2012 Mathematics

Higher

Finalised Marking Instructions

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General Comments

These marking instructions are for use with the 2012 Higher Mathematics Examination.

For each question the marking instructions are in two sections, namely Illustrative Scheme and Generic Scheme. The Illustrative Scheme covers methods which are commonly seen throughout the marking. The Generic Scheme indicates the rationale for which each mark is awarded. In general markers should use the Illustrative Scheme and only use the Generic Scheme where a candidate has used a method not covered in the Illustrative Scheme.

All markers should apply the following general marking principles throughout their marking:

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than deducted for what is wrong.

2. Award one mark for each •. There are no half marks.

3. The mark awarded for each part of a question should be entered in the outer right hand margin, opposite the end of the working concerned. The marks should correspond to those on the question paper and these marking instructions. Only the mark, as a whole number, should be written.

4. Where a candidate has not been awarded any marks for a question, or part of a question, 0 should be written in the right hand margin against their answer. It should not be left blank.

5. Every page of a candidate’s script should be checked for working. Unless blank, every page which is devoid of a marking symbol should have a tick placed in the bottom right hand margin.

6. Where the solution to part of a question is fragmented and continues later in the script, the marks should be recorded at the end of the solution. This should be indicated with a down arrow (↓), in the margin, at the earlier stages.

7. Working subsequent to an error must be followed through, with possible full marks for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.

8. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Throughout this paper, unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.
Marking Symbols

No comments or words should be written on scripts. Please use the following and the symbols indicated on the welcome letter and from comment 6 on the previous page.

A tick should be used where a piece of working is correct and gains a mark. Markers must check through the whole of a response, ticking the work only where a mark is awarded.

At the point where an error occurs, the error should be underlined and a cross used to indicate where a mark has not been awarded. If no mark is lost the error should only be underlined, i.e. a cross is only used where a mark is not awarded.

A cross-tick should be used to indicate “correct” working where a mark is awarded as a result of follow through from an error.

A double cross-tick should be used to indicate correct working which is irrelevant or insufficient to score any marks. This should also be used for working which has been eased.

A tilde should be used to indicate a minor error which is not being penalised, e.g. bad form.

This should be used where a candidate is given the benefit of the doubt.

A roof should be used to show that something is missing, such as part of a solution or a crucial step in the working.

These will help markers to maintain consistency in their marking and will assist the examiners in the later stages of SQA procedures.

The examples below illustrate the use of the marking symbols.

**Example 1**

\[ y = x^3 - 6x^2 \]

\[ \frac{dy}{dx} = 3x^2 - 12 \]

\[ 3x^2 - 12 = 0 \]

\[ x = 2 \]

\[ y = -16 \]

**Example 2**

A(4,4,0), B(2,2,6), C(2,2,0)

\[ \overline{AB} = \overrightarrow{b} + a = \begin{pmatrix} 6 \\ 6 \end{pmatrix} \]

\[ \overline{AC} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \]

Since the remainder is 0, \( x - 4 \) must be a factor. \( x = 4 \) or \( x = -1 \) or \( x = 2 \)

\[ (x^2 - x - 2)\]

\[ (x-4)(x+1)(x-2) \]

\[ x = 4 \text{ or } x = -1 \text{ or } x = 2 \]
In general, as a consequence of an error perceived to be trivial, casual or insignificant, e.g. $6 \times 6 = 12$, candidates lose the opportunity of gaining a mark. But note example 4 in comment 9 and comment 11.

Where a transcription error (paper to script or within script) occurs, the candidate should be penalised, e.g.

This is a transcription error and so the mark is not awarded.

Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt.

Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example: Point of intersection of line with curve

Illustrative Scheme: $^5 \quad x = 2, x = -4$  
$^6 \quad y = 5, y = -7$

Cross marked: $^5 \quad x = 2, y = 5$  
$^6 \quad x = -4, y = -7$

Markers should choose whichever method benefits the candidate, but not a combination of both.

In final answers, numerical values should be simplified as far as possible.

Examples: $\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1 \frac{1}{4}$  
$\frac{43}{7}$ must be simplified to 43  
$\frac{15}{0.3}$ must be simplified to 50  
$\sqrt{64}$ must be simplified to 8

The square root of perfect squares up to and including 100 must be known.

Regularly occurring responses (ROR) are shown in the marking instructions to help mark common and/or non-routine solutions. RORs may also be used as a guide in marking similar non-routine candidate responses.

Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a correct answer;
- Correct working in the wrong part of a question;
- Legitimate variations in numerical answers, e.g. angles in degrees rounded to nearest degree;
- Omission of units;
- Bad form;
- Repeated error within a question, but not between questions or papers.
16 In any ‘Show that . . .’ question, where the candidate has to arrive at a formula, the last mark of that part is not available as a follow through from a previous error.

17 All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate’s response. Marks may still be available later in the question so reference must be made continually to the marking instructions. All working must be checked: the appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.

18 In the exceptional circumstance where you are in doubt whether a mark should or should not be awarded, consult your Team Leader (TL).

19 Scored out working which has not been replaced should be marked where still legible. However, if the scored out working has been replaced, only the work which has not been scored out should be marked.

20 A valid approach, within Mathematical problem solving, is to try different strategies. Where this occurs, all working should be marked. The mark awarded to the candidate is from the highest scoring strategy. This is distinctly different from the candidate who gives two or more solutions to a question/part of a question, deliberately leaving all solutions, hoping to gain some benefit. All such contradictory responses should be marked and the lowest mark given.

21 It is of great importance that the utmost care should be exercised in totalling the marks. The recommended procedure is as follows:

   Step 1  Manually calculate the total from the candidate’s script.
   Step 2  Check this total using the grid issued with these marking instructions.
   Step 3  In EMC, enter the marks and obtain a total, which should now be compared to the manual total.

   This procedure enables markers to identify and rectify any errors in data entry before submitting each candidate’s marks.

22 The candidate’s script for Paper 2 should be placed inside the script for Paper 1, and the candidate’s total score (i.e. Paper 1 Section B + Paper 2) written in the space provided on the front cover of the script for Paper 1.

23 In cases of difficulty, covered neither in detail nor in principle in these instructions, markers should contact their TL in the first instance. A referral to the Principal Assessor (PA) should only be made in consultation with the TL. Further details of PA Referrals can be found in The General Marking Instructions.
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C</td>
</tr>
<tr>
<td>2</td>
<td>D</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
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<tr>
<td>5</td>
<td>A</td>
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<tr>
<td>6</td>
<td>C</td>
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<tr>
<td>7</td>
<td>A</td>
</tr>
<tr>
<td>8</td>
<td>C</td>
</tr>
<tr>
<td>9</td>
<td>A</td>
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<tr>
<td>10</td>
<td>B</td>
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<tr>
<td>11</td>
<td>D</td>
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<tr>
<td>12</td>
<td>B</td>
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<td>13</td>
<td>D</td>
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<td>14</td>
<td>A</td>
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<td>15</td>
<td>D</td>
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<td>16</td>
<td>C</td>
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<td>17</td>
<td>D</td>
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<td>18</td>
<td>B</td>
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<tr>
<td>19</td>
<td>B</td>
</tr>
<tr>
<td>20</td>
<td>A</td>
</tr>
</tbody>
</table>

**Summary**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>6</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
</tr>
<tr>
<td>D</td>
<td>5</td>
</tr>
</tbody>
</table>
21 (a) (i) Show that \((x - 4)\) is a factor of \(x^3 - 5x^2 + 2x + 8\).
(ii) Factorise \(x^3 - 5x^2 + 2x + 8\) fully.
(iii) Solve \(x^3 - 5x^2 + 2x + 8 = 0\).

