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Higher Exam Study Pack

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Differentiation

Note: Some questions overlap two topic areas

For answers, please check SQA marking schemes on the link HERE

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

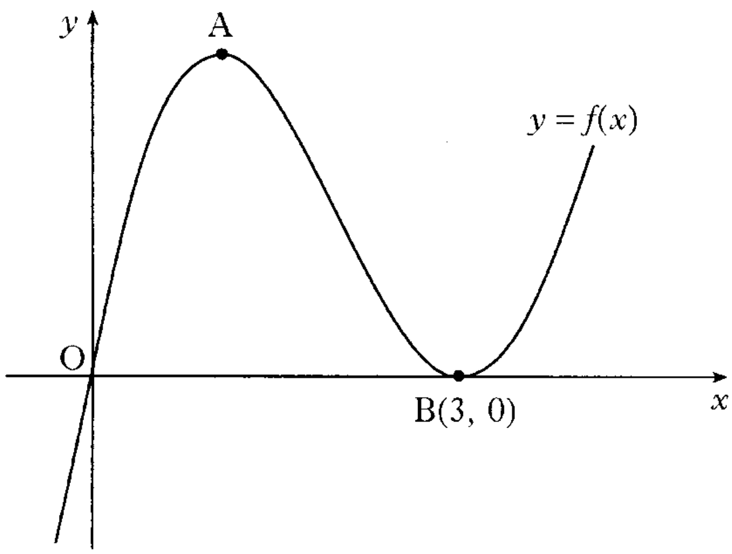
Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

2000

P1 Q2

A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below. The graph has a maximum at A and a minimum at B(3, 0).



(*a*) Find the coordinates of the turning point at A. **4**

(*b*) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x + 2) + 4$.

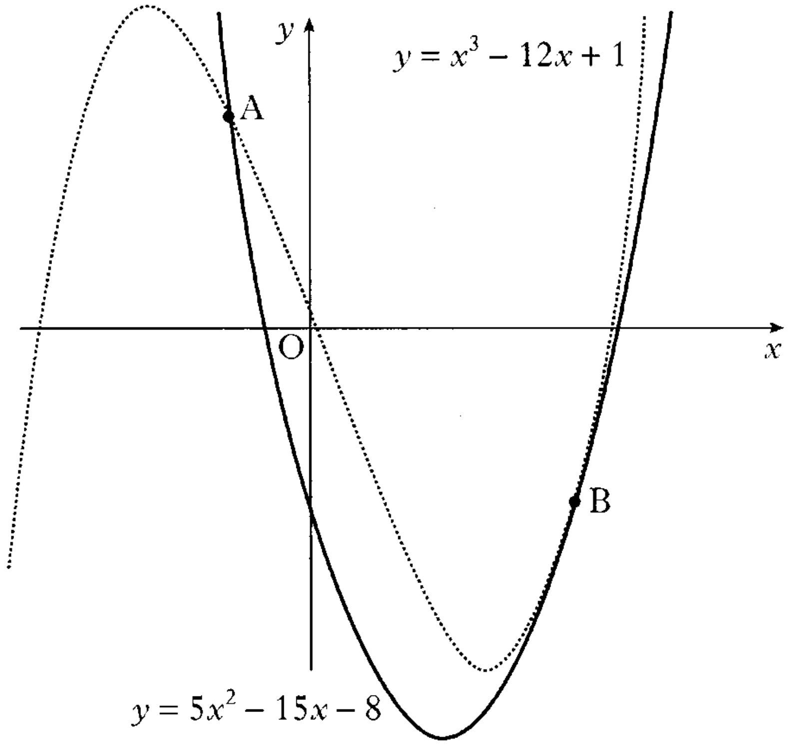
Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. **2**

(*c*) Write down the range of values of k for which $g(x) = k$ has 3 real roots. **1**

2000 P1 Q4

The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$.

The two curves intersect at A and touch at B, ie at B the curves have a common tangent.



(*a*) (i) Find the x-coordinates of the points on the curves where the gradients are equal. **4**

(ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). **1**

(*b*) The point A is $(-1, 12)$ and B is $(3, -8)$.

