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Higher Exam Study Pack

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# Polynomials & Quadratics

Note: Some questions overlap two topic areas

For answers, please check SQA marking schemes on the link HERE

**FORMULAE LIST**

**Circle:**

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a, b)$ and radius $r$.

**Scalar Product:** $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where $\theta$ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

**Trigonometric formulae:**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

**Table of standard derivatives:**

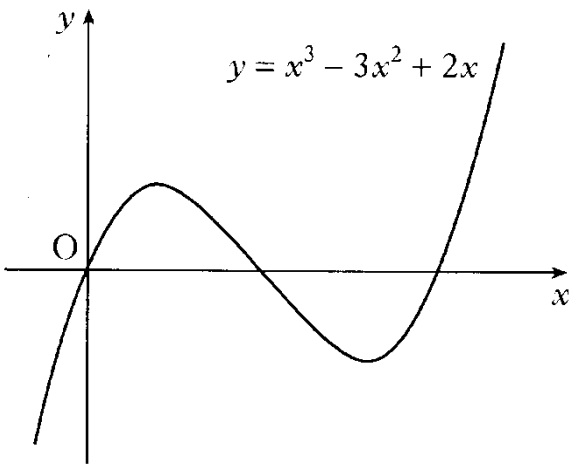
| $f(x)$ | $f'(x)$ |
|---|---|
| $\sin ax$ | $a\cos ax$ |
| $\cos ax$ | $-a\sin ax$ |

**Table of standard integrals:**

| $f(x)$ | $\int f(x)dx$ |
|---|---|
| $\sin ax$ | $-\frac{1}{a}\cos ax + c$ |
| $\cos ax$ | $\frac{1}{a}\sin ax + c$ |

2000 P2 Q1

The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.




(*a*) Find the equation of the tangent to this curve at the point where $x = 1$. **5**

(*b*) The tangent at the point (2, 0) has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. **5**

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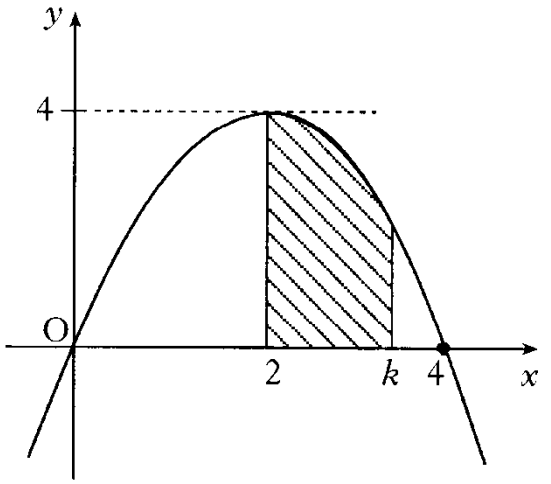
2000 P2 Q4

The parabola shown crosses the $x$-axis at (0, 0) and (4, 0), and has a maximum at (2, 4).

The shaded area is bounded by the parabola, the $x$-axis and the lines $x = 2$ and $x = k$.




(*a*) Find the equation of the parabola. **2**

(*b*) Hence show that the shaded area, A, is given by

$$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}.$$ **3**

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2001 P1 Q2

For what value of $k$ does the equation $x^2 - 5x + (k + 6) = 0$ have equal roots? **3**

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2001 P1 Q4

Given $f(x) = x^2 + 2x - 8$, express $f(x)$ in the form $(x + a)^2 - b$. **2**

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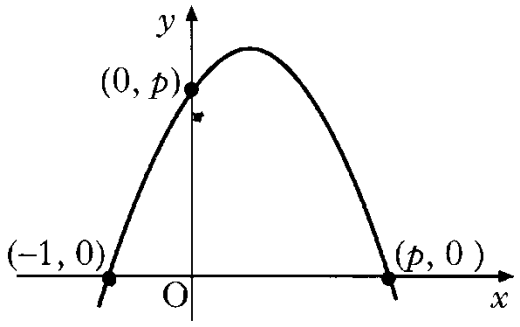
2001 P2 Q1

(*a*) Given that $x + 2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of $k$. **3**

(*b*) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when $k$ takes this value. **2**

2001 P2 Q11

The diagram shows a sketch of a parabola passing through $(-1, 0)$, $(0, p)$ and $(p, 0)$.