<table>
<thead>
<tr>
<th>Generic Scheme</th>
<th>Illustrative Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong> ss know to use (x = 4)</td>
<td>Method 1: Using synthetic division</td>
</tr>
<tr>
<td><strong>2</strong> pd complete evaluation</td>
<td></td>
</tr>
<tr>
<td><strong>3</strong> ic state conclusion</td>
<td><strong>3</strong> 'remainder is zero so ((x - 4)) is a factor' stated, or implied by <strong>5</strong></td>
</tr>
<tr>
<td><strong>4</strong> ic find quadratic factor</td>
<td><strong>4</strong> (x^2 - x - 2) stated explicitly in any order</td>
</tr>
<tr>
<td><strong>5</strong> pd factorise completely</td>
<td><strong>5</strong> ((x - 4)(x - 2)(x + 1)) stated explicitly in any order</td>
</tr>
<tr>
<td><strong>6</strong> ic state solutions</td>
<td><strong>6</strong> ( -1, 2, 4)</td>
</tr>
</tbody>
</table>

Method 2: Using substitution and inspection

| **1** know to use \(x = 4\) |
| **2** \(64 - 80 + 8 + 8 = 0\) |
| **3** \((x - 4)\) is a factor |
| **4** \((x - 4)(x^2 - x - 2)\) stated, or implied by **5** |
| **5** \((x - 4)(x - 2)(x + 1)\) stated explicitly in any order |
| **6** \( -1, 2, 4\) |

**Notes**

1. \(*^3\) is only available as a consequence of the evidence for \(*^1\) and \(*^2\).
2. Communication at \(*^3\) must be consistent with working at \(*^2\).
   i.e. candidate’s working must arrive legitimately at zero before \(*^3\) is awarded.
   If the remainder is not 0 then an appropriate statement would be ‘\((x - 4)\) is not a factor’.
3. Accept any of the following for \(*^5\):
   - ‘\(f(4) = 0\) so \((x - 4)\) is a factor’
   - ‘since remainder is 0, it is a factor’
   - the 0 from table linked to word ‘factor’ by e.g. ‘so’, ‘hence’, ‘\(\Rightarrow\)’.
4. Do not accept any of the following for \(*^5\):
   - double underlining the zero or boxing in the zero, without a comment
   - ‘\(x = 4\) is a factor’, ‘\((x + 4)\) is a factor’, ‘\(x = 4\) is a root’, ‘\((x - 4)\) is a root’
   - the word ‘factor’ only, with no link.
5. To gain \(*^6\), \(4, -1, 2\) must appear together in (a).
6. \((x - 4)(x - 2)(x + 1)\) leading to \((4, 0), (2, 0)\) and \((-1, 0)\) only does not gain \(*^6\).
7. \((x - 2)(x + 1)\) only, leading to \(x = 2, x = -1\) does not gain \(*^5\) as equation solved is not a cubic.
8. Candidates who attempt to solve the cubic equation subsequent to \(x = -1, 2, 4\) and obtain different solutions, or no solutions, cannot gain \(*^6\).
21 (b) The diagram shows the curve with equation \( y = x^3 - 5x^2 + 2x + 8 \).

The curve crosses the x-axis at P, Q and R. Determine the shaded area.

---

**Generic Scheme**

**Illustrative Scheme**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>ic</td>
</tr>
<tr>
<td></td>
<td>identify ( x_Q ) from working in (a)</td>
</tr>
<tr>
<td>8</td>
<td>ic</td>
</tr>
<tr>
<td></td>
<td>interpret appropriate limits</td>
</tr>
<tr>
<td>9</td>
<td>ss</td>
</tr>
<tr>
<td></td>
<td>know and start to integrate</td>
</tr>
<tr>
<td>10</td>
<td>pd</td>
</tr>
<tr>
<td></td>
<td>complete integration</td>
</tr>
<tr>
<td>11</td>
<td>ic</td>
</tr>
<tr>
<td></td>
<td>substitute limits</td>
</tr>
<tr>
<td>12</td>
<td>pd</td>
</tr>
<tr>
<td></td>
<td>state area</td>
</tr>
</tbody>
</table>

**Notes**

9. Evidence for \( \ast^7 \) and \( \ast^8 \) may not appear until \( \ast^{11} \) stage.

10. Where a candidate differentiates one or more terms at \( \ast^9 \), then \( \ast^9 \), \( \ast^{10} \), \( \ast^{11} \) and \( \ast^{12} \) are not available.

11. Candidates who substitute at \( \ast^{11} \), without integrating at \( \ast^9 \), do not gain \( \ast^9 \), \( \ast^{10} \), \( \ast^{11} \) and \( \ast^{12} \).

12. For candidates who make an error in (a), \( \ast^8 \) is only available if 0 is the lower limit and a positive integer value is used for the upper limit.

13. \( \ast^{11} \) is only available where both limits are numerical values.

14. Candidates must show evidence that they have considered the lower limit 0 in their substitution at \( \ast^{11} \) stage.

**Regularly occurring responses**

**Response 1**

Candidates who use Q throughout

**Candidate A**

\[
\int_{0}^{Q} (x^3 - 5x^2 + 2x + 8) \, dx
\]

\[
= \left. \left( \frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{2}{2}x^2 + 8x \right) \right|_{0}^{Q}
\]

\[
= \frac{1}{4}Q^4 - \frac{5}{3}Q^3 + Q^2 + 8Q - 0
\]

However, if Q is replaced by 2 at this stage, and working continues, all 6 marks may still be available.

**Response 2**

Dealing with negatives

**Candidate B**

\[
\int_{0}^{Q} (x^3 - 5x^2 + 2x + 8) \, dx
\]

\[
= \left. \left( \frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{2}{2}x^2 + 8x \right) \right|_{0}^{Q}
\]

\[
= \frac{1}{4}(1)^4 - \frac{5}{3}(1)^3 + \frac{2}{2}(1)^2 + 8(-1) - 0
\]

\[
= \frac{1}{4} - \frac{5}{3} - \frac{2}{2} + 8 - 0
\]

\[
= -\frac{61}{12}
\]

cannot be negative so \( \ast^{11} \)

but

\[
A = \frac{61}{12}
\]
The expression \( \cos x - \sqrt{3} \sin x \) can be written in the form \( k \cos(x + a) \) where \( k > 0 \) and \( 0 \leq a < 2\pi \).

Calculate the values of \( k \) and \( a \).

### Generic Scheme

<table>
<thead>
<tr>
<th>Candidate A</th>
<th>Candidate B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k \cos a )</td>
<td>( k \cos a = \sqrt{3} \times \star )</td>
</tr>
<tr>
<td>( k \sin a = \sqrt{3} \times \star )</td>
<td>( k \sin a = 1 \times \star )</td>
</tr>
<tr>
<td>( \tan a = \frac{1}{\sqrt{3}} ) so ( a = \frac{\pi}{6} \times \star )</td>
<td>( \tan a = \frac{1}{\sqrt{3}} ) so ( a = \frac{\pi}{6} \times \star )</td>
</tr>
</tbody>
</table>

### Notes
1. Treat \( k \cos x \cos a - \sin x \sin a \) as bad form only if the equations at the \( \star \) stage both contain \( k \).
2. \( 2 \cos x \cos a - 2 \sin x \sin a \) or \( 2(\cos x \cos a - \sin x \sin a) \) is acceptable for \( \star \) and \( \star \).
3. Accept \( k \cos a = 1 \) and \( -k \sin a = -\sqrt{3} \) for \( \star \).
4. \( \star \) is not available for \( k \cos x = 1 \) and \( k \sin x = \sqrt{3} \), however, \( \star \) is still available.
5. \( \star \) is only available for a single value of \( a \).
6. Candidates who work in degrees and do not convert to radian measure in \( a \) do not gain \( \star \).
7. Candidates may use any form of the wave equation for \( \star \), \( \star \) and \( \star \), however, \( \star \) is only available if the value of \( a \) is interpreted for the form \( k \cos(x + a) \).