Find the area enclosed between the two curves. **5**

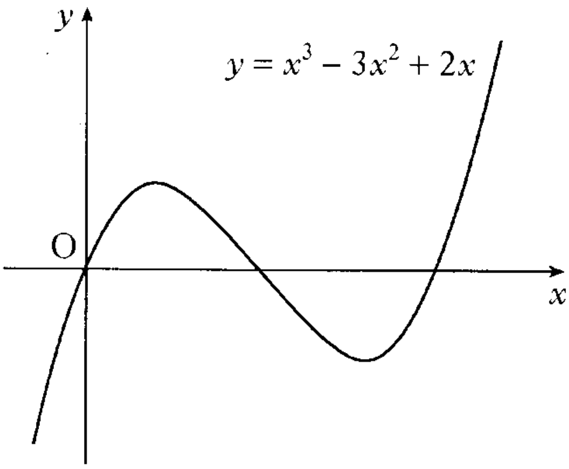
2000 P1 Q8

The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$.

If $f'(x) = \sin(3x)$, express y in terms of x. **4**

2000 P2 Q1

The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.

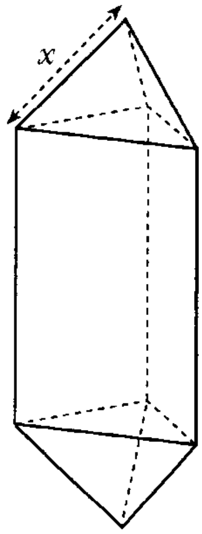


(*a*) Find the equation of the tangent to this curve at the point where $x = 1$. **5**

(*b*) The tangent at the point (2, 0) has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. **5**

2000 P2 Q6

A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.



The surface area, A, of the solid is given by

$$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$

where x is the length of each edge of the tetrahedron.

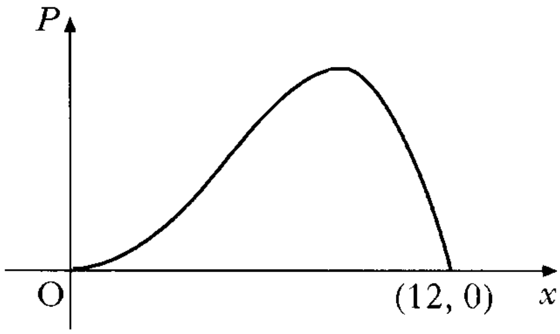
Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid. **6**

2000 P2 Q8

Given that $f(x) = (5x - 4)^{\frac{1}{2}}$, evaluate $f'(4)$. **3**

2001 P1 Q6

A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds. A mathematical model, illustrated in the diagram, suggests that P and x are related by $P = 12x^3 - x^4$ for $0 \le x \le 12$.

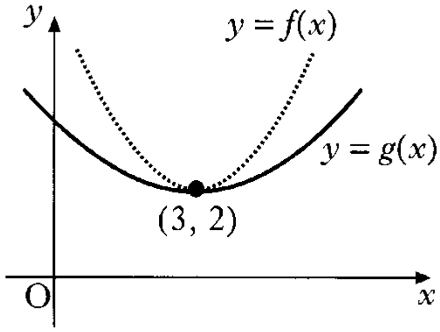


Find the value of x which gives the maximum profit. **5**

2001 P1 Q9

The diagram shows the graphs of two quadratic functions $y = f(x)$ and $y = g(x)$. Both graphs have a minimum turning point at (3, 2).

Sketch the graph of $y = f'(x)$ and on the same diagram sketch the graph of $y = g'(x)$. **2**



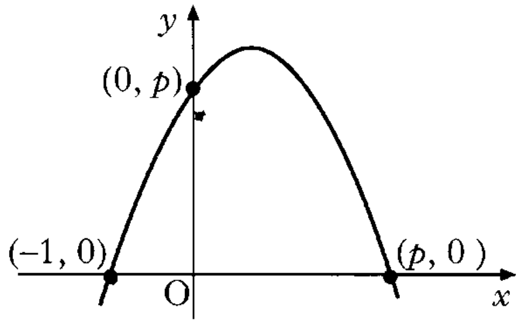
2001 P2 Q2

A curve has equation $y = x - \dfrac{16}{\sqrt{x}}, \quad x > 0$.

Find the equation of the tangent at the point where $x = 4$. **6**

2001 P2 Q11

The diagram shows a sketch of a parabola passing through (–1, 0), (0, p) and (p, 0).



