Grade Booster Paper - Stationary Points

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\,\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Stationary Points

	Stationary Points	
1.	Find the coordinates of the stationary points for the curve $y = 2x^3 - 3x^2 - 12x + 20$ and determine their nature.	7
2.	$f(x)$ is defined by the formula $f(x) = x^3 - 3x + 2$. Find the stationary points for this function and determine their nature	7
3.	A curve has the equation $y = x^4 - 4x^3 + 3$ Algebraically find the coordinates of the stationary points and determine their nature	7
4.	(a) Find the coordinates of the stationary points of the graph with equation $y = x^3 + 3x^2 - 24x$ and determine their nature (b) Hence determine the range of values of x for which the function is strictly decreasing	7 2
5.	(a) Find the x coordinates of the stationary points on the graphs with equation $y = \frac{1}{3}x^3 - 2x^2 - 5x - 4$ (b) Hence determine the range of values of x for which this graph is strictly increasing	4 2

Answers

	Stationary Points - Answers	
1	Differentiate the function: $\frac{dy}{dx} = 6x^2 - 6x - 12$ Set the derivative = 0: $6x^2 - 6x - 12 = 0$ Factorise: $6(x + 1)(x - 2) = 0$ Solve for x: $x = -1, \quad x = 2$ Find values for y by substituting into the original function: $y = 27, \quad y = 0$ Use a nature table: x: -1^-, -1, -1^+, 2, 2^+ $\frac{dy}{dx}$: $+$, 0, $-$, 0, $+$ State a conclusion: Maximum at (-1,27) Minimum at (2,0)	
2	Differentiate the function: $\frac{dy}{dx} = 3x^2 - 3$ Set the derivative = 0: $3x^2 - 3 = 0$ Factorise: $3(x + 1)(x - 1) = 0$ Solve for x: $x = -1, \quad x = 1$ Find values for y by substituting into the original function: $y = 4, \quad y = 0$	

	Use a nature table	x: -1^- -1 -1^+ 1 1^+ $\frac{dy}{dx}$: + 0 - 0 +	
	State a conclusion	Maximum at (-1,4) Minimum at (1,0)	
3	Differentiate the function	$\frac{dy}{dx} = 4x^3 - 12x^2$	
	Set the derivative = 0	$4x^3 - 12x^2 = 0$	
	Factorise	$4x^2 (x - 3) = 0$	
	Solve for x	$x = 0$, $x = 3$	
	Find values for y by substituting into the original function	$y = 3$, $y = -24$	
	Use a nature table	x: 0^- 0 0^+ 3 3^+ $\frac{dy}{dx}$: - 0 - 0 +	
	State a conclusion	Point of inflexion at (0,3) Minimum at (3,-24)	
4	Differentiate the function	$\frac{dy}{dx} = 3x^2 + 6x - 24$	
	Set the derivative = 0	$3x^2 + 6x - 24 = 0$	
	Factorise	$3(x + 4)(x - 2) = 0$	
	Solve for x	$x = -4$, $x = 2$	
	Find values for y by substituting into the original function	$y = 80$, $y = -28$	
	Use a nature table	x: -4^- -4 -4^+ 2 2^+ $\frac{dy}{dx}$: + 0 - 0 +	
	State a conclusion	Maximum at (-4, 80) Minimum at (4,-24)	
	State where the curve is decreasing (dy/dx) is negative	$-4 < x < 2$	
5	Differentiate the function	$\frac{dy}{dx} = x^2 - 4x - 5$	
	Look for stationary points	$x^2 - 4x - 5 = 0$ $(x + 1)(x - 5) = 0$	
	Stationary points at	$x = -1$, $x = 5$	
	Use a nature table to identify the shape of the curve	x: -1^- -1 -1^+ 5 5^+ $\frac{dy}{dx}$: + 0 - 0 +	
	State where the curve is increasing (dy/dx) is positive	$x < -1$ and $x > 5$	