2015 Mathematics

Higher

Finalised Marking Instructions

General Comments

These marking instructions are for use with the 2015 Higher Mathematics Examination.

For each question the marking instructions are in two sections, namely Illustrative Scheme and Generic Scheme. The Illustrative Scheme covers methods which are commonly seen throughout the marking. The Generic Scheme indicates the rationale for which each mark is awarded. In general, markers should use the Illustrative Scheme and only use the Generic Scheme where a candidate has used a method not covered in the Illustrative Scheme.

All markers should apply the following general marking principles throughout their marking:

1 Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than deducted for what is wrong.

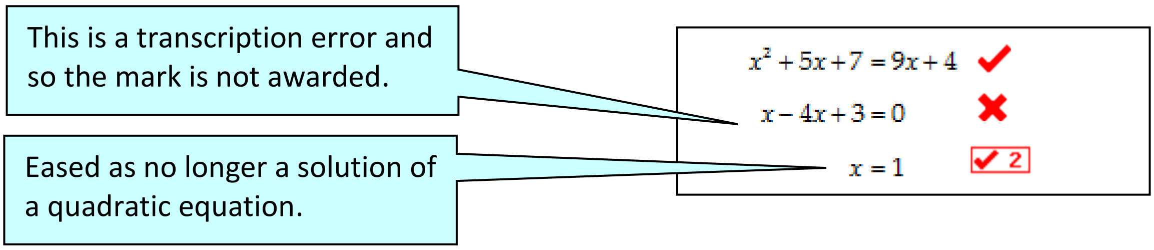
2 One mark is available for each •. There are **no** half marks.

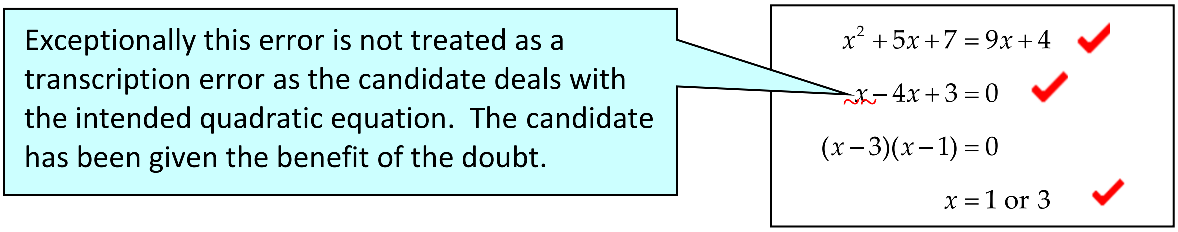
3 Working subsequent to an error **must be followed through**, with possible full marks for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.

4 As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Throughout this paper, unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.

5 In general, as a consequence of an error perceived to be trivial, casual or insignificant, e.g. $6\times 6 = 12$, candidates lose the opportunity of gaining a mark. But note the second example in comment 7.

6 Where a transcription error (paper to script or within script) occurs, the candidate should be penalised, e.g.





7 Vertical/horizontal marking

Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example: Point of intersection of line with curve

Illustrative Scheme: •5 $x=2,\ x=-4$

•6 $y=5,\ y=-7$

	•5	•6
•5	$x=2$	$y=5$
•6	$x=-4$	$y=-7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

8 In final answers, numerical values should be simplified as far as possible, unless specifically mentioned in the detailed marking instructions.

Examples: $\frac{15}{12}$ should be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ should be simplified to 43

$\frac{15}{0\cdot3}$ should be simplified to 50 $\frac{4/5}{3}$ should be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8

The square root of perfect squares up to and including 100 must be known.

9 Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

10 Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a **correct** answer;
- Correct working in the wrong part of a question;
- Legitimate variations in numerical answers, e.g. angles in degrees rounded to nearest degree;
- Omission of units;
- Bad form (bad form only becomes bad form if subsequent working is correct), e.g.

  $(x^3+2x^2+3x+2)(2x+1)$

  written as

  $(x^3+2x^2+3x+2)\times 2x+1$

  $2x^4+4x^3+6x^2+4x+x^3+2x^2+3x+2$

  $2x^4+5x^3+8x^2+7x+2$ gains full credit;
- Repeated error within a question, but not between questions.

11 In any 'Show that . . .' question, where the candidate has to arrive at a required result, the last mark of that part is not available as a follow through from a previous error unless specifically stated otherwise in the detailed marking instructions.

12 All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate's response. Marks may still be available later in the question so reference must be made continually to the marking instructions.

All working must be checked: the appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.

13 If you are in serious doubt whether a mark should or should not be awarded, consult your Team Leader (TL).

14 Scored out working which **has not been replaced** should be marked where still legible. However, if the scored out working **has been replaced**, only the work which has not been scored out should be marked.

15 Where a candidate has made multiple attempts using the same strategy, mark all attempts and award the lowest mark.
Where a candidate has tried different strategies, apply the above ruling to attempts within each strategy and then award the highest resultant mark. For example:

Strategy 1 attempt 1 is worth 3 marks	Strategy 2 attempt 1 is worth 1 mark
Strategy 1 attempt 2 is worth 4 marks	Strategy 2 attempt 2 is worth 5 marks
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

16 In cases of difficulty, covered neither in detail nor in principle in these instructions, markers should contact their TL in the first instance.

Paper 1 Section A

Question	Answer
1	C
2	B
3	D
4	A
5	C
6	B
7	C
8	D
9	B
10	D
11	C
12	C
13	B
14	D
15	A
16	B
17	D
18	C
19	B
20	A

Summary		
	A	3
	B	6
	C	6
	D	5

Question	Generic Scheme	Illustrative Scheme	Max Mark
21(a).			
	•¹ know to use $x=1$	**Method 1** •¹ $1^3-6(1)^2+9(1)-4$	
	•² interpret result and state conclusion	•² $=0 \therefore (x-1)$ is a factor.	
		Method 2 •¹ $\begin{array}{r\|rrrr} 1 & 1 & -6 & 9 & -4 \\ & & 1 & & \\ \hline & 1 & & & \end{array}$	
		•² $\begin{array}{r\|rrrr} 1 & 1 & -6 & 9 & -4 \\ & & 1 & -5 & 4 \\ \hline & 1 & -5 & 4 & 0 \end{array}$ remainder $=0 \therefore (x-1)$ is a factor.	
		Method 3 x^2 •¹ $x-1\overline{)x^3-6x^2+9x+4}$ x^3-x^2 •² $=0 \therefore (x-1)$ is a factor.	
	•³ state quadratic factor	•³ x^2-5x+4 stated or implied by •⁴	
	•⁴ factorise completely	•⁴ $(x-1)^2(x-4)$	4