(*a*) Show that the equation of the parabola is $y = p + (p - 1)x - x^2$. **3**

(*b*) For what value of p will the line $y = x + p$ be a tangent to this curve? **3**

2002 P1 Q4

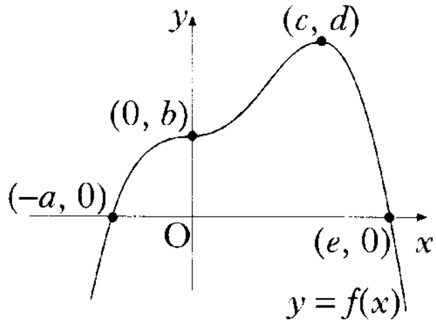
Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of 45° with the positive direction of the x-axis. **4**

2002 P1 Q6

The graph of a function f intersects the x-axis at $(-a, 0)$ and $(e, 0)$ as shown.

There is a point of inflexion at $(0, b)$ and a maximum turning point at (c, d).

Sketch the graph of the derived function f'. **3**



2002 P1 Q10

(*a*) Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}, \ x < 2$. **2**

(*b*) Hence write down $\displaystyle\int \frac{x^2}{(8 - x^3)^{\frac{1}{2}}}\,dx$. **1**

2002 P2 Q6

Find the equation of the tangent to the curve $y = 2\sin\left(x - \dfrac{\pi}{6}\right)$ at the point where $x = \dfrac{\pi}{3}$. **4**

2003 P1 Q5

Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$. **5**

2003 P2 Q4

(*a*) Find the equation of the tangent to the curve with equation $y = x^3 + 2x^2 - 3x + 2$ at the point where $x = 1$. **5**

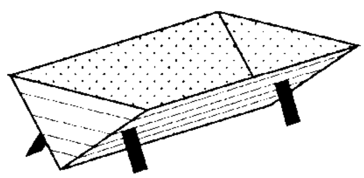
(*b*) Show that this line is also a tangent to the circle with equation $x^2 + y^2 - 12x - 10y + 44 = 0$ and state the coordinates of the point of contact. **6**

2003 P2 Q6

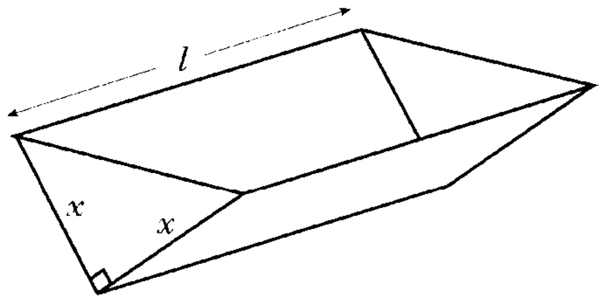
If $f(x) = \cos(2x) - 3\sin(4x)$, find the exact value of $f'\left(\frac{\pi}{6}\right)$. **4**

2003 P2 Q8

An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.



The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.



(*a*) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \frac{432000}{x}$. **3**

(*b*) Find the value of x which minimises this surface area. **5**

2004 P1 Q6

Given that $y = 3\sin(x) + \cos(2x)$, find $\frac{dy}{dx}$. **3**

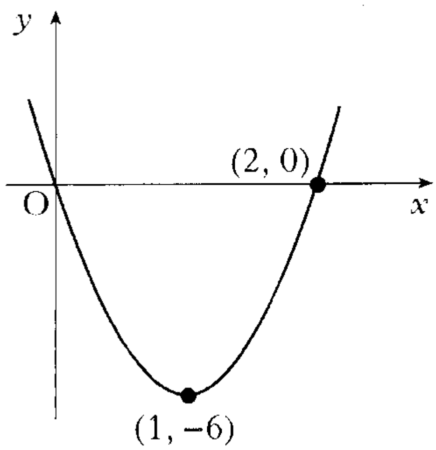
2004 P1 Q8

(*a*) Write $x^2 - 10x + 27$ in the form $(x + b)^2 + c$. **2**

(*b*) Hence show that the function $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ is always increasing. **4**

2004 P1 Q11

The diagram shows a parabola passing through the points (0, 0), (1, −6) and (2, 0).