### Regularly occurring responses

**Response 1:** Missing information in working

- Candidate A
  - \( 2 \cos a = 1 \)
  - \( -2 \sin a = -\sqrt{3} \)
  - \( \tan a = \frac{\sqrt{3}}{1} \)
  - \( a = \frac{\pi}{3} \)

- Candidate B
  - \( \cos a = 1 \)
  - \( \sin a = \sqrt{3} \)
  - \( \tan a = \frac{\pi}{2} \)
  - \( a = \frac{\pi}{3} \)

3 marks out of 4

**Response 2:** Correct expansion of \( k \cos(x + a) \) and possible errors for \( \star \) and \( \star \)

- Candidate C
  - \( k \cos a = 1 \)
  - \( k \sin a = \sqrt{3} \)
  - \( \tan a = \frac{1}{\sqrt{3}} \) so \( a = \frac{\pi}{3} \)

- Candidate D
  - \( k \cos a = \sqrt{3} \)
  - \( k \sin a = 1 \)
  - \( \tan a = \frac{1}{\sqrt{3}} \) so \( a = \frac{\pi}{6} \)

- Candidate E
  - \( k \cos a = 1 \)
  - \( k \sin a = -\sqrt{3} \)
  - \( \tan a = -\sqrt{3} \) so \( a = \frac{5\pi}{3} \)

0 marks out of 4

Not consistent with evidence at \( \star \).

**Response 3:** Labelling incorrect using \( \cos(A + B) = \cos A \cos B - \sin A \sin B \) from formula list

- Candidate F
  - \( k \cos A \cos B - k \sin A \sin B \) \( \times \star \)
  - \( k \cos a = 1 \)
  - \( k \sin a = \sqrt{3} \)
  - \( \tan a = \sqrt{3} \) so \( a = \frac{\pi}{3} \)

- Candidate G
  - \( k \cos A \cos B - k \sin A \sin B \) \( \times \star \)
  - \( k \cos x = 1 \)
  - \( k \sin x = \sqrt{3} \)
  - \( \tan x = \sqrt{3} \) so \( x = \frac{\pi}{3} \)

- Candidate H
  - \( k \cos A \cos B - k \sin A \sin B \) \( \times \star \)
  - \( k \cos B = 1 \)
  - \( k \sin B = \sqrt{3} \)
  - \( \tan B = \sqrt{3} \) so \( B = \frac{\pi}{3} \)

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Page 9
22 (b) Find the points of intersection of the graph of \( y = \cos x - \sqrt{3} \sin x \) with the \( x \) and \( y \) axes, in the interval \( 0 \leq x \leq 2\pi \).

<table>
<thead>
<tr>
<th>Generic Scheme</th>
<th>Illustrative Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bullet^5 ) ic interpret ( y )-intercept</td>
<td>( \bullet^5 ) 1</td>
</tr>
<tr>
<td>( \bullet^6 ) ss strategy for finding roots</td>
<td>( \bullet^6 ) e.g. ( 2 \cos \left( x + \frac{\pi}{3} \right) = 0 ) or ( \sqrt{3} \sin x = \cos x )</td>
</tr>
<tr>
<td>( \bullet^7 ) ic state both roots</td>
<td>( \bullet^7 ) ( \frac{\pi}{6}, \frac{7\pi}{6} )</td>
</tr>
</tbody>
</table>

Notes

8. Candidates should only be penalised once for leaving their answer in degrees in (a) and (b).

9. If the expression used in (b) is not consistent with (a) then only \( \bullet^5 \) and \( \bullet^7 \) are available.

10. Correct roots without working cannot gain \( \bullet^6 \) but will gain \( \bullet^7 \).

11. Candidates should only be penalised once for not simplifying \( \sqrt{4} \) in (a) and (b).

Regularly occurring responses

Response 4: Communication for \( \bullet^5 \)

Candidate I

(1, 0) without working. \( \times \bullet^5 \)

Candidate J

\( \cos 0 - \sqrt{3} \sin 0 = 1 \) \( \checkmark \bullet^5 \)

so (1, 0).

Response 5: Follow through from a wrong value of \( a \)

Candidate K

From (a) \( a = \frac{\pi}{6} \) \( \checkmark \bullet^6 \)

then in (b) \( x = \frac{\pi}{3}, \frac{4\pi}{3} \) only \( \checkmark \bullet^7 \)

Candidate L

From (a) \( a = 60^\circ \) \( \checkmark \bullet^6 \)

then in (b) \( x = 30^\circ, 210^\circ \) only \( \checkmark \bullet^7 \)

Note 10

Response 6: Root or graphical approach

Candidate M

\( \frac{\pi}{2} - \frac{\pi}{3} \) and \( \frac{3\pi}{2} - \frac{\pi}{3} \) \( \checkmark \bullet^6 \)

\( = \frac{\pi}{6} \) and \( \frac{7\pi}{6} \) \( \checkmark \bullet^7 \)

Candidate N

(a) \( 60^\circ \) \( \checkmark \bullet^4 \)

(b) \( 90^\circ \) \( 270^\circ \)

When \( x = 30^\circ, 210^\circ \) \( \times \bullet^7 \)

Response 7: Circular argument not leading anywhere

Candidate P

\( 2 \cos x \times \frac{1}{2} - 2 \sin x \times \frac{\sqrt{3}}{2} = 0 \) \( \checkmark \bullet^6 \)

\( \cos x - \sqrt{3} \sin x = 0 \) \( \times \bullet^7 \)

Response 8: Transcription error in (b)

Candidate Q

(a) correct

(b) \( 2 \cos \left( x - \frac{\pi}{3} \right) = 0 \) so \( x = \frac{5\pi}{6}, \frac{11\pi}{6} \) \( \checkmark \bullet^6 \)

\( y = 2 \cos \left( 0 - \frac{\pi}{3} \right) = 2 \cos \left( \frac{\pi}{3} \right) = 1 \) \( \checkmark \bullet^7 \)

\( x - \frac{\pi}{3} \) is penalised as \( x + \frac{\pi}{3} \) obtained in (a).

However \( \bullet^5 \) and \( \bullet^7 \) are still available as follow through. See Note 9.
23 (a) Find the equation of \( \ell_1 \), the perpendicular bisector of the line joining \( P(3, 3) \) to \( Q(-1, 9) \).

### Generic Scheme

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>SS find midpoint of ( PQ )</td>
</tr>
<tr>
<td>2.</td>
<td>SS find gradient of ( PQ )</td>
</tr>
<tr>
<td>3.</td>
<td>IC interpret perpendicular gradient</td>
</tr>
<tr>
<td>4.</td>
<td>IC state equation of perp. bisector</td>
</tr>
</tbody>
</table>

### Illustrative Scheme

\[ m_{PQ} = \frac{9 - 3}{-1 - 3} = -\frac{3}{2} \]

\[ m_1 = \frac{1}{3} \]

\[ y - 3 = \frac{1}{3}(x - 1) \]

### Notes

1. \( \bullet^4 \) is only available if a midpoint and a perpendicular gradient are used.

