

Unit 3 Practice NAB B - Solutions

Question	Points of expected responses	Illustrative scheme
1	<ul style="list-style-type: none"> •¹ knows to integrate and states limits •² integrates •³ substitute limits •⁴ process limits 	<ul style="list-style-type: none"> •¹ $\int_0^2 \dots \dots dx$ •² $\frac{x^4}{4} - \frac{5x^3}{3} + x^2 + 8x$ •³ $\left(\frac{2^4}{4} - \frac{5 \times 2^3}{3} + 2^2 + 8 \times 2\right) - (0)$ •⁴ $10\frac{2}{3}$ units² <p>NB candidates who substitute without integrating, only •¹ is available</p> <p>candidates who differentiate at •², only •¹ is available</p>
2	<ul style="list-style-type: none"> •¹ knows to integrate and states limits •² use ‘upper – lower’ •³ integrate •⁴ substitute limits •⁵ process limits 	<ul style="list-style-type: none"> •¹ $\int_{-3}^0 \dots \dots \dots dx$ •² $\int_{-3}^0 (x^3 + 3x^2 + 2x + 3) - (2x + 3) dx$ •³ $\frac{x^4}{4} + x^3$ (or equivalent) •⁴ $(0) - \left(\frac{(-3)^4}{4} + (-3)^3\right)$ •⁵ $\frac{27}{4}$ or $6\frac{3}{4}$ or 6.75 units² <p>NB candidates who differentiate at •³, only •¹ and •² are available</p> <p>candidates who substitute without integrating, only •¹ and •² are available</p> <p>candidates who integrate ‘lower–upper’ lose •² and also •⁵ unless final area is expressed as A = 15.75</p>
3	<ul style="list-style-type: none"> •¹ prepares to integrate •² integrates first term correctly •³ integrates second term correctly •⁴ includes constant of integration 	<ul style="list-style-type: none"> •¹ $\int (4x^{\frac{1}{3}} + x^{-3}) dx$ •² $= 3x^{\frac{4}{3}} \dots$ •³ $\dots - \frac{1}{2x^2}$ •⁴ $+ c$

4	<ul style="list-style-type: none"> •¹ prepares to integrate •² integrates correctly 	<ul style="list-style-type: none"> •¹ $\int(x^{-\frac{3}{2}} dx$ •² $= -2x^{-\frac{1}{2}} + c$ or $-\frac{2}{\sqrt{x}} + c$
5	<ul style="list-style-type: none"> •¹ process missing side •² expand $\sin(p + q)$ and begin substitution •³ complete substitution •⁴ process 	<ul style="list-style-type: none"> •¹ $\sqrt{5}$ •² $= \sin p \cos q + \cos p \sin q$ $= \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{3} + \dots$ •³ $= \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{3} + \frac{1}{\sqrt{5}} \times \frac{2}{3}$ •⁴ $= \frac{2}{3} + \frac{2}{3\sqrt{5}} = \frac{2+2\sqrt{5}}{3\sqrt{5}}$
6	<ul style="list-style-type: none"> •¹ expand L.H.S. •² substitutes for $\cos^2 x$ •³ simplifies and completes <p>#2.1 valid strategy</p> <p>Alternatively use R.H.S.</p>	<ul style="list-style-type: none"> •¹ $9 - 16 \cos^2 x$ •² $9 - 16(1 - \sin^2 x)$ •³ $9 - 16 + 16 \sin^2 x = 16 \sin^2 x - 7 = \text{RHS}$ <p>#2.1 know to use identities</p> <p>NB do not award •² if candidate omits brackets, ie $16 - 25 + 25 \cos^2 x$</p>
7	<ul style="list-style-type: none"> •¹ starts to solve •² factorises •³ solves for one factor •⁴ solves for second factor 	<ul style="list-style-type: none"> •¹ $2 \sin x \cos x + \cos x = 0$ •² $\cos x (2 \sin x + 1) = 0$ •³ $\cos x = 0 \Rightarrow 90^\circ, 270^\circ$ •⁴ $2 \sin x + 1 = 0 \Rightarrow \sin x = -\frac{1}{2} \Rightarrow x = 210^\circ, 330^\circ$ <p>NB •³ and •⁴ can be marked horizontally or vertically</p>
8	<ul style="list-style-type: none"> •¹ starts to solve •² factorises •³ solves for one factor •⁴ solves for second factor 	<ul style="list-style-type: none"> •¹ $1 - 2 \sin^2 x - 4 \sin x + 5 = 0$ •² $2(\sin x - 1)(\sin x + 3) = 0$ •³ $\sin x = -3 \Rightarrow \text{no solution}$ •⁴ $\sin x - 1 = 0 \Rightarrow x = 90^\circ$

9	<ul style="list-style-type: none"> •¹ finds centre •² finds radius 	<ul style="list-style-type: none"> •¹ $(-3, 1)$ •² $\sqrt{27}$ or $3\sqrt{3}$
10	<ul style="list-style-type: none"> •¹ interprets congruent •² correct substitution 	<ul style="list-style-type: none"> •¹ $r = 10$ units, $C(0, 10)$ •² $(x - 0)^2 + (y - 10)^2 = 100$
11	<ul style="list-style-type: none"> •¹ set up intersection equation •² simplifies •³ solves #2.1 interprets solution 	<ul style="list-style-type: none"> •¹ $(x - 8)^2 + (2x + 3 + 2)^2 = 45$ •² $5x^2 + 4x + 44 = 0$ •³ No solution (check with discriminant) $b^2 - 4ac \Rightarrow 4^2 - 4 \times 5 \times 44$ $16 - 880 = -864$ <p>#2.1 zero points of contact or a negative discriminant implies line is not a tangent to the curve</p>
		Total of 37marks plus 2 x #2.1