(*a*) The equation of the parabola is of the form $y = ax(x - b)$.
Find the values of a and b. **3**

(*b*) This parabola is the graph of $y = f'(x)$.
Given that $f(1) = 4$, find the formula for $f(x)$. **5**

2004 P2 Q5

The point P(x, y) lies on the curve with equation $y = 6x^2 - x^3$.

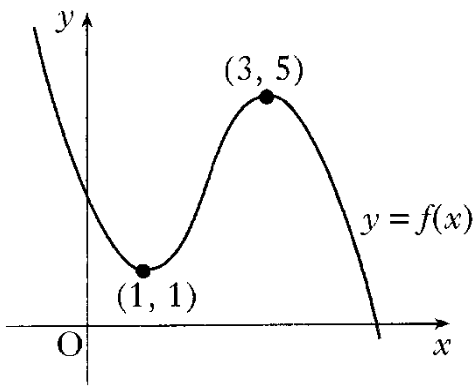
(*a*) Find the value of x for which the gradient of the tangent at P is 12. **5**

(*b*) Hence find the equation of the tangent at P. **2**

2004 P2 Q7

The graph of the cubic function $y = f(x)$ is shown in the diagram. There are turning points at (1, 1) and (3, 5).

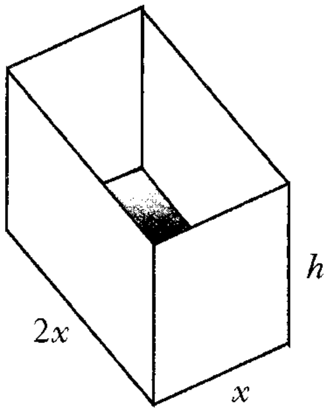
Sketch the graph of $y = f'(x)$. **3**



2004 P2 Q9

Marks

An open cuboid measures internally x units by $2x$ units by h units and has an inner surface area of 12 units^2.



(*a*) Show that the volume, V units^3, of the cuboid is given by $V(x) = \frac{2}{3}x(6 - x^2)$. **3**

(*b*) Find the exact value of x for which this volume is a maximum. **5**

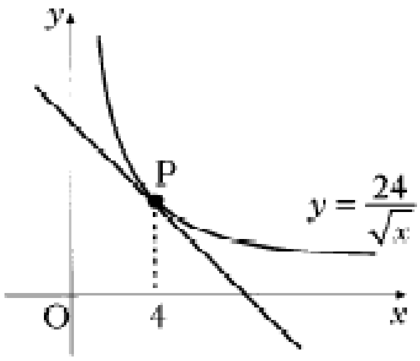
2005 P1 Q5

Differentiate $(1 + 2\sin x)^4$ with respect to x. **2**

2005 P2 Q6

The diagram shows the graph of $y = \frac{24}{\sqrt{x}}$, $x > 0$.

Find the equation of the tangent at P, where $x = 4$. **6**



2006 P1 Q5

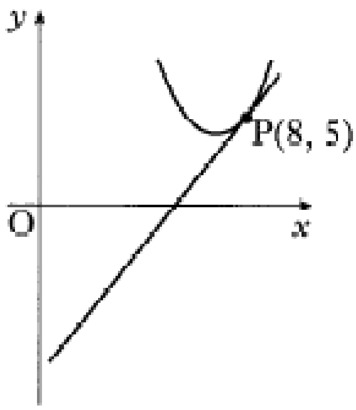
A function f is defined by $f(x) = (2x - 1)^5$.

Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature. **7**

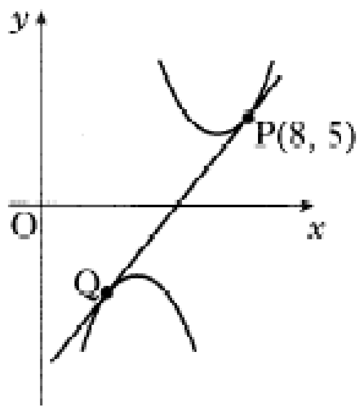
2006 P1 Q3

The parabola with equation $y = x^2 - 14x + 53$ has a tangent at the point P(8, 5).