2. Candidates who use \( y = mx + c \) must obtain a numerical value for \( c \) before \( \bullet^4 \) is available.

### Regularly occurring responses

**Response 1**: Candidates who use wrong midpoint or no midpoint

<table>
<thead>
<tr>
<th>Candidate A</th>
<th>Candidate B</th>
</tr>
</thead>
<tbody>
<tr>
<td>midpoint ( M(2, -6) )</td>
<td>( m_{PQ} = -3 )</td>
</tr>
<tr>
<td>( m_{MQ} = -5 )</td>
<td>( m_1 = \frac{1}{3} )</td>
</tr>
<tr>
<td>( y = \frac{1}{5}(x - 2) )</td>
<td>( \text{using } R, \ y = \frac{1}{3}(x - 1) )</td>
</tr>
</tbody>
</table>

23 (b) Find the equation of \( \ell_2 \) which is parallel to \( PQ \) and passes through \( R(1, -2) \).

### Generic Scheme

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>IC use parallel gradients</td>
</tr>
<tr>
<td>2.</td>
<td>IC state equation of line</td>
</tr>
</tbody>
</table>

### Illustrative Scheme

\[ m = -3 \]

\[ y = \frac{1}{3}(x - 1) \]

### Notes

3. \( \bullet^6 \) is only available to candidates who use \( R \) and their gradient of \( PQ \) from (a).

### Regularly occurring responses

**Response 2**: Not using parallel gradient for equation

<table>
<thead>
<tr>
<th>Candidate C</th>
<th>Candidate D</th>
<th>Candidate E</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = \frac{1}{3}(x - 1) )</td>
<td>Parallel so same gradients</td>
<td></td>
</tr>
<tr>
<td>( m = \frac{1}{3} )</td>
<td>( m = -3 )</td>
<td></td>
</tr>
</tbody>
</table>

If \( m_{PQ} = -3 \) only do not award \( \bullet^5 \)
### 23 (c) Find the point of intersection of \( \ell_1 \) and \( \ell_2 \).

**Generic Scheme**

- \( \bullet^7 \) ss use valid approach
- \( \bullet^8 \) pd solve for one variable
- \( \bullet^9 \) pd solve for other variable

**Illustrative Scheme**

- e.g. \( x - 3y = -8 \) and \( 9x + 3y = 3 \)
- or \( -3x + 1 = \frac{1}{3}x + \frac{8}{3} \)
- or \( 3(3y - 8) + y = 1 \)
- e.g. \( x = -\frac{1}{2} \)
- e.g. \( y = \frac{5}{2} \)

**Notes**

4. Neither \( x - 3y = -8 \) and \( 3x + y = 1 \) nor \( y = -3x + 1 \) and \( 3y = x + 8 \) are sufficient to gain \( \bullet^7 \).

5. \( \bullet^7, \bullet^8 \) and \( \bullet^9 \) are not available to candidates who:
   - Equate zeros
   - Give answers only, without working
   - Use R for equations in both (a) and (b)
   - Use the same gradient for the lines in (a) and (b).

### 23 (d) Hence find the shortest distance between PQ and \( \ell_2 \).

**Generic Scheme**

- \( \bullet^{10} \) ss identify appropriate points
- \( \bullet^{11} \) pd calculate distance

**Illustrative Scheme**

- (1, 3), (1, 2) \( \bullet^{10} \)
- \( \bullet^{11} \sqrt{\frac{5}{2}} \) accept \( \sqrt{\frac{10}{2}} \) or \( \sqrt{2.5} \)

**Notes**

6. \( \bullet^{10} \) and \( \bullet^{11} \) are only available for considering the distance between the midpoint of PQ and the candidate’s answer from (c) or for considering the perpendicular distance from P or Q to \( \ell_2 \).

7. At least one coordinate at \( \bullet^{10} \) stage must be a fraction for \( \bullet^{11} \) to be available.

8. There should only be one calculation of a distance to gain \( \bullet^{11} \).

**Regularly occurring responses**

**Response 3**: Following through from correct (a), (b) and (c)

**Candidate F**

(1, 3), (1, -2) \( \times \) \( \bullet^{10} \)

\[ d = 5 \] \( \times \) \( \bullet^{11} \)

**Response 4**: Following through from correct (a), (b) and (c)

**Candidate G**

(1, 3), \( \left(-\frac{1}{2}, \frac{5}{2}\right) \) \( \checkmark \) \( \bullet^{10} \)

\[ PR = \sqrt{5}, \text{QR}=\sqrt{125}, \text{ d } = \sqrt{2.5} \]

so \( \sqrt{2.5} \) is shortest distance. \( \times \) \( \bullet^{11} \)

If reference was made to this being the perpendicular distance then \( \bullet^{11} \) would be available.
1 Functions \( f \) and \( g \) are defined on the set of real numbers by

- \( f(x) = x^2 + 3 \)
- \( g(x) = x + 4 \)

(a) Find expressions for:

(i) \( f(g(x)) \);

(ii) \( g(f(x)) \).

<table>
<thead>
<tr>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Candidates must clearly identify which of their answers are ( f(g(x)) ) and ( g(f(x)) ); the minimum evidence for this could be as little as using (i) and (ii) as labels.</td>
</tr>
<tr>
<td>2. Candidates who interpret the composite functions as either ( f(x) \times g(x) ) or ( f(x) + g(x) ), do not gain any marks.</td>
</tr>
</tbody>
</table>

### Regularly occurring responses

**Response 1:** The first two marks are for either \( f(g(x)) \) or \( g(f(x)) \) correct. The third mark is for the other composite function.

<table>
<thead>
<tr>
<th>Candidate A</th>
<th>Candidate B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(g(x)) )</td>
<td>( f(g(x)) )</td>
</tr>
<tr>
<td>( = (x + 4)^2 + 3 )</td>
<td>( = (x + 7)^2 \times )</td>
</tr>
<tr>
<td>( \checkmark \bullet \checkmark \bullet )</td>
<td>( \checkmark \bullet \bullet )</td>
</tr>
</tbody>
</table>

2 marks out of 3

<table>
<thead>
<tr>
<th>Candidate C</th>
<th>Candidate D</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(g(x)) )</td>
<td>( f(g(x)) )</td>
</tr>
<tr>
<td>( = x^2 + 7 \times )</td>
<td>( = x^2 + 7 \times )</td>
</tr>
<tr>
<td>( \times \bullet \checkmark \bullet \bullet )</td>
<td>( \times \bullet \checkmark \bullet \bullet )</td>
</tr>
</tbody>
</table>

2 marks out of 3

<table>
<thead>
<tr>
<th>Candidate E</th>
<th>Candidate F</th>
<th>Candidate G</th>
<th>Candidate H</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (x + 4)^2 + 3 \times \bullet \checkmark \bullet \bullet )</td>
<td>( x^2 + 7 \times \bullet \bullet \bullet \bullet )</td>
<td>( x^2 + 7 ) ONLY</td>
<td></td>
</tr>
<tr>
<td>( x^2 + 7 \times \bullet \bullet \bullet \bullet )</td>
<td>( (x + 4)^2 + 3 \times \bullet \bullet \bullet \bullet )</td>
<td>( (x + 4)^2 + 3 ) ONLY</td>
<td></td>
</tr>
<tr>
<td>( x^2 + 7 \times \bullet \bullet \bullet \bullet )</td>
<td>( 0 ) marks out of 3</td>
<td>( 3 ) marks out of 3</td>
<td></td>
</tr>
<tr>
<td>( 2 ) marks out of 3</td>
<td>( 1 ) mark out of 3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1 (b) Show that \( f(g(x)) + g(f(x)) = 0 \) has no real roots.

