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Proving Trig Identities

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Essential Skills 19

The skills in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Proving Trigonometric Identities **(Non Calculator)**

Prove that:

1. $tanA - tanB = \frac{\sin(A-B)}{cosAcosB}$

2. $(sinA + cosA)^2 = 1 + sin2A$

3. $sin3A = 3sinA - 4sin^3A$

4. $cos3A = 4cos^3A - 3cosA$

5. $(cosA + cosB)^2 + (sinA - sinB)^2 = 2(1 + \cos(A + B))$

6. $sin^2A = \frac{1}{2}(1 - cos2A)$

7. $cos^2A = \frac{1}{2}(1 + cos2A)$

8. $(cosA + sinA)(cosA - sinA) = cos2A$

9. $\frac{\sin(A-B)}{cosAcosB} = tanA - tanB$

10. $\cos(A - B) - \cos(A + B) = 2sinAsinB$

APPLYING QUESTIONS

1. (a) Prove that: $\frac{\cos(A+B)}{cosAcosB} = 1 - tanAtanB$

   (b) Hence evaluate $\frac{\cos(\frac{7\pi}{12})}{cos\frac{\pi}{3}cos\frac{\pi}{4}}$

Tip: $\frac{\pi}{3} + \frac{\pi}{4} = \frac{7\pi}{12}$

2. (a) Show that: $\cos(A + B)\cos(A - B) = cos^2B - sin^2A$

   (b) Hence evaluate: $\cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right)\cos\left(\frac{\pi}{6} - \frac{\pi}{4}\right)$