -5 \times \text{ss}$
- $\tan a = -\frac{3}{5}$
- $a = 5.74 \times \text{pd}$

---

**Response 2**

- $R \sin(x - a) = R \sin x \cos a - R \cos x \sin a \times \text{ic}$
- $R \cos a = 3$  $R \sin a = 5 \times \text{ss}$
- $R = \sqrt{34} \times \text{ss}$
- $a = 1.03 \times \text{ss}$

See note 7

3 marks out of 4

**Response 3**

- $k \sin x \cos a + k \cos x \sin a \times \text{ss}$
- $\cos a = 3$  $\sin a = -5 \times \text{ss}$
- $R = \sqrt{34} \times \text{ss}$
- $a = 5.3 \times \text{ss}$

Not consistent with working at $\text{pd}$

2 marks out of 4
6 (b) Hence find the value of $t$, where $0 \leq t \leq 2$, for which

\[ \int_{0}^{t} (3\cos x + 5\sin x) \, dx = 3 \]

**Generic Scheme**

- £5 pd integrate given expression
- £6 ic substitute limits
- £7 pd process limits
- £8 ss know to use wave equation
- £9 ic write in standard format
- £10 ss start to solve equation
- £11 pd complete and state solution

**Illustrative Scheme**

- £5 $3\sin x - 5\cos x$
- £6 $(3\sin t - 5\cos t) - (3\sin 0 - 5\cos 0)$
- £7 $3\sin t - 5\cos t + 5$
- £8 $\sqrt{34}\sin(t + 5\cdot 3) + 5$
- £9 $\sin(t + 5\cdot 3) = -\frac{2}{\sqrt{34}}$
- £10 $t + 5\cdot 3 = 3.5$ and $5.9$
- £11 $t = 0.6$

**Notes**

8. The inclusion of “+ c” at £5 stage should be treated as bad form.

9. For those candidates who use $a$ as $5\cdot 253$ or $5\cdot 25...$, follow through their working for £8 to £11.

10. Candidates who use degree measure in (a) lose £4 and if they continue to do so in (b), only £5, £6, £7 and £8 are available (see also response 6A and 6B below.)

**Regularly occurring responses**

**Response 4 (No integration)**

\[ \int_{0}^{t} (3\cos x + 5\sin x) \, dx = \sqrt{34}\sin(x + 5\cdot 3) \]

lose £5, £6, £7 and £8

then

\[ \sin(x + 5\cdot 3) = \frac{3}{\sqrt{34}} \]

\[ x + 5\cdot 3 = 0.5, 2.6, 6.8 \]

\[ x = 1.5 \]

Needs to be in terms of $t$

**Response 5**

\[ \ldots 3\sin x - 5\cos x \]

\[ 3\sin t - 5\cos t - 0 \]

\[ \sqrt{34}\sin(t + 5\cdot 3) \]

\[ \sin(t + 5\cdot 3) = -\frac{2}{\sqrt{34}} \]

\[ t + 5\cdot 3 = 0.5, 2.6, 6.8 \]

\[ t = 1.5 \]

**Response 6A (Misreading question)**

\[ \int_{0}^{t} \sqrt{34}\sin(x + 5\cdot 3) \, dx \]

\[ = -\sqrt{34}\cos(t + 5\cdot 3) \]

\[ \cos(t + 5\cdot 3) = -\frac{3}{\sqrt{34}} \]

\[ t + 5\cdot 3 = 2.1, 4.2, 8.4 \]

\[ t = 3.1 \text{ i.e. no solution} \]

3 marks out of 7

**Response 6B (Misreading question)**

\[ \int_{0}^{t} \sqrt{34}\sin(x + 5\cdot 3) \, dx \]

\[ = \left[ -\sqrt{34}\cos(x + 5\cdot 3) \right]_{0}^{t} \]

\[ = -\sqrt{34}\cos(t + 5\cdot 3) + \sqrt{34}\cos 5\cdot 3 \]

\[ = -\sqrt{34}\cos(t + 5\cdot 3) + 3.2 \]

\[ \cos(t + 5\cdot 3) = -0.2 \]

\[ t + 5\cdot 3 = 1.5, 4.7, 7.8 \]

\[ t = 2.5 \text{ i.e. no solution} \]

5 marks out of 7
Circle $C_1$ has equation $(x+1)^2 + (y-1)^2 = 121$.