(*a*) Find the equation of this tangent. **4**



(*b*) Show that the tangent found in (*a*) is also a tangent to the parabola with equation $y = -x^2 + 10x - 27$ and find the coordinates of the point of contact Q. **5**



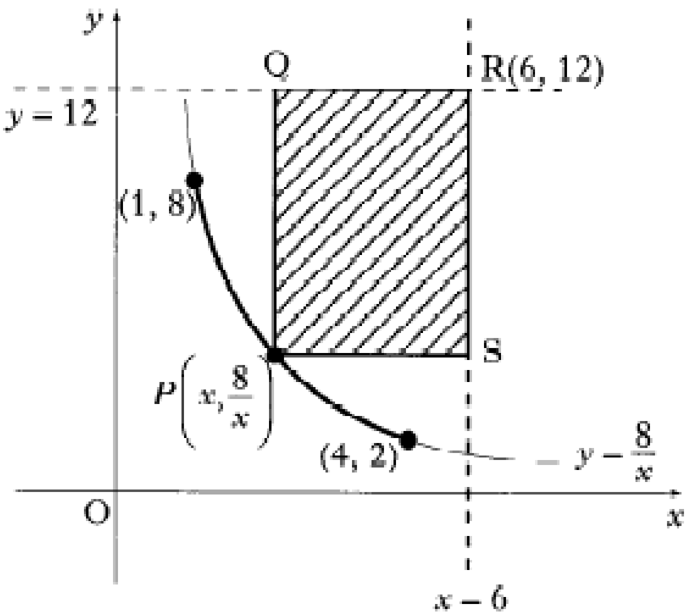
2006 P2 Q9

If $y = \frac{1}{x^3} - \cos 2x$, $x \neq 0$, find $\frac{dy}{dx}$. **4**

2006 P2 Q12

PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- P lies on the curve with equation $y = \frac{8}{x}$ between (1, 8) and (4, 2)
- R is the point (6, 12).



(*a*) (i) Express the lengths of PS and RS in terms of x, the x-coordinate of P.

(ii) Hence show that the area, A square units, of PQRS is given by

$$A = 80 - 12x - \frac{48}{x}.$$ **3**

(*b*) Find the greatest and least possible values of A and the corresponding values of x for which they occur. **8**

2007 P1 Q9

A function f is defined by the formula $f(x) = 3x - x^3$.

(*a*) Find the exact values where the graph of $y = f(x)$ meets the x- and y-axes. **2**

(*b*) Find the coordinates of the stationary points of the function and determine their nature. **7**

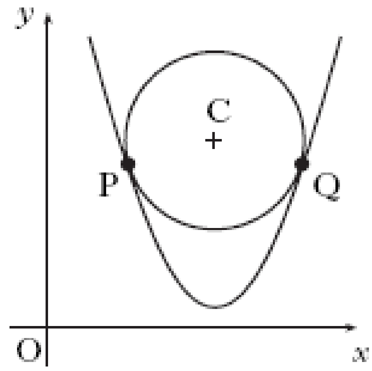
(*c*) Sketch the graph of $y = f(x)$. **1**

2007 P1 Q10

Given that $y = \sqrt{3x^2 + 2}$, find $\frac{dy}{dx}$. **3**

2007 P2 Q5

A circle centre C is situated so that it touches the parabola with equation $y = \frac{1}{2}x^2 - 8x + 34$ at P and Q.



(*a*) The gradient of the tangent to the parabola at Q is 4. Find the coordinates of Q. **5**

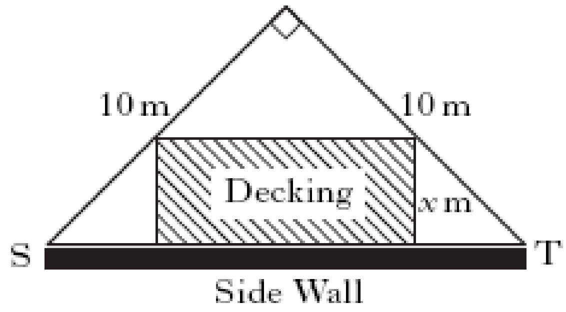
(*b*) Find the coordinates of P. **2**

(*c*) Find the coordinates of C, the centre of the circle. **2**

2007 P2 Q6

A householder has a garden in the shape of a right-angled isosceles triangle.

It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.



(*a*) (i) Find the exact value of ST.