Notes:
1. Communication at $\bullet^2$ must be consistent with working at that stage i.e. a candidate's working must arrive legitimately at 0 before $\bullet^2$ is awarded. 2. Accept any of the following for $\bullet^2$: • '$f(1)=0$ so $(x-1)$ is a factor' • 'since remainder is 0, it is a factor' • the 0 from the table linked to the word 'factor' by e.g. 'so', 'hence', '$\therefore$', '$\rightarrow$', '$\Rightarrow$' 3. Do not accept any of the following for $\bullet^2$: • double underlining the zero or boxing the zero without comment • '$x=1$ is a factor', '$(x+1)$ is a factor', '$x=1$ is a root', '$(x+1)$ is a root', "$(x-1)$ is a root" • the word 'factor' **only,** with no link 4. At $\bullet^4$ the expression may be written as $(x-1)(x-1)(x-4)$ in any order. 5. An incorrect quadratic correctly factorised may gain $\bullet^4$. 6. Where the quadratic factor obtained is irreducible, candidates must clearly demonstrate that $b^2-4ac<0$ to gain $\bullet^4$. 7. $=0$ must appear at $\bullet^1$ or $\bullet^2$ for $\bullet^2$ to be awarded. 8. For candidates who do not arrive at 0 at the $\bullet^2$ stage $\bullet^2\bullet^3\bullet^4$ are not available. 9. Do not penalise candidates who attempt to solve a cubic equation. However, within this working there may be evidence of the correct factorisation of the cubic. 10. Evidence for $\bullet^3$&$\bullet^4$ may appear in part (b).
Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
21(b)(i).			
	•5 know to and differentiate •6 find gradient •7 state equation of tangent	•5 $3x^2-12x+11$ •6 2 •7 $y=2x+1$	**3**

Notes:

11. •7 is only available if an attempt has been made to find the gradient from differentiation.
12. At mark •7 accept $y-3=2(x-1)$, $y-2x=1$ or any other rearrangement of the equation.

Commonly Observed Responses:

Candidate A

•5 ✓ •6 ✓

using $y=mx+c$

$x=1 \quad y=3 \quad m=2$

$\Rightarrow 3=2\times1+c$

$\Rightarrow c=1$ •7 ✓

$y=2x+1$

Question	Generic Scheme	Illustrative Scheme	Max Mark
21(b)(ii).			
	•8 set $y_{\text{CURVE}}=y_{\text{LINE}}$ •9 arrange equation in standard cubic form •10 identify x coordinate of B and calculate y coordinate	•8 $x^3-6x^2+11x-3=2x+1$ •9 $x^3-6x^2+9x-4=0$ •10 $(4,9)$	**3**

Notes:

13. •9 is only available if '$=0$' appears in at least one arrangement of the equation.
14. Solutions at •10 must be consistent with working at •4 and •7.
15. Candidates who obtain three distinct factors at •4 can gain •8 and •9 but •10 is unavailable.
16. For •10 accept $x=4, y=9$.
17. Do not penalise the appearance of $(1,3)$.

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
22.			
	•[1] arrange in differentiable form	•[1] $f(x)=4x^{-2}+x$	
	•[2] start differentiation	•[2] $-8x^{-3}$ or 1	
	•[3] complete differentiation and set $f'(x)=0$	•[3] $-8x^{-3}+1=0$	
	•[4] evaluate f at stationary point	•[4] $x=2,\ f(x)=3$	
	•[5] consider end-points	•[5] $f(1)=5,\ \ f(4)=\frac{17}{4}$ (or $4\cdot25$)	
	•[6] state max and min values	•[6] $\max 5,\ \min 3$	6

Notes:

1. Candidates must attempt to differentiate a term with a -ve or fractional power by •[3] for •[3] to be awarded.
2. •[3] is not available for simply stating '$f'(x)=0$'. A clear link between the candidates derivative and '$f'(x)=0$' is required.
3. For candidates who integrate, but clearly believe they are finding the derivative •[1] •[4] •[5] •[6] are available (see CORs - K, L, and M). In other instances where candidates have integrated then only •[1] and •[5] are available. A numerical approach can only gain •[5].
4. •[5] and •[6] are not available to candidates who consider stationary points only.
5. Treat maximum $(1,5)$ and minimum $(2,3)$ as bad form.
6. Vertical marking is **not** applicable to •[5] and •[6].
7. The appearance of $(2,3)$ following any 2nd derivative or nature table gains •[4].
8. If at the •[4] stage a value of x is obtained outwith the given interval then •[4] is unavailable, but •[6] may still be gained (see CORs - F and G).
9. Candidates who consider the end values but do not evaluate the stationary value cannot gain •[6].

Commonly Observed Responses:

Candidate A	Candidate B	Candidate C	Candidate D
$f(x)=4x^{-2}+x$ $f'(x)=-8x^{-3}+1$ •[1] ✓ •[2] ✓ for stationary •[3] ✓ points $f'(x)=0$ **With no further working**	$f(x)=4x^{-2}+x$ •[1] ✓ •[2] ^ for stationary •[3] ^ points $f'(x)=0$ **With no further working**	$f(x)=4x^{-2}+x$ •[1] ✓ $f'(x)=-8x^{-3}+1$ •[2] ✓ for stationary •[3] ^ points $\frac{dy}{dx}=0$ **With no further working**	$f(x)=4x^{-2}+x$ •[1] ✓ $-8x^{-3}+1$ •[2] ✓ for stationary •[3] ^ points $f'(x)=0$ **With no further working**

Candidate E **Solely a numerical attempt** $f(1)=5,\ f(2)=3,$ $f(3)=\frac{31}{9},\ f(4)=\frac{17}{4}$ Award only •5 ✓ For any similar attempt which includes the evaluation of f for a value outwith the range award 0.	**Candidate F** $f(x)=4x^{-2}+x$ $f'(x)=8x..........$ $f'(x)=..........+1=0$ $x=-\frac{1}{8}\quad f(-\frac{1}{8})=255\frac{7}{8}$ $f(1)=5,\ f(4)=4\frac{1}{4}$ $\min\ 4\frac{1}{4},\ \max\ 255\frac{7}{8}$ •1 ✓ •2 × •3 ✓ •4 ✓ 2 •5 ✓ •6 ✓1	**Candidate G** $f(x)=4x^{-2}+x$ $f'(x)=-8x..........$ $f'(x)=..........+1=0$ $x=\frac{1}{8}\quad f(\frac{1}{8})=256\frac{1}{8}$ $f(1)=5,\ f(4)=4\frac{1}{4}$ $\min\ 4\frac{1}{4},\ \max\ 256\frac{1}{8}$ •1 ✓ •2 × •3 ✓ •4 ✓ 2 •5 ✓ •6 ✓1
Candidate H $f(x)=4x^{-2}+x$ $f'(x)=-8x^{-1}..........$ $f'(x)=..........+1=0$ $x=8\quad f(x)=8\frac{1}{16}$ $f(1)=5,\ f(4)=4\frac{1}{4}$ $\min\ 4\frac{1}{4},\ \max\ 8\frac{1}{16}$ •1 ✓ •2 × •3 ✓ •4 ✓ 2 •5 ✓ •6 ✓1	**Candidate I** $f(x)=4+x^3$ $f'(x)=0+3x^2=0$ $x=0\quad f(x)=\text{undefined}$ •1 × •2 × •3 ✓ 2 •4 × •5 ^ •6 ^	**Candidate J** $f'(x)=\frac{4}{x^2}+x=0$ $f(1)=5,\ f(4)=4\frac{1}{4}$ •1 × •2 × •3 × •4 × •5 ✓ •6 ^
Candidate K $f(x)=4x^{-2}+x$ $f'(x)=-4x^{-1}+\frac{x^2}{2}$ $x=2\quad f(2)=3$ $f(1)=5,\ f(4)=4\frac{1}{4}$ min 3, max 5 •1 ✓ •2 × •3 × •4 ✓1 •5 ✓ •6 ✓1	**Candidate L** $f(x)=4x^{-2}+x$ $\int 4x^{-2}+x=-4x^{-1}+\frac{x^2}{2}=0$ $x=2\quad f(2)=3$ $f(1)=5,\ f(4)=4\frac{1}{4}$ min 3, max 5 •1 ✓ •2 × •3 × •4 × •5 ✓ •6 ×	**Candidate M** $4x^{-2}+x$ $-4x^{-1}+\frac{x^2}{2}=0$ $x=2\quad f(2)=3$ $f(1)=5,\ f(4)=4\frac{1}{4}$ min 3, max 5 •1 ✓ •2 × •3 × •4 × •5 ✓ •6 ×