### Generic Scheme

<table>
<thead>
<tr>
<th>Method 1: Discriminant</th>
<th>Method 2: Quadratic Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>•^i pd obtain a quadratic expression</td>
<td>•^i pd obtain a quadratic expression</td>
</tr>
<tr>
<td>•^v ss know to and use discriminant</td>
<td>•^v ss know to and use quadratic formula</td>
</tr>
<tr>
<td>•^v ic interpret result</td>
<td>•^v ic interpret result</td>
</tr>
</tbody>
</table>

### Illustrative Scheme

<table>
<thead>
<tr>
<th>Method 1: Discriminant</th>
<th>Method 2: Quadratic Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>•^i ( 2x^2 + 8x + 26 )</td>
<td>•^i ( 2x^2 + 8x + 26 )</td>
</tr>
<tr>
<td>•^v ( 8^2 - 4 \cdot 2 \cdot 26 ) or ( 4^2 - 4 \cdot 1 \cdot 13 ) stated, or implied by •^v</td>
<td>•^v ( -8 \pm \sqrt{8^2 - 4 \cdot 2 \cdot 26} ) ( \frac{2 \times 2}{2} ) stated, or implied by •^v</td>
</tr>
<tr>
<td>•^v (-144 &lt; 0 ) or (-36 &lt; 0 ) so no real roots</td>
<td>•^v ( \sqrt{-144} ) not possible so no real roots</td>
</tr>
</tbody>
</table>

### Notes

3. Candidates who use \( f(x) \times g(x) \) can gain no marks in (b) as a cubic will be obtained.

4. Candidates who use \( f(x) + g(x) \) do not gain •^i (eased) but •^v and •^v are available as follow through marks.

5. In method 1, any other formula masquerading as a discriminant cannot gain •^v and •^v.

6. •^i, •^v and •^v are only available if \( f(g(x)) + g(f(x)) \) simplifies to a quadratic expression of the form \( ax^2 + bx + c \), with \( a \) and \( c \) both non-zero.

7. •^v is only available for a numerical value, calculated correctly from the candidate’s response at •^i, and leading to no real roots.

8. Do not accept for •^v:
   • ‘no roots’ in lieu of ‘no real roots’
   • ‘maths error’ or ‘ma error’.

9. Candidates who use the word derivative instead of discriminant should not be penalised.

### Regularly occurring responses

**Response 4:** Candidates who do not simplify the value of their discriminant

**Candidate I**

\[
8^2 - 4 \cdot 2 \cdot 26 \quad \checkmark \\
= 64 - 208 < 0 \quad \text{so no real roots} \quad \checkmark \\
\]

**Candidate J**

\[
\sqrt{8^2 - 4 \cdot 2 \cdot 26} \quad \checkmark \\
= \sqrt{-144} \quad \text{not valid} \quad \checkmark \\
\text{so no real roots} \quad \checkmark \\
\]

**Candidate L**

no real roots if \( b^2 - 4ac < 0 \)

\[
64 - 208 = -144 \quad \checkmark \\
\]

**Response 5:** Acceptable communication marks

**Method 1**

**Candidate K**

\[
\text{Discriminant} = \sqrt{8^2 - 4 \cdot 2 \cdot 26} \quad \checkmark \\
= \sqrt{-144} \quad \text{can’t find root of negative} \quad \checkmark \\
\text{so no real roots} \quad \checkmark \\
\]

**Method 2**

**Candidate M**

\[
\frac{-(-4) \pm \sqrt{8^2 - 4 \cdot 2 \cdot 26}}{2 \times 2} \quad \checkmark \\
= \frac{4 \pm \sqrt{-144}}{4} \quad \text{no} \sqrt{-ve} \quad \checkmark \\
\text{so no real roots} \quad \checkmark \\
\]
2 (a) Relative to a suitable set of coordinate axes, diagram 1 shows the line

\[ 2x - y + 5 = 0 \]

intersecting the circle \( x^2 + y^2 - 6x - 2y = 30 = 0 \) at the points P and Q.

Find the coordinates of P and Q.

Diagram 1

---

**Generic Scheme**

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ss rearrange linear equation</td>
</tr>
<tr>
<td>2</td>
<td>ss substitute into circle</td>
</tr>
<tr>
<td>3</td>
<td>pd express in standard form</td>
</tr>
<tr>
<td>4</td>
<td>pd start to solve</td>
</tr>
<tr>
<td>5</td>
<td>ic state roots</td>
</tr>
<tr>
<td>6</td>
<td>pd determine corresponding y-coordinates</td>
</tr>
</tbody>
</table>

**Illustrative Scheme**

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Substituting for ( y ) stated, or implied by ( \star^2 )</td>
</tr>
<tr>
<td>2</td>
<td>( y = 2x + 5 )</td>
</tr>
<tr>
<td>3</td>
<td>( \ldots (2x + 5)^2 \ldots - 2(2x + 5) \ldots )</td>
</tr>
<tr>
<td>4</td>
<td>( 5x^2 + 10x - 15 = 0 ) must appear at the ( \star^3 )</td>
</tr>
<tr>
<td>5</td>
<td>e.g. ( 5(x + 3)(x - 1) ) or ( \star^4 ) stage to gain ( \star^3 )</td>
</tr>
<tr>
<td>6</td>
<td>( x = -3 ) and ( x = 1 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Substituting for ( x ) stated, or implied by ( \star^2 )</td>
</tr>
<tr>
<td>2</td>
<td>( x = \frac{y - 5}{2} )</td>
</tr>
<tr>
<td>3</td>
<td>( \left( \frac{y - 5}{2} \right)^2 \ldots - 6 \left( \frac{y - 5}{2} \right) \ldots )</td>
</tr>
<tr>
<td>4</td>
<td>( 5y^2 - 30y - 35 = 0 ) must appear at the ( \star^3 )</td>
</tr>
<tr>
<td>5</td>
<td>e.g. ( 5(y + 1)(y - 7) ) or ( \star^4 ) stage to gain ( \star^3 )</td>
</tr>
<tr>
<td>6</td>
<td>( y = -1 ) and ( y = 7 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( x = -3 ) and ( x = 1 )</td>
</tr>
</tbody>
</table>

---

**Notes**

1. At \( \star^4 \) the quadratic must lead to two real distinct roots for \( \star^5 \) and \( \star^6 \) to be available.
2. Cross marking is available here for \( \star^5 \) and \( \star^6 \).
3. Candidates do not need to distinguish between points P and Q.

**Regularly occurring responses**

**Response 1** : Solving quadratic equation

<table>
<thead>
<tr>
<th>Candidate A</th>
<th>5x^2 + 10x + 5 = 0</th>
<th>x = -1</th>
<th>y = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Candidate B</td>
<td>y = 2x + 5</td>
<td>( x^2 + (2x + 5)^2 - 6x - 2(7x + 5) - 30 = 0 )</td>
<td>( x = \pm \sqrt{3} )</td>
</tr>
<tr>
<td>Candidate C</td>
<td>5x^2 + 10x - 15 = 0</td>
<td>5x^2 + 10x = 15</td>
<td>x(x + 2) = 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>x = 3 ( x = 1 )</td>
<td>y = 11 ( y = 7 )</td>
</tr>
</tbody>
</table>

Cross marking is **not** available here for \( \star^5 \) and \( \star^6 \), as there are no distinct roots. See Note 1.
Diagram 2 shows the circle from (a) and a second congruent circle, which also passes through P and Q.