(ii) Given that the breadth of the decking is x metres, show that the area of the decking, A square metres, is given by

$$A = \left(10\sqrt{2}\right)x - 2x^2.$$ **3**

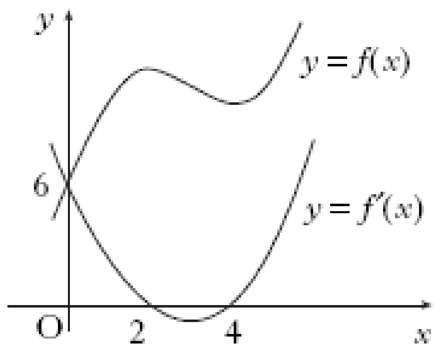
(*b*) Find the dimensions of the decking which maximises its area. **5**

2007 P2 Q10

The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.

Both graphs pass through the point (0, 6).

The graph of $y = f'(x)$ also passes through the points (2, 0) and (4, 0).



(*a*) Given that $f'(x)$ is of the form $k(x-a)(x-b)$:

(i) write down the values of a and b;

(ii) find the value of k. **3**

(*b*) Find the equation of the graph of the cubic function $y = f(x)$. **4**

2008 P2 Q15

What is the derivative of $(x^3 + 4)^2$?

A $(3x^2 + 4)^2$

B $\frac{1}{3}(x^3 + 4)^3$

C $6x^2(x^3 + 4)$

D $2(3x^2 + 4)^{-1}$

2008 P2 Q21

A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

(a) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. **6**

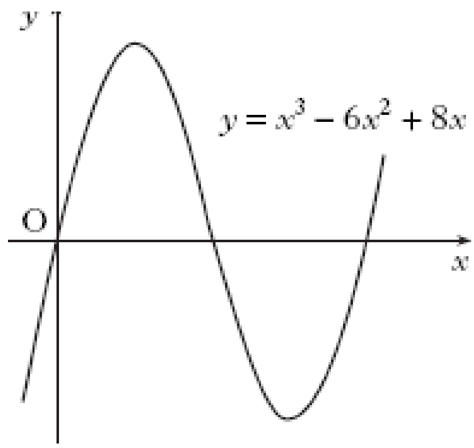
(b) (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$.

(ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. **5**

(c) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve. **4**

2008 P2 Q22

The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.



(a) Find the coordinates of the points on the curve where the gradient of the tangent is -1. **5**

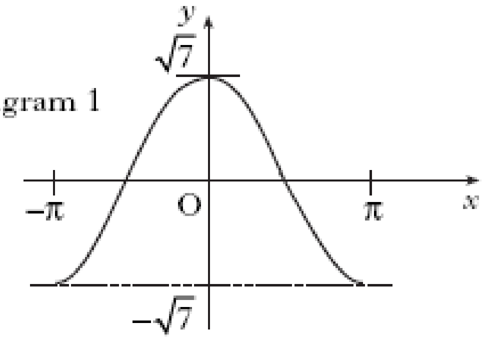
(b) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A. **2**

2008 P2 Q3

(*a*) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$.

Write down the value of p.

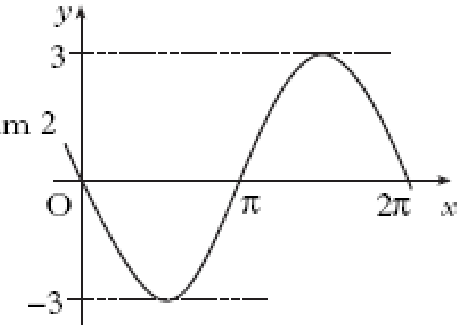
Diagram 1



(ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$.

Write down the value of q.

Diagram 2



2

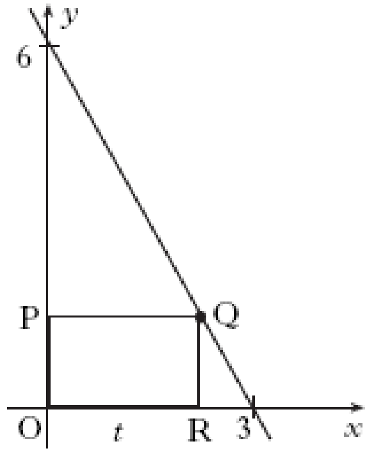
(*b*) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. **4**

(*c*) Hence find $f'(x) + g'(x)$ as a single trigonometric expression. **2**

2008 P2 Q6

In the diagram, Q lies on the line joining (0, 6) and (3, 0).