Question	Generic Scheme	Illustrative Scheme	Max Mark
23.			
	•[1] collect log terms •[2] use laws of logs •[3] use laws of logs •[4] solve for x	**Method 1** •[1] $\log_2(3x+7)-\log_2(x-1)=3$ **stated or implied by •[2]** •[2] $\log_2\frac{(3x+7)}{(x-1)}=3$ •[3] $\frac{(3x+7)}{(x-1)}=2^3$ •[4] $x=3$ **Method 2** •[1] $\log_2(3x+7)=3\log_2 2+\log_2(x-1)$ **stated or implied by •[2]** •[2] $\log_2(3x+7)=\log_2 2^3+\log_2(x-1)$ •[3] $\log_2(3x+7)=\log_2 8(x-1)$ •[4] $x=3$	4

Notes:

1. For •[3] accept $\log_2\frac{(3x+7)}{(x-1)}=\log_2 8$.

Commonly Observed Responses:

Candidate A	Candidate B
$\log_2(3x+7)=3+\log_2(x-1)$ $-3=\log_2(x-1)-\log_2(3x+7)$ •[1] ✓ $-3=\log_2\frac{x-1}{3x+7}$ •[2] ✓ $\frac{x-1}{3x+7}=2^{-3}$ •[3] ✓ $8x-8=3x+7$ •[4] ✓ $x=3$	$\log_2(3x+7)=3+\log_2(x-1)$ $\log_2(3x+7)+\log_2(x-1)=3$ •[1] ✗ $\log_2(3x+7)(x-1)=3$ •[2] ✓ $(3x+7)(x-1)=2^3$ •[3] ✓ $3x^2+4x-15=0$ •[4] ✓ $(3x-5)(x+3)=0$ $x=\frac{5}{3}$ or $x=-3$ discard as $x>1$ $x=\frac{5}{3}$ •[4] is not available for candidates who do not discard $x=-3$

Question	Generic Scheme	Illustrative Scheme	Max Mark
24.			
	•[1] interpret the values of *a*, *b* and *c* and substitute	•[1] $3^2 - 4\times k \times 9k$	
	•[2] know to use discriminant ≥ 0	•[2] $\ldots \geq 0$	
	Method 1	**Method 1**	
	•[3] simplify or factorise quadratic inequation	•[3] $k^2 \leq \frac{9}{36}$ or $9(1-2k)(1+2k) \geq 0$	
	•[4] state range of values of *k*	•[4] $-\frac{1}{2} \leq k \leq \frac{1}{2}$	
	Method 2	**Method 2**	
	•[3] simplify or factorise quadratic expression	•[3] $9 - 36k^2 = 0 \Rightarrow k = -\frac{1}{2}, \frac{1}{2}$	
	•[4] evidence and range of values of *k*	•[4] graph or other evidence leading to $-\frac{1}{2} \leq k \leq \frac{1}{2}$	4

Notes:

1. The "≥ 0" must appear at least once at the •[1] or •[2] stage for •[2] to be awarded.
2. If an x appears in the candidate's 'discriminant' only •[2] may be awarded.
3. The use of any expression masquerading as the discriminant can gain only •[2] at most.
4. Award •[2] to candidates who write **BOTH** $9 - 36k^2 > 0$ **AND** $9 - 36k^2 = 0$.
5. For candidates who at •[3] simplify or factorise an equation •[4] can only be awarded if evidence of solving an inequation (for example a graph) appears.
6. At •[2] stage, quoting $b^2 - 4ac \geq 0$ is not sufficient.
7. At •[3] stage, in Method 2, solutions for k need not be simplified.

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
25(a)			
	•[1] know to use Theorem of Pythagoras' or distance formula •[2] process to obtain result	•[1] $D=\sqrt{(2t-5)^2+(t-10)^2}$ •[2] $D=\sqrt{4t^2-20t+25+t^2-20t+100}$ $=\sqrt{5t^2-40t+125}$	2

Notes:

Be wary of fudged solutions!

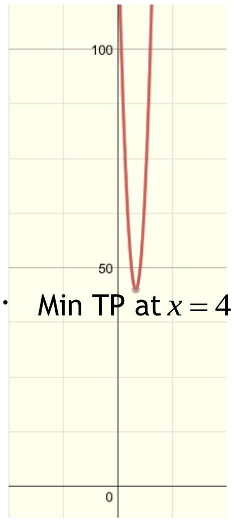
Commonly Observed Responses:

Candidate A	**Beware**
$D^2=(y_2-y_1)^2+(x_2-x_1)^2$ $D=\sqrt{((2t-5)+0)^2+(0+(t-10))^2}$ •[1] ✗ $D=\sqrt{5t^2-40t+125}$ •[2] ✗ **See note 12 in the general instructions**	$A(2t-5,0)\ B(0,t-10)$ •[1] ✗ $\overrightarrow{AB}=\begin{pmatrix}0\\t-10\end{pmatrix}-\begin{pmatrix}2t-5\\0\end{pmatrix}=\begin{pmatrix}2t-5\\t-10\end{pmatrix}$ •[2] ✓1 $D=\sqrt{(2t-5)^2+(t-10)^2}$ $D=\sqrt{4t^2+t^2-20t-20t+25+100}$ $D=\sqrt{5t^2-40t+125}$ **NB: This is an exception to note 12 in the general instructions**

Question	Generic Scheme	Illustrative Scheme	Max Mark
25(b)			
	•[3] write in differentiable form •[4] start differentiation •[5] complete differentiation •[6] substitute t = 5 and interpret result	•[3] $(5t^2-40t+125)^{\frac{1}{2}}$ •[4] $\frac{1}{2}(5t^2-40t+125)^{-\frac{1}{2}}$..... •[5] $.....\times(10t-40)$ •[6] $D'(5)=\frac{10}{2\sqrt{50}}>0\ \therefore\text{increasing}$	4

Notes:

1. •[4] is only available for differentiating an expression of the form (trinomial)$^{\text{proper fraction}}$.
2. Do not penalise the use of x instead of t.
3. •[6] is only available to candidates who substitute into a derivative.