Determine the equation of this second circle.

Diagram 2

---

**Generic Scheme**

1. Centre of original circle
2. Radius of original circle

**Illustrative Scheme**

1. (3, 1)
2. \( \sqrt{40} \)
   
   Accept \( r^2 = 40 \)

---

**Method 1:** Using midpoint

- \( \bullet^7 \) ss midpoint of chord
- \( \bullet^8 \) evidence for finding new centre
- \( \bullet^9 \) ic centre of new circle
- \( \bullet^{10} \) ic equation of new circle

**Method 2:** Stepping out using P and Q

- \( \bullet^7 \) evidence of \( C \) to \( P \)
- \( \bullet^8 \) evidence of \( Q \) to \( C \)
- \( \bullet^9 \) ss evidence of centre
- \( \bullet^{10} \) ss evidence of \( Q \) to \( C \), or \( P \) to \( C \)
- \( \bullet^{11} \) ic equation of new circle

---

**Notes**

4. The evidence for \( \bullet^7 \) and \( \bullet^8 \) may appear in (a).

5. Centre \((−5, 5)\) **without working** in method 1 may still gain \( \bullet^{12} \) but not \( \bullet^{10} \) or \( \bullet^{11} \), in method 2 may still gain \( \bullet^{12} \) but not \( \bullet^9 \), \( \bullet^{10} \) or \( \bullet^{11} \).

   Any other centre **without working** in method 1 does not gain \( \bullet^{10} \), \( \bullet^{11} \) or \( \bullet^{12} \), in method 2 does not gain \( \bullet^9 \), \( \bullet^{10} \), \( \bullet^{11} \) or \( \bullet^{12} \).

6. The centre must have been clearly indicated before it is used at the \( \bullet^{12} \) stage.

7. Do not accept e.g. \( \sqrt{40} \) or 39.69, or any other decimal approximations for \( \bullet^{12} \).

8. The evidence for \( \bullet^8 \) may not appear until the candidate states the radius or equation of the second circle.

---

**Regularly occurring responses**

**Response 2:** Examples of evidence for stepping out for \( \bullet^{10} \) in method 1 or \( \bullet^9 \) or \( \bullet^{10} \) in method 2

**Response 3:** Examples of evidence which do not gain \( \bullet^{10} \) in method 1 for stepping out

---

Page 16
A function \( f \) is defined on the domain \( 0 \leq x \leq 3 \) by \( f(x) = x^3 - 2x^2 - 4x + 6 \).

Determine the maximum and minimum values of \( f \).
3 continued

Regularly occurring responses

Response 3: Solving quadratic equation

**Candidate H**

\[ f(x) = 3x^2 - 4x - 4 \]

- \( x = 4, \frac{4}{3} \)

**Candidate I**

\[ 3x^2 - 4x - 4 = 0 \]

- \( x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 3 \cdot (-4)}}{2 \cdot 3} \)

- Ignore omission of negative sign at square here.

Due to ‘method’ chosen, \( \bullet^3, \bullet^4, \bullet^3 \) and \( \bullet^7 \) are not available.

Response 4: Numerical approach

**Candidate J**

\[ f(0) = 6 \]

\[ f(3) = 3 \, \checkmark \, \bullet^6 \]

**Candidate K**

\[ f(0) = 6 \]

\[ f(1) = 1 \]

\[ f(2) = -2 \, \times \, \bullet^5 \]

\[ f(3) = 3 \, \checkmark \, \bullet^6 \]

**Candidate L**

\[ f(0) = 6 \]

\[ f(1) = 1 \]

\[ f(2) = -2 \, \times \, \bullet^5 \]

\[ f(3) = 3 \, \times \, \bullet^6 \]

\[ f(4) = 22 \]

For \( \bullet^6 \), \( f(2) \) must come from calculus and not from any other approach.

This candidate has stayed within the interval \( 0 \leq x \leq 3 \).

This candidate has gone out with the interval \( 0 \leq x \leq 3 \).
The diagram below shows the graph of a quartic \( y = h(x) \), with stationary points at \( x = 0 \) and \( x = 2 \).

On separate diagrams sketch the graphs of:

(a) \( y = h'(x) \);
(b) \( y = 2 - h(x) \).

**Generic Scheme**

**Illustrative Scheme**

4 (a)

1. ic identify roots
2. ic interpret point of inflection
3. ic complete cubic curve

4 (b)

1. ic reflection in \(-x\)-axis
2. ic translation \([0 \ 2]\)
3. ic annotation of ‘transformed’ graph

**Notes**

1. All graphs must include both the \( x \) and \( y \) axes (labelled or unlabelled), however the origin need not be labelled.
2. No marks are available unless a graph is attempted.
3. No marks are available to a candidate who makes several attempts at a graph on the same diagram, unless it is clear which is the final graph.
4. A linear graph gains no marks in both (a) and (b).

5. ‘Transformed’ here means a reflection followed by a translation.
6. \( \bullet^1 \) and \( \bullet^3 \) apply to the entire curve.
7. In each of the following circumstances:
   - Candidates who transform the original graph
   - Candidates who sketch a parabola in (a)
     - mark the candidate’s attempt as normal and unless a mark of 0 has been scored, deduct the last mark awarded. Indicate this with \( \times \) (see Regular occurring response G).
8. A reflection in any line parallel to the \( y \)-axis does not gain \( \bullet^1 \) or \( \bullet^6 \).
9. A translation other than \([0 \ 2] \) does not gain \( \bullet^5 \) or \( \bullet^6 \).
4 continued

Regularly occurring responses

In (a)

Curve does not only pass through 0 and 2 on x-axis.

Treat cusp as turning point.

In (a) and (b)

See Note 7

See Note 7

y-values of 2 should be roughly in line – too far apart here
A is the point \((3, -3, 0)\), B is \((2, -3, 1)\) and C is \((4, k, 0)\).

(a) (i) Express \(\overrightarrow{BA}\) and \(\overrightarrow{BC}\) in component form.

(ii) Show that \(\cos \angle ABC = \frac{3}{\sqrt{2(k^2 + 6k + 14)}}\).

---

### Notes

1. If the evidence for \(\bullet^3\) does not appear explicitly, then \(\bullet^3\) is only awarded if working for \(\bullet^7\) is attempted.

2. \(\bullet^7\) is dependent on gaining \(\bullet^3\), \(\bullet^5\) and \(\bullet^6\).