OPQR is a rectangle, where P and R lie on the axes and OR = t.



(*a*) Show that QR = $6 - 2t$. **3**

(*b*) Find the coordinates of Q for which the rectangle has a maximum area. **6**

2009 P1 Q4

A curve has equation $y = 5x^3 - 12x$.

What is the gradient of the tangent at the point $(1, -7)$?

A -7

B -5

C 3

D 5

2009 P1 Q8

What is the derivative of $\dfrac{1}{4x^3}$, $x \neq 0$?

A $\dfrac{1}{12x^2}$

B $-\dfrac{1}{12x^2}$

C $\dfrac{4}{x^4}$

D $-\dfrac{3}{4x^4}$

2009 P1 Q20

$A = 2\pi r^2 + 6\pi r$.

What is the rate of change of A with respect to r when $r = 2$?

A 10π

B 12π

C 14π

D 20π

2009 P2 Q1

Find the coordinates of the turning points of the curve with equation $y = x^3 - 3x^2 - 9x + 12$ and determine their nature. **8**

2009 P2 Q2

Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$.

(*a*) (i) Find $p(x)$ where $p(x) = f(g(x))$.

(ii) Find $q(x)$ where $q(x) = g(f(x))$. **3**

(*b*) Solve $p'(x) = q'(x)$. **3**

2010 P1 Q12

If $f(x) = \frac{1}{\sqrt[5]{x}}$, $x \neq 0$, what is $f'(x)$?

A $-\frac{1}{5}x^{-\frac{6}{5}}$

B $-\frac{1}{5}x^{-\frac{4}{5}}$

C $-\frac{5}{2}x^{-\frac{7}{2}}$

D $-\frac{5}{2}x^{-\frac{3}{2}}$

2010 P1 Q15

The derivative of a function f is given by $f'(x) = x^2 - 9$.

Here are two statements about f:

(1) f is increasing at $x = 1$;

(2) f is stationary at $x = -3$.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

2010 P1 Q17

If $s(t) = t^2 - 5t + 8$, what is the rate of change of s with respect to t when $t = 3$?

A -5

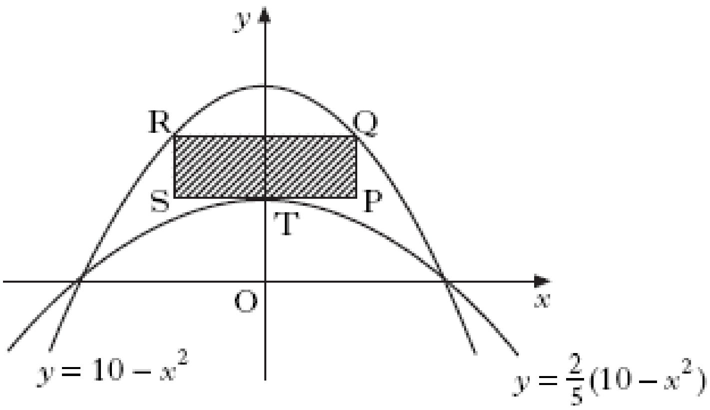
B 1

C 2

D 9

2010 P2 Q5

The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.



A rectangle PQRS is placed between the two parabolas as shown, so that:

- Q and R lie on the upper parabola;
- RQ and SP are parallel to the x-axis;
- T, the turning point of the lower parabola, lies on SP.

(a) (i) If TP = x units, find an expression for the length of PQ.

(ii) Hence show that the area, A, of rectangle PQRS is given by

$$A(x) = 12x - 2x^3.$$ **3**

(b) Find the maximum area of this rectangle. **6**

2011 P1 Q4

A tangent to the curve with equation $y = x^3 - 2x$ is drawn at the point $(2, 4)$.

What is the gradient of this tangent?

A 2

B 3

C 4

D 10

2011 P1 Q13

Given that $f(x) = 4 \sin 3x$, find $f'(0)$.

A 0

B 1

C 12

D 36

2011 P1 Q22

A function f is defined on the set of real numbers by $f(x) = (x - 2)(x^2 + 1)$.