Commonly Observed Responses:	
Candidate A $D(t)=\left(5t^2-40t+125\right)^{-1}$ $D'(t)=-\left(5t^2-40t+125\right)^{-2}\ldots$ $=\ldots\ldots\times(10t-40)$ $D'(5)=\frac{-10}{50^2}<0\therefore$ decreasing •3 ✗ •4 ✗ see note 1. •5 ✓ •6 ✓1	**Candidate B** $D(t)=\left(5t^2-40t+125\right)^{-\frac{1}{2}}$ $D'(t)=-\frac{1}{2}\left(5t^2-40t+125\right)^{-\frac{3}{2}}\ldots$ $=\ldots\ldots\times(10t-40)$ $D'(5)=\frac{-5}{\sqrt{50^3}}<0\therefore$ decreasing •3 ✗ •4 ✓1 •5 ✓ •6 ✓1
Candidate C $D(t)=\left(5t^2-40t+125\right)^{\frac{1}{2}}$ $D'(t)=\frac{1}{2}\left(5t^2-40t+125\right)^{\frac{3}{2}}\ldots$ $=\ldots\ldots\times(10t-40)$ $D'(5)=5\sqrt{50^3}>0\therefore$ increasing •3 ✓ •4 ✗ •5 ✓ •6 ✓1	**Candidate D** $D(t)=\left(5t^2-40t+125\right)^{\frac{1}{2}}$ $D'(t)=\frac{1}{2}\left(5t^2-40t+125\right)^{-\frac{1}{2}}\times10t-40$ $D'(t)=5t\left(5t^2-40t+125\right)^{-\frac{1}{2}}-40$ $D'(5)=\frac{25}{\sqrt{50}}-40<0\therefore$ decreasing •3 ✓ •4 ✓ •5 ✗ •6 ✓1
Candidate E $D(t)=\left(5t^2-40t+125\right)^{\frac{1}{2}}$ $D'(t)=\frac{1}{2}\left(5t^2-40t+125\right)^{-\frac{1}{2}}\times10t-40$ $D'(5)=\frac{1}{\sqrt{2}}>0\ \therefore$increasing •3 ✓ •4 ✓ •5 ✓ •6 ✓	**Candidate F - Alternative Method** $D^2=5t^2-40t+125$ $\frac{dD^2}{dt}=10t-40$ $t=5\Rightarrow\frac{dD^2}{dt}=50-40$ $\frac{dD^2}{dt}=10>0\quad\therefore D^2$ is increasing $\therefore$ D is increasing •3 ✓ •4 ✓ •5 ✓ •6 ✓
Calculating Distance $t=5\quad D=\sqrt{50}$ $t=4\quad D=\sqrt{45}$ so distance is increasing •3 ✓2 •4 ✗ •5 ✗ •6 ✗ **Award 0 marks as answer is not from differentiation.**	**Alternative Response** $D^2=5(t^2-8t+25)$ $D^2=5[(t-4)^2+9]$ Graph together with a statement indicating that when $t=5$, D^2 is increasing and therefore D is increasing.  •3 ✓ •4 ✓ •5 ✓ •6 ✓

Paper 2

Question	Generic Scheme	Illustrative Scheme	Max Mark
1(a)			
	•[1] calculate gradient of AB	•[1] $m_{AB} = -3$	
	•[2] use property of perpendicular lines	•[2] $m_{alt} = \frac{1}{3}$	
	•[3] substitute into general equation of a line	•[3] $y-3=\frac{1}{3}(x-13)$	
	•[4] demonstrate result	•[4] $\ldots \Rightarrow x-3y=4$	4

Notes:

1. •[3] is only available as a consequence of trying to find and use a perpendicular gradient.
2. •[4] is only available if there is/are appropriate intermediate lines of working between •[3] and •[4].
3. The ONLY acceptable variations for the final equation for the line in •[4] are $4=x-3y,\ -3y+x=4,\ 4=-3y+x$.

Commonly Observed Responses:

Candidate A

$m_{AB} = \frac{-1-(-5)}{-5-7} = \frac{4}{-12} = -\frac{1}{3}$ •[1] ✗

$m_{alt} = 3$ •[2] ✓1

$y-3=3(x-13)$ •[3] ✓1

•[4] ✗

•[4] is not available

Candidate B

For •[4]

$y-3=\frac{1}{3}x-\frac{13}{3}$

$y=\frac{1}{3}x-\frac{4}{3}$

$3y=x-4$ - not acceptable

$3y-x=-4$ - not acceptable

$x-3y=4$ ✓

Question	Generic Scheme	Illustrative Scheme	Max Mark
1(b)			
	•[5] calculate midpoint of AC •[6] calculate gradient of median •[7] determine equation of median	•[5] $M_{AC} = (4,5)$ •[6] $m_{BM} = 2$ •[7] $y = 2x - 3$	3

Notes:

4. •[6] and •[7] are not available to candidates who do not use a midpoint.
5. •[7] is only available as a consequence of using a non-perpendicular gradient and a midpoint.
6. Candidates who find either the median through A or the median through C or a side of the triangle gain 1 mark out of 3.
7. At •[7] accept $y-(-5)=2(x-(-1))$, $y-5=2(x-4)$, $y-2x+3=0$ or any other rearrangement of the equation.

Commonly Observed Responses:

Median through A	Median through C
$M_{BC} = (6,-1)$ $m_{AM} = \frac{-8}{11}$ $y+1=\frac{-8}{11}(x-6)$ or $y-7=\frac{-8}{11}(x+5)$ **Award 1/3**	$M_{AB} = (-3,1)$ $m_{CM} = \frac{1}{8}$ $y-3=\frac{1}{8}(x-13)$ or $y-1=\frac{1}{8}(x+3)$ **Award 1/3**

Question	Generic Scheme	Illustrative Scheme	Max Mark
1(c)			
	•[8] calculate x or y coordinate •[9] calculate remaining coordinate of the point of intersection	•[8] $x=1$ or $y=-1$ •[9] $y=-1$ or $x=1$	2

Notes:

8. If the candidate's 'median' is either a vertical or horizontal line then award 1 out of 2 if both coordinates are correct, otherwise award 0.

Commonly Observed Responses:

For candidates who find the altitude through B in part (b)	Candidate A
$x=-\frac{1}{5}$ •[8] ✓1 $y=-\frac{7}{5}$ •[9] ✓1	(b) $y-5=2(x-4)$ •[7] ✓ $y=2x-13$ - error (c) $x-3y=4$ $y=2x-13$ •[8] ✗ Leading to $x=7$ and $y=1$ •[9] ✓1

Question	Generic Scheme	Illustrative Scheme	Max Mark
2(a)			
	•¹ interpret notation •² state a correct expression	•¹ $f((1+x)(3-x)+2)$ stated or implied by •² •² $10+(1+x)(3-x)+2$ stated or implied by •³	2

Notes:

1. •¹ is not available for $g\left(f(x)\right)=g(10+x)$ but •² may be awarded for $(1+10+x)\left(3-(10+x)\right)+2$.

Commonly Observed Responses:

Candidate A		Candidate B	
(a) $f(g(x))='g(f(x))'$		$f(g(x))$	•¹ ^
$=(1+10+x)(3-(10+x))+2$	•¹ ✗ •² ✓1	$=10((1+x)-(3-x))+2$	•² ✗
(b) $=-75-18x-x^2$ or $-x^2-18x-75$	•³ ✓1	**Candidate C**	
$=-(x^2+18x$	•⁴ ✓1	$f(g(x))$	•¹ ^
$=-(x+9)^2$	•⁵ ✓1	$=10((1+x)(3-x)+2)$	•² ✗
$=-(x+9)^2+6$			
(c) $-(x+9)^2+6=0$	•⁶ ✓1		
$x=-9+\sqrt{6}$ or $-9-\sqrt{6}$	•⁷ ✓1		

Question	Generic Scheme	Illustrative Scheme	Max Mark
2(b)			
	•³ write $f(g(x))$ in quadratic form	•³ $15+2x-x^2$ or $-x^2+2x+15$	
	Method 1	**Method 1**	
	•⁴ identify common factor	•⁴ $-1(x^2-2x$ stated or implied by •⁵	
	•⁵ complete the square	•⁵ $-1(x-1)^2+16$	
	Method 2	**Method 2**	
	•⁴ expand completed square form and equate coefficients	•⁴ $px^2+2pqx+pq^2+r$ and $p=-1$,	
	•⁵ process for q and r and write in required form	•⁵ $q=-1$ and $r=16$ Note if $p=1$ •⁵ is not available	3

Notes:
2. Accept $16-(x-1)^2$ or $-\left[(x-1)^2-16\right]$ at $\bullet^5$.