### Regularly occurring responses

**Response 1:** Calculating wrong angle

**Candidate A**

\[
\cos \angle AOC = \frac{\overrightarrow{OA} \cdot \overrightarrow{OC}}{|\overrightarrow{OA}| |\overrightarrow{OC}|} \times \bullet^3
\]

\[
\overrightarrow{OA} \cdot \overrightarrow{OC} = 3 \times 4 + (-3) \times k + 0 \times 0 = 12 - 3k
\]

\[
|\overrightarrow{OA}| = \sqrt{18} \times \bullet^5
\]

\[
|\overrightarrow{OC}| = \sqrt{16 + k^2} \times \bullet^6
\]

\[
\cos \angle ABC = \frac{12 - 3k}{\sqrt{18} \sqrt{16 + k^2}} \times \bullet^7
\]

**Candidate B**

\[
\cos \angle AOB = \frac{\overrightarrow{OA} \cdot \overrightarrow{OB}}{|\overrightarrow{OA}| |\overrightarrow{OB}|} \times \bullet^3
\]

\[
\overrightarrow{OA} \cdot \overrightarrow{OB} = 3 \times 2 + (-3) \times 0 \times 1 = 15
\]

\[
|\overrightarrow{OA}| = \sqrt{18} \times \bullet^5
\]

\[
|\overrightarrow{OB}| = \sqrt{14} \times \bullet^6
\]

\[
\cos \angle ABC = \frac{15}{\sqrt{18} \sqrt{14}} \times \bullet^7
\]
5 (b) If angle ABC = 30°, find the possible values of \( k \).

### Generic Scheme

**5(b)**

- Method 1: Squaring first
  - **ic** link with (a)
  - **ss** square both sides
  - **pd** rearrange into 'non-fractional' format
  - **pd** write in standard form
  - **pd** solve for \( k \)

- Method 2: Dealing with fractions first
  - **ic** link with (a)
  - **pd** rearrange into 'non-fractional' format
  - **ss** square both sides
  - **pd** write in standard form
  - **pd** solve for \( k \)

### Illustrative Scheme

**Method 1 : Squaring first**

\[
\frac{3}{\sqrt{2(k^2 + 6k + 14)}} = \cos 30°
\]

\[
\left( \frac{3}{\sqrt{2(k^2 + 6k + 14)}} \right)^2 = \left( \frac{\sqrt{3}}{2} \right)^2
\]

\[
k^2 + 6k + 14 = 6 \quad \text{or equivalent}
\]

\[
k^2 + 6k + 8 = 0 \quad \text{or equivalent}
\]

\[
k = -2 \text{ or } -4
\]

**Method 2 : Dealing with fractions first**

\[
\frac{3}{\sqrt{2(k^2 + 6k + 14)}} = \cos 30°
\]

\[
\sqrt{3} \cdot \frac{3}{\sqrt{2(k^2 + 6k + 14)}} = 6
\]

\[
6(k^2 + 6k + 14) = 36
\]

\[
k^2 + 6k + 8 = 0 \quad \text{or equivalent}
\]

\[
k = -2 \text{ or } -4
\]

### Notes

3. The evidence for \( \bullet^9 \) may appear in the working for \( \bullet^{10} \) in both methods.

4. \( \bullet^9 \) is the only mark available to candidates who replace \( \cos 30° \) by 30 in method 1 and \( \bullet^{10} \) in method 2.

5. All 5 marks are available to candidates who use 0.87 for \( \cos 30° \) but 0.9 can gain a maximum of 4 marks.

### Regularly occurring responses

**Response 2 :** Working with \( \cos 30° \) throughout the question

<table>
<thead>
<tr>
<th>Candidate C (Method 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \cos 30° = \frac{3}{\sqrt{2(k^2 + 6k + 14)}} )</td>
</tr>
<tr>
<td>( (\cos 30°)^2 = \left( \frac{3}{\sqrt{2(k^2 + 6k + 14)}} \right)^2 )</td>
</tr>
<tr>
<td>( (\cos 30°)^2 = \frac{9}{2(k^2 + 6k + 14)} )</td>
</tr>
<tr>
<td>( 2(\cos 30°)^2(k^2 + 6k + 14) = 9 )</td>
</tr>
</tbody>
</table>

If \( \cos 30° \) is subsequently evaluated then **\( \bullet^{11} \) and **\( \bullet^{12} \) may still be available.

**Response 3 :** Using the wrong value for \( \cos 30° \)

<table>
<thead>
<tr>
<th>Candidate D (Method 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{\sqrt{2(k^2 + 6k + 14)}} = \frac{1}{2} )</td>
</tr>
<tr>
<td>( \sqrt{2(k^2 + 6k + 14)} = 6 )</td>
</tr>
<tr>
<td>( 2(k^2 + 6k + 14) = 36 )</td>
</tr>
<tr>
<td>( k^2 + 6k + 14 = 18 )</td>
</tr>
<tr>
<td>( k = -6 \pm \frac{6^2 - 4 \times 1 \times (-4)}{2 \times 1} )</td>
</tr>
<tr>
<td>( = 0.61, -6.61 )</td>
</tr>
</tbody>
</table>
6. For $0 < x < \frac{\pi}{2}$, sequences can be generated using the recurrence relation

$$u_{n+1} = (\sin x)u_n + \cos 2x, \text{ with } u_0 = 1.$$  

(a) Why do these sequences have a limit?

**Regularly occurring responses**

**Response 1:** Attempts at giving a reason for limit

- **Candidate A:**
  
  This sequence has a limit because $-1 < a < 1$, $-1 < \sin x < 1$ within the domain.

- **Candidate B:**
  
  Since $\sin x$ in this domain will always be greater than 0 and less than 1.

- **Candidate C:**
  
  $\sin \frac{\pi}{2} = 1$ and $\sin 0 = 0$ so the multiplier of $u_n$ is between 0 and 1, so it has a limit.

- **Candidate D:**
  
  $-1 \leq \sin x \leq 1$,
  
  for $0 < x < \frac{\pi}{2}, 0 < \sin x < 1$ so limit exists

**Response 2:** Minimum response for both marks

- **Candidate E:**
  
  for $0 < x < \frac{\pi}{2}, 0 < \sin x < 1$ so limit
  
  - $-1 < \sin x < 1$  

- **Candidate F:**
  
  if limit, $-1 < \sin x < 1$  
  
  for $0 < x < \frac{\pi}{2}, 0 < \sin x < 1$ so limit
6 (b) The limit of one particular sequence generated by this recurrence relation is \( \frac{1}{2} \sin x \).

Find the value(s) of \( x \).

**Generic Scheme**

- **\( \bullet^3 \)** ss appropriate limit method
- **\( \bullet^4 \)** ic substitute for limit
- **\( \bullet^5 \)** ss use appropriate double angle formula
- **\( \bullet^6 \)** pd express in standard form
- **\( \bullet^7 \)** pd start to solve quadratic equation
- **\( \bullet^8 \)** pd reduce to equations in \( \sin x \) only
- **\( \bullet^9 \)** ic select valid solution

**Illustrative Scheme**

- **\( \bullet^3 \)** limit = \( \frac{\cos 2x}{1 - \sin x} \) or \( l = \sin x \times l + \cos 2x \)
- **\( \bullet^4 \)** \( \frac{\sin x}{2} = \frac{\cos 2x}{1 - \sin x} \) or \( \frac{\sin x}{2} = \sin x \times \frac{1}{2} \sin x + \cos 2x \)
- **\( \bullet^5 \)** (may be stated, or implied by \( \bullet^1 \) in both methods)
- **\( \bullet^6 \)** \( \ldots -2 \sin^2 x \ldots \)
- **\( \bullet^7 \)** e.g. \( 3 \sin^2 x + \sin x - 2 \)
- **\( \bullet^8 \)** e.g. \( (3 \sin x - 2)(\sin x + 1) \) = 0 must appear at \( \bullet^6 \) or \( \bullet^7 \)
- **\( \bullet^9 \)** \( \sin x = \frac{2}{3} \) or \( \sin x = -1 \)
- **\( \bullet^9 \)** \( x = 0.730 \) or outwith interval

**Notes**

5. \( \bullet^7 \), \( \bullet^6 \) and \( \bullet^9 \) are only available if a quadratic equation is obtained at \( \bullet^6 \) stage.
6. Candidates may express the quadratic equation at the \( \bullet^8 \) stage in the form \( 3s^2 + s - 2 = 0 \). For candidates who do not solve a trigonometric quadratic equation at \( \bullet^8 \) \( \sin x \) must appear explicitly to gain \( \bullet^8 \).
7. \( \bullet^7 \), \( \bullet^6 \) and \( \bullet^9 \) are not available to candidates who 'solve' a quadratic equation in the form \( ax^2 + bx + c = 0 \), \( c \neq 0 \).
8. For \( \bullet^9 \) there must be one valid solution, and one solution outwith interval which is rejected.
9. \( \bullet^9 \) is not available to candidates who leave their answer in degree measure.
10. Cross marking is available for \( \bullet^8 \) and \( \bullet^9 \).

