

Angle between Vectors

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product:

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$

Essential Skills 30

The skills in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Angle between Vectors

Calculate the angle between each pair of vectors:



1. $\underline{u} = \underline{i} + 2\underline{j} + 3\underline{k}$ and $\underline{v} = 4\underline{i} + \underline{j} + 5\underline{k}$
2. $\underline{u} = 4\underline{i} + 2\underline{k}$ and $\underline{v} = -2\underline{i} + 2\underline{j} + 2\underline{k}$
3. $\underline{u} = 3\underline{i} - 2\underline{j} + \underline{k}$ and $\underline{v} = 5\underline{j} - \underline{k}$
4. $\underline{u} = \underline{i} + \underline{j} + \underline{k}$ and $\underline{v} = \underline{i} + 2\underline{j} - 2\underline{k}$
5. $\underline{u} = 2\underline{i} + 3\underline{j}$ and $\underline{v} = -\underline{i} + 2\underline{j} + \underline{k}$
6. $\underline{u} = 3\underline{i} + \underline{k}$ and $\underline{v} = 2\underline{i} + 2\underline{j} - 7\underline{k}$
7. $\underline{u} = 5\underline{i} + \underline{j} - \underline{k}$ and $\underline{v} = 2\underline{i} - \underline{j} + 2\underline{k}$
8. $\underline{u} = 2\underline{j} - 6\underline{k}$ and $\underline{v} = 3\underline{i} - 2\underline{j} + 2\underline{k}$
9. $\underline{u} = 7\underline{i} + \underline{j} - \underline{k}$ and $\underline{v} = 2\underline{i} - \underline{j} + 3\underline{k}$
10. $\underline{u} = \underline{i} - 7\underline{j}$ and $\underline{v} = \underline{i} + 4\underline{j} + \underline{k}$

APPLYING QUESTION



- (a) C divides the line joining A (2, 1, -1) and B (8, 4, 8) in the ratio 2:1.
Find the coordinates of C.
- (b) D has coordinates (9, -3, 11) and C divides \overline{DE} in the ratio 3:1
Find the coordinates of E.
- (c) Hence, calculate the size of angle BCE.