Commonly Observed Responses:

Candidate A	Candidate B	Candidate C
$-(x^2-2x-15)$ $\bullet^4$ ✓ $-(x^2-2x+1-1-15)$ $-(x-1)^2-16$ $\bullet^5$ ✗	$15+2x-x^2$ $\bullet^3$ ✓ $x^2-2x-15$ $\bullet^4$ ✗ $px^2+2pqx+pq^2+r$ and $p=1$ $q=-1$ $r=-16$ $\bullet^5$ ✓2 eased	$-x^2+2x+15$ $\bullet^3$ ✓ $-(x+1)^2\ldots$ $\bullet^4$ ✗ $-(x+1)^2+14$ $\bullet^5$ ✗
Candidate D	**Candidate E**	**Candidate F**
$15+2x-x^2$ $\bullet^3$ ✓ $x^2-2x-15$ $\bullet^4$ ✗ $(x-1)^2-16$ $\bullet^5$ ✓2 eased Eased, unitary coefficient of x^2 (lower level skill)	$15+2x-x^2$ $\bullet^3$ ✓ $x^2-2x-15$ $\bullet^4$ ✓ $(x-1)^2-16$ so $15+2x-x^2=-(x-1)^2+16$ $\bullet^5$ ✓	$-x^2+2x+15$ $\bullet^3$ ✓ $-(x+1)^2\ldots$ $\bullet^4$ ✗ $-(x+1)^2+16$ $\bullet^5$ ✓1

2(c)		
$\bullet^6$ identify critical condition $\bullet^7$ identify critical values	$\bullet^6$ $-1(x-1)^2+16=0$ or $f((g(x))=0$ $\bullet^7$ 5 and -3	2

Notes:
3. Any communication indicating that the denominator cannot be zero gains $\bullet^6$.
4. Accept $x=5$ and $x=-3$ or $x\neq5$ and $x\neq-3$ at $\bullet^7$.
5. If $x=5$ and $x=-3$ appear without working award 1/2.

Commonly Observed Responses:

Candidate A	Candidate B
$\dfrac{1}{-(x-1)^2+16}$ $\bullet^6$ ✓ $x\neq5$ $\bullet^7$ ∧	$\dfrac{1}{f(g(x))}$ $f(g(x))>0$ $\bullet^6$ ✗ $x=-3,\ x=5$ $\bullet^7$ ✓ $-3<x$ $x<5$

3(a)		
$\bullet^1$ determine the value of the required term	$\bullet^1$ $22\frac{3}{4}$ or $\frac{91}{4}$ or $22\cdot75$	1

Notes:
1. Do not penalise the inclusion of incorrect units.
2. Accept rounded and unsimplified answers following evidence of correct substitution.

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
3(b)			
	Method 1 (Considering both limits)	**Method 1**	
	•2 know how to calculate limit	•2 $\frac{32}{1-\frac{1}{3}}$ or $L=\frac{1}{3}L+32$	
	•3 know how to calculate limit	•3 $\frac{13}{1-\frac{3}{4}}$ or $L=\frac{3}{4}L+13$	
	•4 calculate limit	•4 48	
	•5 calculate limit	•5 52	
	•6 interpret limits and state conclusion	•6 $52>50$ $\therefore$ toad will escape	
	Method 2 (Frog first then numerical for toad)	**Method 2**	
	•2 know how to calculate limit	•2 $\frac{32}{1-\frac{1}{3}}$ or $L=\frac{1}{3}L+32$	
	•3 calculate limit	•3 48	
	•4 determine the value of the highest term less than 50	•4 $49\cdot803...$	
	•5 determine the value of the lowest term greater than 50	•5 $50\cdot352...$	
	•6 interpret information and state conclusion	•6 $50\cdot352>50$ $\therefore$ toad will escape	
	Method 3 (Numerical method for toad only)	**Method 3**	
	•2 continues numerical strategy	•2 numerical strategy	
	•3 exact value	•3 $30\cdot0625$	
	•4 determine the value of the highest term less than 50	•4 $49\cdot803...$	
	•5 determine the value of the lowest term greater than 50	•5 $50\cdot352...$	
	•6 interpret information and state conclusion	•6 $50\cdot352>50$ $\therefore$ toad will escape	
	Method 4 (Limit method for toad only)	**Method 4**	
	•2 & •3 know how to calculate limit	•2 & •3 $\frac{13}{1-\frac{3}{4}}$ or $L=\frac{3}{4}L+13$	
	•4 & •5 calculate limit •	•4 & •5 52	
	•6 interpret limit and state conclusion	•6 $52>50$ $\therefore$ toad will escape	5

Notes:			
3. •6 is unavailable for candidates who do not consider the toad in their conclusion. 4. For candidates who only consider the frog numerically award 1/5 for the strategy.			
Commonly Observed Responses:			
Error with frogs limit - Frog Only	**Using Method 3 - Toad Only**	**Using Method 3- Toad Only**	**Using Method 3 - Toad Only**
$L_F = \frac{34}{1-\frac{1}{3}}$ •2 ✗ •3 ✗ $L_F = 51$ •4 ✓1 •5 ✓1 $51 > 50$ •6 ✓1 $\therefore$ frog will escape.	•2 ✓ •3 ✓ •4 missing ^ •5 $50 \cdot 352...$ ✓ •6 $50.352 > 50$ so the toad escapes. ✓	•2 ✓ •3 ✓ •4 missing ^ •5 $50 \cdot 1..$rounding error ✗ •6 $50.1 > 50$ ✓1 so the toad escapes.	•2 ✓ •3 ✓ •4 $49 \cdot 7..$rounding error ✗ •5 $50 \cdot 1...$ ✓1 •6 $50.1 > 50$ ✓1 so the toad escapes.

Toad Conclusions
Limit = 52
This is greater than the height of the well and so the toad will escape - award •6.

However

Limit =52 and so the toad escapes - •6 ^.

