# Trig Formulae and Equations

## Higher Maths Exam Questions

### Source: 2019 P1 Q13 Higher Maths

(1) Triangles ABC and ADE are both right angled. Angles $p$ and $q$ are as shown in the diagram.

(a) Determine the value of

(i) $\cos p$

(ii) $\cos q$.

(b) Hence determine the value of $\sin (p + q)$.

**Answers:**

(a) (i) $\frac{2}{\sqrt{5}}$ (ii) $\frac{3}{\sqrt{10}}$ (b) $\frac{1}{\sqrt{2}}$

### Source: 2019 P1 Q15 Higher Maths

(2) Solve the equation $\sin 2x^\circ + 6 \cos x^\circ = 0$ for $0 \leq x < 360$.

(b) Hence solve $\sin 4x^\circ + 6 \cos 2x^\circ = 0$ for $0 \leq x < 360$.

**Answers:**

(a) $x = 90^\circ, 270^\circ$ (b) $x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$
(3) (a) Express \((\sin x - \cos x)^2\) in the form \(p + q \sin rx\) where \(p, q\) and \(r\) are integers.
(b) Hence, find \(\int (\sin x - \cos x)^2 \, dx\).

Answers: 
(a) \(1 - \sin 2x\) 
(b) \(x + \frac{1}{2} \cos 2x + c\)

(4) (a) Express \(2 \cos x^\circ - 3 \sin x^\circ\) in the form \(k \cos(x + a)^\circ\) where \(k > 0\) and \(0 \leq a < 360\).
(b) Hence solve \(2 \cos x^\circ - 3 \sin x^\circ = 3\) for \(0 \leq x < 360\).

Answers: 
(a) \(\sqrt{13} \cos(x + 56.3)^\circ\) 
(b) \(x = 270^\circ, 337.38 \ldots^\circ\)

(5) The right-angled triangle in the diagram is such that \(\sin x = \frac{2}{\sqrt{11}}\) and \(0 < x < \frac{\pi}{4}\).

(a) Find the exact value of:
(i) \(\sin 2x\)
(ii) \(\cos 2x\).
(b) By expressing \(\sin 3x\) as \(\sin(2x + x)\), find the exact value of \(\sin 3x\).

Answers: 
(a) (i) \(\frac{4\sqrt{7}}{11}\) 
(ii) \(\frac{3}{11}\) 
(b) \(\frac{34}{11\sqrt{11}}\)
(6) Solve \(5 \sin x - 4 = 2 \cos 2x\) for \(0 \leq x < 2\pi\).

Answers: \(x = 0.848, 2.29\)

(7) (a) Show that \(\frac{\sin 2x}{2 \cos x} - \sin x \cos^2 x = \sin^3 x\), where \(0 < x < \frac{\pi}{2}\).

(b) Hence, differentiate \(\frac{\sin 2x}{2 \cos x} - \sin x \cos^2 x\), where \(0 < x < \frac{\pi}{2}\).

Answers: (a) Proof (b) \(3\sin^2 x \cos x\)

(8) Triangle ABD is right-angled at B with angles \(BAC = p\) and \(BAD = q\) and lengths as shown in the diagram below.

![Diagram](image)

Show that the exact value of \(\cos(q - p)\) is \(\frac{19\sqrt{17}}{85}\).

Answer: Proof
(9) Express $5 \cos x - 2 \sin x$ in the form $k \cos(x + a)$, where $k > 0$ and $0 < a < 2\pi$.

(b) The diagram shows a sketch of part of the graph of $y = 10 + 5 \cos x - 2 \sin x$ and the line with equation $y = 12$.

The line cuts the curve at the points P and Q.

Find the x-coordinates of P and Q.

Answers: (a) $\sqrt{29} \cos(x + 0.38)$ \hspace{1cm} (b) $x = 0.8097, 4.712$

(10) Show that $\sin 2x \tan x = 1 - \cos 2x$, where $\frac{\pi}{2} < x < \frac{3\pi}{2}$.

(b) Given that $f(x) = \sin 2x \tan x$, find $f'(x)$.

Answers: (a) Proof \hspace{1cm} (b) $f'(x) = 2 \sin 2x \hspace{1cm} \text{or} \hspace{1cm} f'(x) = 4 \sin x \cos x$
### Source: 2015 P1 Q10 Higher Maths

(11) \[ \tan 2x = \frac{3}{4}, \quad 0 < x < \frac{\pi}{4}, \] find the exact value of
(a) \[ \cos 2x \]
(b) \[ \cos x. \]

**Answers:** \( a \) \( \frac{4}{5} \) \( b \) \( \frac{3}{\sqrt{10}} \)

### Source: 2015 P2 Q7b Higher Maths

(12) (a) Find \( \int (3\cos 2x + 1)\, dx \).
(b) Show that \( 3\cos 2x + 1 = 4\cos^2 x - 2\sin^2 x \).
(c) Hence, or otherwise, find \( \int (\sin^2 x - 2\cos^2 x)\, dx \)

**Answers:** (a) \( \frac{3}{2} \sin 2x + c \) \( b \) Proof \( c \) \( -\frac{3}{4} \sin 2x - \frac{1}{2} x + c \)

### Source: Specimen P1 Q6 Higher Maths

(13) (a) Find an equivalent expression for \( \sin(x + 60)^\circ \).
(b) Hence, or otherwise, determine the exact value of \( \sin 105^\circ \).

**Answers:** (a) \( \sin x \cos 60 + \cos x \sin 60 \)

\( b \) \( \frac{1 + \sqrt{3}}{2\sqrt{2}} \) or \( \frac{\sqrt{2} + \sqrt{6}}{4} \)
(14) The voltage, \( V(t) \), produced by a generator is described by the function \( V(t) = 120 \sin 100 \pi t \), \( t > 0 \), where \( t \) is the time in seconds.

(a) Determine the period of \( V(t) \).

(b) Find the first three times for which \( V(t) = -60 \).

Answers: (a) Period = \( \frac{1}{50} = 0.02 \)  
(b) \( \frac{7}{600} \), \( \frac{11}{600} \), \( \frac{19}{600} \)

(15) For the function \( f(x) = 2 - 3 \sin \left( x - \frac{\pi}{3} \right) \) in the interval \( 0 \leq x < 2\pi \), determine which two of the following statements are true and justify your answer.

Statement A  The maximum value of \( f(x) \) is \(-1\).

Statement B  The maximum value of \( f(x) \) is \(5\).

Statement C  The maximum value occurs when \( x = \frac{5\pi}{6} \).

Statement D  The maximum value occurs when \( x = \frac{11\pi}{6} \).

Answers: Statements B & D are true

(16) (a) Solve \( \cos 2x^\circ - 3 \cos x^\circ + 2 = 0 \) for \( 0 \leq x < 360 \).

(b) Hence solve \( \cos 4x^\circ - 3 \cos 2x^\circ + 2 = 0 \) for \( 0 \leq x < 360 \).

Answers: (a) \( 0^\circ, 60^\circ, 300^\circ \)  
(b) \( 0^\circ, 30^\circ, 150^\circ, 180^\circ, 210^\circ, 330^\circ \)