(*a*) Find where the graph of $y = f(x)$ cuts:

(i) the x-axis;

(ii) the y-axis. **2**

(*b*) Find the coordinates of the stationary points on the curve with equation $y = f(x)$ and determine their nature. **8**

(*c*) On separate diagrams sketch the graphs of:

(i) $y = f(x)$;

(ii) $y = -f(x)$. **3**

2012 P1 Q2

What is the gradient of the tangent to the curve with equation $y = x^3 - 6x + 1$ at the point where $x = -2$?

A -24

B 3

C 5

D 6

2012 P1 Q6

If $y = 3x^{-2} + 2x^{\frac{3}{2}}$, $x > 0$, determine $\frac{dy}{dx}$.

A $-6x^{-3} + \frac{4}{5}x^{\frac{5}{2}}$

B $-3x^{-1} + 3x^{\frac{1}{2}}$

C $-6x^{-3} + 3x^{\frac{1}{2}}$

D $-3x^{-1} + \frac{4}{5}x^{\frac{5}{2}}$

2012 P1 Q8

The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$.

What is the rate of change of V with respect to r, at $r = 2$?

A $\frac{16\pi}{3}$

B $\frac{32\pi}{3}$

C 16π

D 32π

2012 P1 Q16

If $y = 3\cos^4 x$, find $\frac{dy}{dx}$.

A $12\cos^3 x \sin x$

B $12\cos^3 x$

C $-12\cos^3 x \sin x$

D $-12\sin^3 x$

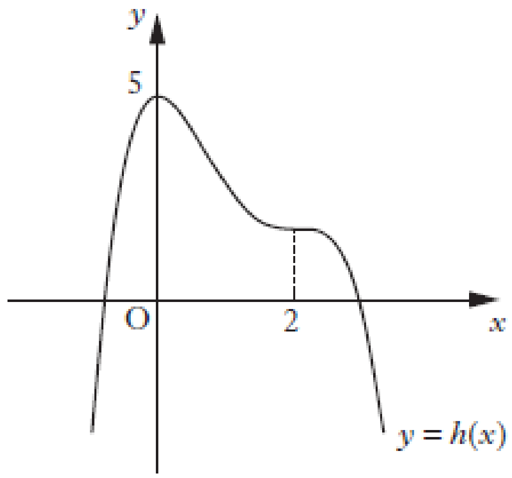
2012 P2 Q3

A function f is defined on the domain $0 \le x \le 3$ by $f(x) = x^3 - 2x^2 - 4x + 6$.

Determine the maximum and minimum values of f. **7**

2012 P2 Q4

The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $x = 0$ and $x = 2$.



On separate diagrams sketch the graphs of:

(*a*) $y = h'(x)$; **3**

(*b*) $y = 2 - h'(x)$. **3**

2013 P1 Q2

The point P (5, 12) lies on the curve with equation $y = x^2 - 4x + 7$.

What is the gradient of the tangent to this curve at P?

A 2

B 6

C 12

D 13

2013 P1 Q18

Given that $y = \sin(x^2 - 3)$, find $\dfrac{dy}{dx}$.

A $\sin 2x$

B $\cos 2x$

C $2x\sin(x^2 - 3)$

D $2x\cos(x^2 - 3)$

2013 P2 Q3

(*a*) Given that $(x - 1)$ is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic fully. **4**

(*b*) Show that the curve with equation

$$y = x^4 + 4x^3 + 2x^2 - 20x + 3$$

has only one stationary point.

Find the x-coordinate and determine the nature of this point. **5**

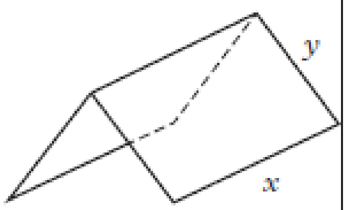
2013 P2 Q7

A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions.

Condition 1

The frame of a shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.



Condition 2

The frame is to be covered by a rectangular sheet of material.

The total area of the sheet is $24\ \text{m}^2$.

(*a*) Show that the total length, L metres, of the rods used in a shelter is given by

$$L = 3x + \frac{48}{x}.$$

3

(*b*) These rods cost £8·25 per metre.

To minimise production costs, the total length of rods used for a frame should be as small as possible.

(i) Find the value of x for which L is a minimum.

(ii) Calculate the minimum cost of a frame. **7**