Iterations		
$f_1 = 32$	$t_1 = 13$	
$f_2 = 42 \cdot 667$	$t_2 = 22 \cdot 75$	
$f_3 = 46 \cdot 222$	$t_3 = 30 \cdot 0625$	
$f_4 = 47 \cdot 407$	$t_4 = 35 \cdot 547$	
$f_5 = 47 \cdot 802$	$t_5 = 39 \cdot 660$	
$f_6 = 47 \cdot 934$	$t_6 = 42 \cdot 745$	
$f_7 = 47 \cdot 978$	$t_7 = 45 \cdot 059$	
$f_8 = 47 \cdot 993$	$t_8 = 46 \cdot 794$	
$f_9 = 47 \cdot 998$	$t_9 = 48 \cdot 096$	
	$t_{10} = 49 \cdot 072$	
	$t_{11} = 49 \cdot 804$	
	$t_{12} = 50 \cdot 353$	

Question	Generic Scheme	Illustrative Scheme	Max Mark
4(a)			
	•[1] know to equate $f(x)$ and $g(x)$ •[2] solve for x	•[1] $\frac{1}{4}x^2 - \frac{1}{2}x + 3 = \frac{1}{4}x^2 - \frac{3}{2}x + 5$ •[2] $x = 2$	2

Notes:

1. •[1] and •[2] are not available to candidates who: (i) equate zeros, (ii) give answer only without working, (iii) arrive at $x = 2$ with erroneous working.

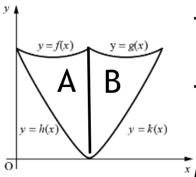
Commonly Observed Responses:

Candidate A	Candidate B
$y = \frac{1}{4}x^2 - \frac{1}{2}x + 3$ $y = \frac{1}{4}x^2 - \frac{3}{2}x + 5$ •[1] ✓ subtract to get $0 = x - 2$ $x = 2$ •[2] ✓	$\frac{1}{4}x^2 - \frac{1}{2}x = -3$ $\frac{1}{4}x^2 - \frac{3}{2}x = -5$ •[1] × $x = 2$ •[2] × *In this case the candidate has equated zeros*
Candidate C $f(x) = \frac{1}{4}x^2 - \frac{1}{2}x + 3$ $g(x) = \frac{1}{4}x^2 - \frac{3}{2}x + 5$ $f'(x) = \frac{1}{2}x - \frac{1}{2}$ $g'(x) = \frac{1}{2}x - \frac{3}{2}$ . . $x = 1$ $x = 3$ •[1] ✓ •[2] ✓ $\therefore x = 2$	

Question	Generic Scheme	Illustrative Scheme	Max Mark
4(b)			
	•[3] know to integrate	•[3] $\int$	
	•[4] interpret limits	•[4] $\int_0^2$	
	•[5] use 'upper - lower'	•[5] $\int_0^2 (\frac{1}{4}x^2 - \frac{1}{2}x + 3) - (\frac{3}{8}x^2 - \frac{9}{4}x + 3)\, dx$	
	•[6] integrate	•[6] $-\frac{1}{24}x^3 + \frac{7}{8}x^2$ accept unsimplified integral	
	•[7] substitute limits	•[7] $\left(-\frac{1}{24}\times 2^3 + \frac{7}{8}\times 2^2\right) - 0$	
	•[8] evaluate area between $f(x)$ and $h(x)$	•[8] $\frac{19}{6}$	
	•[9] state total area	•[9] $\frac{19}{3}$	**7**

Notes:

2. If limits $x = 0$ and $x = 2$ appear ex nihilo award •[4].
4. If a candidate differentiates at •[6] then •[6], •[7] and •[8] are not available. However, •[9] is still available.
5. Candidates who substitute at •[7], without attempting to integrate at •[6], cannot gain •[6], •[7] or •[8]. However, •[9] is still available.
6. Evidence for •[8] may be implied by •[9].
7. •[9] is a strategy mark and should be awarded for correctly multiplying their solution at •[8], or for any other valid strategy applied to previous working.
8. For •[5] both a term containing a variable and the constant term must be dealt with correctly.
9. In cases where •[5] is not awarded, •[6] may be gained for integrating an expression of equivalent difficulty i.e. a polynomial of at least degree two. •[6] is unavailable for the integration of a linear expression.
10. •[8] must be as a consequence of substituting into a term where the power of x is not equalto 1 or 0.

<table>
<tr><th colspan="2">Commonly Observed Responses:</th></tr>
<tr>
<td>
Candidate A - Valid Strategy
Candidates who use the strategy:


Total Area = Area A + Area B

Then mark as follows:

Mark Area A for $\bullet^3$ to $\bullet^8$ then mark Area B for $\bullet^3$ to $\bullet^8$ and award the higher of the two.
$\bullet^9$ is available for correctly adding two equal areas.
</td>
<td>
Candidate B - Invalid Strategy
For example, candidates who integrate each of the four functions separately within an invalid strategy

$\bullet^3$ ✓
Gain $\bullet^4$ if limits correct on
$\int f(x)$ and $\int h(x)$
or
$\int g(x)$ and $\int k(x)$
$\bullet^5$ is unavailable

Gain $\bullet^6$ for calculating either
$\int f(x)$ or $\int g(x)$
and
$\int h(x)$ or $\int k(x)$

Gain $\bullet^7$ for correctly substituting at least twice
Gain $\bullet^8$ for evaluating at least two integrals correctly
$\bullet^9$ is unavailable
</td>
</tr>
<tr>
<td>
Candidate C
$\int_0^2 (\frac{1}{4}x^2 - \frac{1}{2}x + 3 - \frac{3}{8}x^2 - \frac{9}{4}x + 3)\ dx$
$\int_0^2 (-\frac{1}{8}x^2 - \frac{11}{4}x)\ dx$ $\bullet^5$ ✓
$\frac{-1}{24}x^3 - \frac{11}{8}x^2$ $\bullet^6$ ✗
</td>
<td>
Candidate D
$\int_0^2 (\frac{1}{4}x^2 - \frac{1}{2}x + 3 - \frac{3}{8}x^2 - \frac{9}{4}x + 3)\ dx$
$\int_0^2 (-\frac{1}{8}x^2 - \frac{11}{4}x + 6)\ dx$ $\bullet^5$ ✗
$-\frac{1}{24}x^3 - \frac{11}{8}x^2 + 6x$ $\bullet^6$ ✓1
</td>
</tr>
<tr>
<td>
Candidate E

$\int \ldots = -\frac{1}{3}$ cannot be negative so $= \frac{1}{3}$ $\bullet^8$ ✗

however, $= -\frac{1}{3}$ so Area $= \frac{1}{3}$ $\bullet^8$ ✓
</td>
<td>
Candidate F
$\int_0^2 (\frac{1}{4}x^2 - \frac{1}{2}x + 3 - \frac{3}{8}x^2 - \frac{9}{4}x + 3)\ dx$
$\int_0^2 (-\frac{1}{8}x^2 + \frac{7}{4}x)\ dx$ $\bullet^5$ ✓
$-\frac{1}{24}x^3 + \frac{7}{8}x^2$ $\bullet^6$ ✓
</td>
</tr>
</table>

Question	Generic Scheme	Illustrative Scheme	Max Mark
5(a)			
	•[1] state centre of C_1 •[2] state radius of C_1 •[3] calculate distance between centres of C_1 and C_2 •[4] calculate radius of C_2	•[1] $(-3,-5)$ •[2] 5 •[3] 20 •[4] 15	**4**

Notes:

1. For •[4] to be awarded radius of C_2 must be greater than the radius of C_1.
2. Beware of candidates who arrive at the correct solution by finding the point of contact by an invalid strategy.
3. •[4] is for $\text{Distance}_{c1c2} - r_{c1}$ but only if the answer obtained is greater than r_{c1}.