**Regularly occurring responses**

**Response 3**: Evidence for identification of \( \bullet \) appearing in (b)

**Candidate G**

- (a) \( -1 < a < 1 \)
- (b) \( L = \frac{c}{1-a} = \frac{\sin 2x}{1-\sin x} \)

**Response 4**: Error in algebra and subsequent quadratic equation solution

**Candidate H**

\[
L = \frac{b}{1-a} = \frac{\sin 2x}{1-\sin x} \]

\[
\cos 2x = \frac{1}{2} \sin x \quad \checkmark \quad \checkmark
\]

\[
\cos 2x = -\frac{1}{2} \sin^2 x \quad \times \quad \bullet^6
\]

\[
\cos 2x = 0
\]

\[
\frac{1}{2} \sin^2 x + \cos 2x = 0
\]

\[
\frac{1}{2} \sin^2 x + (1-2 \sin^2 x) = 0 \quad \times \quad \bullet^5
\]

\[
-\frac{3}{2} \sin^2 x + 1 = 0
\]

\[
\sin^2 x = \frac{2}{3} \quad \times \quad \bullet^7
\]

\[
\sin x = \frac{\sqrt{2}}{3} \text{ and } \sin x = -\frac{\sqrt{2}}{3} \quad \times \quad \bullet^8
\]

\[
x = 0.955, 2.146, 4.097, 5.328 \quad \times \quad \bullet^9
\]

**Candidate I**

\[
\frac{\cos 2x}{1-\sin x} = \frac{\sin x}{2} \quad \checkmark \quad \checkmark \quad \checkmark
\]

\[
\frac{\sin x}{2} \quad \times \quad \bullet^6
\]

\[
\sin^2 x + \sin x - 2 = 0 \quad \times \quad \bullet^5
\]

\[
(\sin x - 1)(\sin x + 2) = 0 \quad \times \quad \bullet^7
\]

\[
\sin x = 1 \text{ and } \sin x = -2 \quad \times \quad \bullet^8
\]

\[
x = \frac{\pi}{2} \quad \text{not possible} \quad \times \quad \bullet^9
\]

See Note 8
7 The diagram shows the curves with equations \( y = 4^x \) and \( y = 3^{3-x} \).

The graphs intersect at the point T.
(a) Show that the x-coordinate of T can be written in the form \( \frac{\log_a p}{\log_a q} \), for all \( a > 1 \).

\[
\begin{align*}
\log 4 (2 \log 3) & = \log 4 (2 \log 3) \\
x \log_y 4 & = (2-x) \log_y 3 \\
x & = \frac{2}{1 + \log_y 4} \\
\frac{\log_y 9}{\log_y 12} & = \frac{\log_y 9}{\log_y 12} \\
\end{align*}
\]

\[
\begin{align*}
\log y & = \log 12 \log 9 \\
x & = \frac{2}{1 + \log 4} \\
\frac{\log 9}{\log 12} & = \frac{\log 9}{\log 12} \\
\end{align*}
\]

**Notes**

1. In methods 1 and 2, if no base is indicated then \( \bullet^1 \) is not available, however \( \bullet^3 \), \( \bullet^4 \) and \( \bullet^5 \) are still available.

   In method 3, if no base is indicated then \( \bullet^1 \) is not available, however \( \bullet^5 \) is still available.

2. In all methods, if a numerical base is used then \( \bullet^6 \) is not available.

3. In method 1, the omission of brackets at the \( \bullet^5 \) stage is treated as bad form, see Response 1.

4. \( p \) and \( q \) must be numerical values.

**Regularly occurring responses**

<table>
<thead>
<tr>
<th>Response 1: Omission of brackets around ( 2-x )</th>
<th>Candidate A</th>
<th>Candidate B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 4^x = 3^{3-x} ) ( \checkmark ) ( \bullet^1 )</td>
<td>( x \log_y 4 = 2-x \log_y 3 ) ( \checkmark ) ( \checkmark ) ( \checkmark ) ( \bullet^3 )</td>
<td>( x \log_y 4 - \log_y 3 = 2 ) ( \checkmark ) ( \bullet^4 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Response 2: Using different bases</th>
<th>Candidate C</th>
<th>Candidate D</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 4^x = 3^{3-x} ) ( \checkmark ) ( \bullet^1 )</td>
<td>( \log_y y = \log_a 4 ) ( \checkmark ) ( \checkmark ) ( \checkmark ) ( \bullet^2 )</td>
<td>( \log_y 4 = (2-x) \log_y 3 ) ( \checkmark ) ( \bullet^4 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Response 3: Taking logs first</th>
<th>Candidate D</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 4^x ) and ( y = 3^{3-x} )</td>
<td>( x = \log_a 12 ) ( \checkmark ) ( \checkmark ) ( \checkmark ) ( \bullet^5 )</td>
</tr>
</tbody>
</table>
### Generic Scheme

7(b)

- \(7\) ic substitute in for \(x\)
- \(8\) pd process \(y\)

<table>
<thead>
<tr>
<th>Generic Scheme</th>
<th>Illustrative Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>(y)</td>
</tr>
<tr>
<td>(y)</td>
<td>(x)</td>
</tr>
</tbody>
</table>

**Log in log**

- \(\log_{9} 4 = 3.41\) stated, or implied by \(\cdot 8\)
- \(\log_{12} 4 = 0.8842 \approx 3.4\)

**Notes**

5. Candidates must work to at least two significant figures in (b) e.g. \(4^{0.9} = 3.5\) does not gain \(\cdot 8\), but \(\cdot 7\) is available.

6. \(\cdot 8\) is only available if the power used comes from \(\log_{a} p / \log_{a} q\) in (a).

### Regularly occurring responses

**Response 4**: Using \(p\) and \(q\) as integer values without working

<table>
<thead>
<tr>
<th>Candidate E</th>
<th>Candidate F</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p = 4)</td>
<td>(p = 3)</td>
</tr>
<tr>
<td>(q = 3)</td>
<td>(q = 4)</td>
</tr>
<tr>
<td>(y = 4^{1.26} = 5.74) or (5.75)</td>
<td>(y = 4^{0.79} = 2.99) or (3)</td>
</tr>
</tbody>
</table>

**Response 5**: Using integer values calculated in (a)

<table>
<thead>
<tr>
<th>Candidate G</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p = 10)</td>
</tr>
<tr>
<td>(q = 4)</td>
</tr>
<tr>
<td>(y = 4^{3.5} = 32)</td>
</tr>
</tbody>
</table>