

# Differential Equations

## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

### Scalar Product:

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

### Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

### Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

### Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$

## Essential Skills 25

The skills in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed



### Differential Equations

Find the equations of the curves ( $y$  or  $f(x)$ ) that satisfy each of the following conditions:

1.  $\frac{dy}{dx} = 6x + 5$ , passing (2, 21)
2.  $\frac{dy}{dx} = 4x - 4$ , passing (-1, 6)
3.  $f'(x) = x^2$ , where  $f(3) = 13$
4.  $f'(x) = 3x^2 - 6$ , where  $f(-1) = 8$
5.  $\frac{dy}{dx} = 6x^2 + 8x + 5$ , passing (-2, -12)
6.  $f'(x) = 2(2 - 3x)$ , where  $f(1) = 1$
7.  $\frac{dy}{dx} = \frac{9}{2}x^2 - 6x$ , passing (2, 3)
8.  $\frac{dy}{dx} = \frac{4}{x^3}$ , passing (1, 1)
9.  $\frac{dy}{dx} = 9(3x - 5)^2 + 5$ , passing (2, 6)
10.  $f'(x) = 6\cos 2x$ , where  $f\left(\frac{\pi}{12}\right) = \frac{5}{2}$



### APPLYING QUESTIONS

1. The gradient of a tangent to a curve at each point  $(x, y)$  is given by  $\frac{dy}{dx} = 3x(2x - 1)$ .

If the curve passes through the point  $(-1, 10)$ , find its equation.

2. The velocity of an object is given by  $\frac{ds}{dt} = 9\sqrt{t} - 12$ , where  $s$  is the distance in metres and  $t$  is the time in seconds.

Find an expression for the displacement  $s$ , given that when  $t = 0, s = 2$ .