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
5(b)			
	•5 find ratio in which centre of C_3 divides line joining centres of C_1 and C_2 •6 determine centre of C_3 •7 calculate radius of C_3 •8 state equation of C_3	•5 $3:1$ •6 $(6,7)$ •7 $r=20$ (answer must be consistent with distance between centres) •8 $(x-6)^2+(y-7)^2=400$	4

Notes:

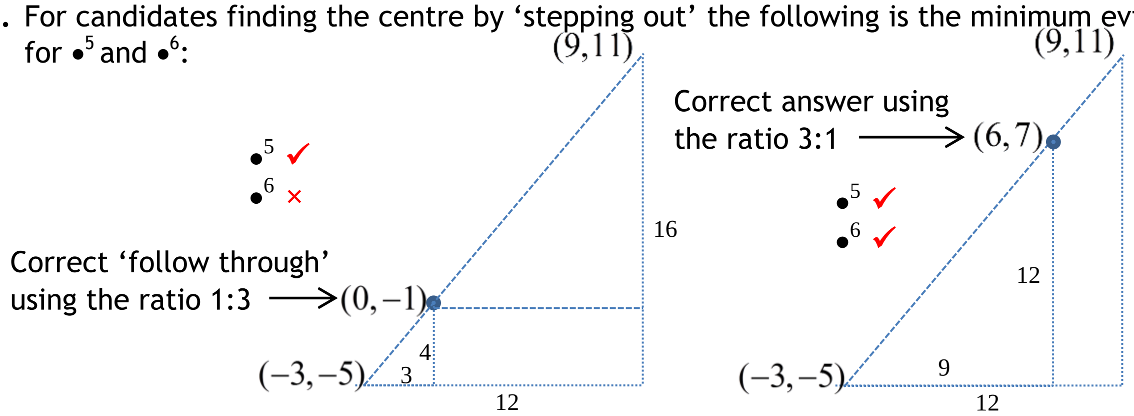
4. For •5 accept ratios $\pm3:\pm1$, $\pm1:\pm3$, $\mp3:\pm1$, $\mp1:\pm3$ (or the appearance of $\frac{3}{4}$).

5. •7 is for $r_{c2}+r_{c1}$.

6. Where candidates arrive at an incorrect centre or radius from working then •8 is available. However •8 is not available if either centre or radius appear ex nihilo (see note 5).

7. Do not accept 20^2 for •8.

8. For candidates finding the centre by 'stepping out' the following is the minimum evidence for •5 and •6:



Commonly Observed Responses:

Candidate A

using the mid-point of centres: •5 ×

centre $C_3=(3,3)$ •6 ✓ 2

radius of $C_3=20$ •7 ✓

$(x-3)^2+(y-3)^2=400$ •8 ✓1

Candidate B

$C_1=(-3,-5)$ ⟷ $C_2(9,11)$ $r=20$

$1:3$

$C_3=\frac{1}{4}\begin{pmatrix}0\\-4\end{pmatrix}$ •5 ✓ note 4

$C_3=(0,-1)$ •6 ✓ 2

•7 ✓

$x^2+(y+1)^2=400$ •8 ✓1

Candidate C - touches C_1 internally only

•5 ×

•6 centre $C_3=(3,3)$ ×

•7 radius of C_3 = radius of $C_2=15$ ✓1

•8 $(x-3)^2+(y-3)^2=225$ ✓1

Candidate D - touches C_2 internally only

•5 ×

•6 centre $C_3=(3,3)$ ×

•7 radius of C_3 = radius of $C_1=5$ ✓1

•8 $(x-3)^2+(y-3)^2=25$ ✓1

Candidate E - centre C_3 collinear with C_1, C_2

•5 ×

•6 e.g. centre $C_3=(21,27)$ ×

•7 radius of $C_3=45$ (touch C_1 internally only) ✓1

•8 $(x-21)^2+(y-27)^2=2025$ ✓1

Question	Generic Scheme	Illustrative Scheme	Max Mark
6(a)			
•[1] Expands		•[1] $\mathbf{p.q}+\mathbf{p.r}$	
•[2] Evaluate $\mathbf{p.q}$		•[2] $4\frac{1}{2}$	
•[3] Completes evaluation		•[3] $...+0=4\frac{1}{2}$	3

Notes:

1. For $\mathbf{p.(q+r)}=\mathbf{pq}+\mathbf{pr}$ with no other working •[1] is not available.

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
6(b)			
•[4] correct expression		•[4] $\mathbf{-q}+\mathbf{p}+\mathbf{r}$ or equivalent	1
6(c)			
•[5] correct substitution		•[5] $\mathbf{-q.q}+\mathbf{q.p}+\mathbf{q.r}$	
•[6] start evaluation		•[6] $-9+....+3\lvert\mathbf{r}\rvert\cos 30^{\circ}=9\sqrt{3}-\frac{9}{2}$	
•[7] find expression for $\lvert\mathbf{r}\rvert$		•[7] $\lvert\mathbf{r}\rvert=\frac{3\sqrt{3}}{\cos 30}$	3

Notes:

2. Award •[5] for $\mathbf{-q}^2+\mathbf{q.p}+\mathbf{q.r}$

Commonly Observed Responses:

Candidate A

$\mathbf{-q.q}+\mathbf{q.p}+\mathbf{q.r}=9\sqrt{3}-\frac{9}{2}$ •[5] ✓

$-9+\frac{9}{2}+3\lvert\mathbf{r}\rvert\cos 150^{\circ}=9\sqrt{3}-\frac{9}{2}$ •[6] ✗

$\lvert\mathbf{r}\rvert=\frac{3\sqrt{3}}{\cos 150}$ •[7] ✓1

Candidate B

$\mathbf{-q.q}+\mathbf{q.p}+\mathbf{q.r}=9\sqrt{3}-\frac{9}{2}$ •[5] ✓

$-9+\frac{9}{2}+3\lvert\mathbf{r}\rvert\cos 30^{\circ}=9\sqrt{3}-\frac{9}{2}$ •[6] ✓

$\lvert\mathbf{r}\rvert=6$ •[7] ✓

Question	Generic Scheme	Illustrative Scheme	Max Mark
7(a)			
	•[1] integrate a term •[2] complete integration with constant	•[1] $\frac{3}{2}\sin 2x$ OR x •[2] $x+c$ $\frac{3}{2}\sin 2x+c$	2

Notes:

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
7(b)			
	•[3] substitute for cos 2*x* •[4] substitute for 1 and complete	•[3] $3\left(\cos^2 x-\sin^2 x\right)\ldots$ or $\ldots(\sin^2 x+\cos^2 x)$ •[4] $\ldots\left(\sin^2 x+\cos^2 x\right)=4\cos^2 x-2\sin^2 x$	2

Notes:

1. Any valid substitution for $\cos 2x$ is acceptable for •[3].
2. Candidates who show that $4\cos^2 x-2\sin^2 x=3\cos 2x+1$ may gain both marks.
3. Candidates who quote the formula for $\cos 2x$ in terms of A but do not use in the context of the question cannot gain •[3].