A circle $C_2$ with equation $x^2 + y^2 - 4x + 6y + p = 0$ is drawn inside $C_1$.

The circles have no points of contact.

What is the range of values of $p$?

### Generic Scheme

| 7 | ic state centre of $C_1$ | $(-1, 1)$ | 1
| 8 | ic state radius of $C_1$ | $\sqrt{121}$ | 2
| 9 | pd find radius of $C_2$ in terms of $p$ | $\sqrt{13-p}$ | 3
| 10 | ic interpret upper bound for $p$ | $p < 13$ | 4
| 11 | ic find distance between centres ($d$) | $5$ | 5
| 12 | ss identify relevant relationship | Stated explicitly | 6
| 13 | ic develop relationship by squaring | 7
| 14 | pd find lower bound for $p$ | $p > -23$ | 8

### Illustrative Scheme

| 7 | 1 | 2
| 8 | 3
| 9 | 4

### Notes

1. Treat as bad form the use of $c$ in lieu of $p$.
2. The evidence for $\star^5$ must involve an inequality, but may be in words.
3. Treat $\sqrt{13-p}$ as bad form as long as it is clear that the candidate is using $\sqrt{13-p}$.
4. Candidates who are only working with an equation lose both $\star^5$ and $\star^8$, however, $\star^8$ may still be available.
5. $\star^9$ is only available to candidates who solve an inequality involving a negative coefficient of $p$.

### Regularly occurring responses

<table>
<thead>
<tr>
<th>Response 1A</th>
<th>Response 1B</th>
<th>Response 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marks 1 to 3 gained</td>
<td>$C_1 = (-1, 1)$</td>
<td>For marks 7 to 9</td>
</tr>
<tr>
<td>$\star^5$</td>
<td>$C_2 = (2, -3)$</td>
<td>$\sqrt{13-p} &lt; 6$</td>
</tr>
<tr>
<td>$\star^6$</td>
<td>$r_1 = 11 \checkmark \star^2$</td>
<td>$\sqrt{13-p} &lt; 6$</td>
</tr>
<tr>
<td>$\star^7$</td>
<td>$r_2 = \sqrt{13+p} \times \star^4$</td>
<td>$169-p &lt; 36$</td>
</tr>
<tr>
<td>$\star^8$</td>
<td>$d = 5 \checkmark \star^6$</td>
<td>$-p &lt; -133$</td>
</tr>
<tr>
<td>$\star^9$</td>
<td>$\sqrt{13+p} &lt; 11 \checkmark \star^7$</td>
<td>$p &gt; 133$ \times \star$</td>
</tr>
<tr>
<td>$\star^8$</td>
<td>$13+p &lt; 121 \checkmark \star^8$</td>
<td>penalise the use of $\leq$ and/or $\geq$ only.</td>
</tr>
<tr>
<td>$\star^9$</td>
<td>$p &lt; 108 \times \star^9$</td>
<td></td>
</tr>
</tbody>
</table>

**Response 3** (see note 4)

\[
\begin{align*}
\sqrt{13-p} &= 0 \\
p &= 13 \times \star^5 \\
\sqrt{13-p} &= 6 \times \star^7 \\
13-p &= 36 \times \star^8 \\
p &= -23 \times \star^9
\end{align*}
\]

**Response 4**

\[
\begin{align*}
\sqrt{13-p} &\geq 0 \\
p &\leq 13 \times \star^5 \\
\sqrt{13-p} &\leq 6 \times \star^7 \\
p &\geq -23 \times \star^9
\end{align*}
\]

**Response 5**

\[
\begin{align*}
0 &< \sqrt{13-p} < 6 \checkmark \star^7 \\
0 &< 13-p < 36 \checkmark \star^8 \\
-13 &< -p < 23 \\
\checkmark \star^5 \\
\text{so } p &< 13 \text{ and } p > -23 \checkmark \star^9 \\
or &-23 < p < 13 \checkmark \star^5
\end{align*}
\]
### Regularly occurring responses

**Response 6**

\[(x - 2)^2 + (y + 3)^2 = 13 - p\]

1. \(13 - p < 121\) ✗ ✗ ✗
2. \(p > -108\) ✗ ✗ ✗