Commonly Observed Responses:

Candidate A

$3\cos 2x+1=(2\cos^2 x-1)+(2\cos^2 x-1)+(1-2\sin^2 x)+1$ •[3] ✓

$=4\cos^2 x-2\sin^2 x$ •[4] ✓

Candidate B

$4\cos^2 x-2\sin^2 x=2(\cos 2x+1)-(1-\cos 2x)$ •[3] ✓

$=3\cos 2x+1$ •[4] ✓

Question	Generic Scheme	Illustrative Scheme	Max Mark
7(c)			
	•[5] interpret link •[6] state result	•[5] $-\frac{1}{2}\int\ldots$ •[6] $-\frac{3}{4}\sin 2x-\frac{1}{2}x+c$	2

Notes:

Commonly Observed Responses:

Candidate A

$\int \sin^2 x-2\cos^2 x\,dx$

$=\int(3\cos 2x+1)\ dx$ •[5] ✗

$\frac{3}{2}\sin 2x+x+c$ •[6] ✗

Question	Generic Scheme	Illustrative Scheme	Max Mark
8.			
	•1 use compound angle formula	•1 $k\sin 1{\cdot}5t\cos a - k\cos 1{\cdot}5t\sin a$	
	•2 compare coefficients	•2 $k\cos a = 36$, $k\sin a = 15$ **stated explicitly**	
	•3 process for k	•3 $k = 39$	
	•4 process for a	•4 $a = 0{\cdot}39479\ldots$ rad or $22{\cdot}6^\circ$	
	•5 equates expression for h to 100	•5 $39\sin\left(1{\cdot}5t - 0{\cdot}39479\ldots\right) + 65 = 100$	
	•6 write in standard format and attempt to solve	•6 $\sin\left(1{\cdot}5t - 0{\cdot}39479\ldots\right) = \frac{35}{39}$ $\Rightarrow 1{\cdot}5t - 0{\cdot}39479\ldots = \sin^{-1}\left(\frac{35}{39}\right)$	
	•7 solve equation for $1{\cdot}5t$	•7 $1{\cdot}5t = 1{\cdot}508$ (•7) and $2{\cdot}422$ (•8)	
	•8 process solutions for t	•8 $t = 1{\cdot}006$ and $1{\cdot}615$	8

Notes:

1. Treat $k\sin 1{\cdot}5t\cos a - \cos 1{\cdot}5t\sin a$ as bad form only if the equations at the •2 stage both contain *k*.
2. $39\sin 1{\cdot}5t\cos a - 39\cos 1{\cdot}5t\sin a$ or $39\left(\sin 1{\cdot}5t\cos a - \cos 1{\cdot}5t\sin a\right)$ is acceptable for •1 and •3.
3. Accept $k\cos a = 36$ and $-k\sin a = -15$ for •2.
4. •2 is not available for $k\cos 1{\cdot}5t = 36$ and $k\sin 1{\cdot}5t = 15$, however, •4 is still available.
5. •3 is only available for a single value of k, $k > 0$.
6. •4 is only available for a single value of a.
7. The angle at •4 must be consistent with the equations at •2 even when this leads to an angle outwith the required range.
8. Candidates who identify and use any form of the wave equation may gain •1 , •2 and •3 , however, •4 is only available if the value of a is interpreted for the form $k\sin(1{\cdot}5t - a)$.
9. Candidates who work consistently in degrees cannot gain •8.
10. Do not penalise additional solutions at •8.
11. On this occasion accept any answers which round to $1{\cdot}0$ and $1{\cdot}6$ (2 significant figures required).

Commonly Observed Responses:

Response 1: Missing information in working.

Candidate A

$39\cos a = 36$

$-39\sin a = -15$

$\tan a = \frac{15}{36}$

$a = 0{\cdot}39479\ldots\text{rad or } 22{\cdot}6^\circ$

$\bullet^1$ ∧ $\bullet^2$ ✓ $\bullet^3$ ✓ $\bullet^4$ ✓

Candidate B

$\cos a = 36$

$\sin a = 15$

$\tan a = \frac{15}{36}$

$a = 0{\cdot}39479\ldots\text{rad or } 22{\cdot}6^\circ$

Does not satisfy equations at $\bullet^2$

$\bullet^1$ ∧ $\bullet^2$ ✗ $\bullet^3$ ∧ $\bullet^4$ ✗

Candidate C

$k\sin 1{\cdot}5t\cos a - k\cos 1{\cdot}5t\sin a$

$k\cos a = 36, \quad k\sin a = 15$

$k = 39 \text{ or } -39$

$\tan a = \frac{15}{36}$

$a = 0{\cdot}39479\ldots\text{rad or } 22{\cdot}6^\circ$

or

$a = 3{\cdot}53638\ldots\text{rad or } 202{\cdot}6^\circ$

$\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✗ $\bullet^4$ ✗

Response 2: Correct expansion of $k\sin(x + a)^\circ$ and possible errors for $\bullet^2$ and $\bullet^4$

Candidate D

$k\cos a = 36$

$k\sin a = 15$

$\tan a = \frac{36}{15}$

$a = 1{\cdot}176\ldots\text{rad or } 67{\cdot}4^\circ$

$\bullet^2$ ✓ $\bullet^4$ ✗

Candidate E

$k\cos a = 15$

$k\sin a = 36$

$\tan a = \frac{36}{15}$

$a = 1{\cdot}176\ldots\text{rad or } 67{\cdot}4^\circ$

$\bullet^2$ ✗ $\bullet^4$ [✓1]

Candidate F

$k\cos a = 36$

$k\sin a = -15$

$\tan a = \frac{-15}{36}$

$a = 5{\cdot}888\ldots\text{rad or } 337{\cdot}4^\circ$

$\bullet^2$ ✗ $\bullet^4$ [✓1]

Response 3: Labelling incorrect, $\sin(A - B) = \sin A\cos B - \cos A\sin B$ from formula list.

Candidate G

$k\sin A\cos B - k\cos A\sin B$

$k\cos a = 36$

$k\sin a = 15$

$\tan a = \frac{15}{36}$

$a = 0{\cdot}39479\ldots\text{rad or } 22{\cdot}6^\circ$

$\bullet^1$ ✗ $\bullet^2$ ✓ $\bullet^4$ ✓

Candidate H

$k\sin A\cos B - k\cos A\sin B$

$k\cos 1{\cdot}5t = 36$

$k\sin 1{\cdot}5t = 15$

$\tan 1{\cdot}5t = \frac{15}{36}$

$1{\cdot}5t = 0{\cdot}39479\ldots\text{rad or } 22{\cdot}6^\circ$

$\bullet^1$ ✗ $\bullet^2$ ✗ $\bullet^4$ [✓1]

Candidate I

$k\sin A\cos B - k\cos A\sin B$

$k\cos B = 36$

$k\sin B = 15$

$\tan B = \frac{15}{36}$

$B = 0{\cdot}39479\ldots\text{rad or } 22{\cdot}6^\circ$

$\bullet^1$ ✗ $\bullet^2$ [✓1] $\bullet^4$ [✓1]

Candidate J

$39\sin(1{\cdot}5t - 0{\cdot}395) = 100$

$\sin(1{\cdot}5t - 0{\cdot}395) = \frac{100}{39}$

$1{\cdot}5t - 0{\cdot}395 = \sin^{-1}\frac{100}{39}$

$\bullet^5$ ✗ $\bullet^6$ [✓1] $\bullet^7$ ✗ $\bullet^8$ ✗

Candidate K

$39\sin(1{\cdot}5t - 0{\cdot}395) = 100$

$1{\cdot}5t - 0{\cdot}395 = \sin^{-1}\frac{39}{100}$

$\bullet^6$ ✗ $\bullet^7$ ✗ $\bullet^8$ ✗

[END OF MARKING INSTRUCTIONS]