(*a*) Show that the equation of the parabola is $y = p + (p - 1)x - x^2$. **3**

(*b*) For what value of $p$ will the line $y = x + p$ be a tangent to this curve? **3**

---

2002 P1 Q7

(*a*) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$. **2**

(*b*) On the same diagram sketch:

(i) the graph of $y = f(x)$;

(ii) the graph of $y = 10 - f(x)$. **4**

(*c*) Find the range of values of $x$ for which $10 - f(x)$ is positive. **1**

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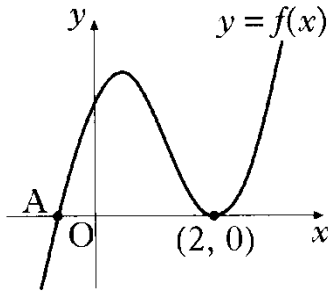
2002 P2 Q3

The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.




(*a*) Find the $x$-coordinate of the maximum turning point. **5**

(*b*) Factorise $2x^3 - 7x^2 + 4x + 4$. **3**

(*c*) State the coordinates of the point A and hence find the values of $x$ for which $2x^3 - 7x^2 + 4x + 4 < 0$. **2**

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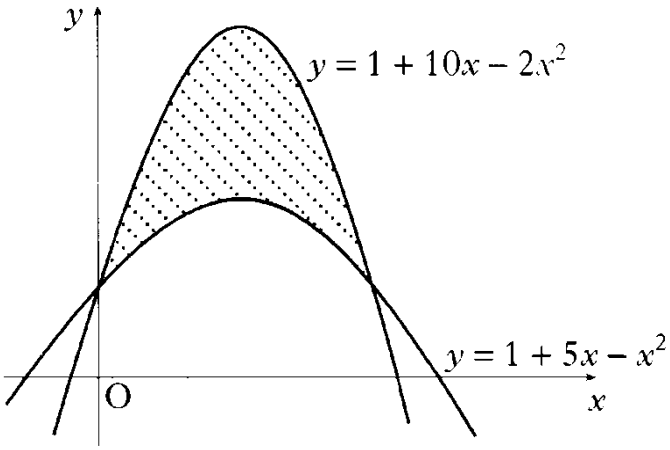
2002 P2 Q5

Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$. **6**

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2002 P2 Q9

Show that the equation $(1 - 2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of $k$. **5**

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2003 P1 Q2

(*a*) Write $f(x) = x^2 + 6x + 11$ in the form $(x + a)^2 + b$. **2**

(*b*) Hence or otherwise sketch the graph of $y = f(x)$. **2**

2003 P1 Q7

Show that the line with equation $y = 2x + 1$ does not intersect the parabola with equation $y = x^2 + 3x + 4$. **5**

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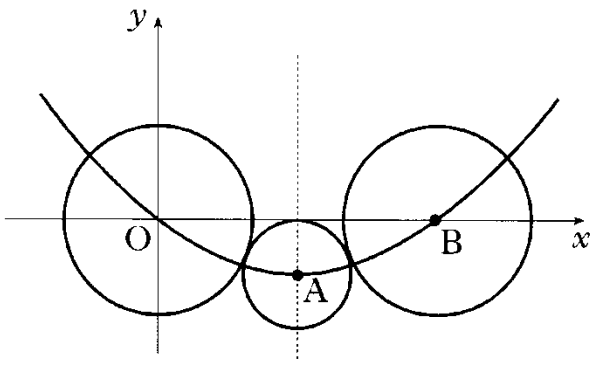
2003 P1 Q11

- O, A and B are the centres of the three circles shown in the diagram below.
- The two outer circles are congruent and each touches the smallest circle.
- Circle centre A has equation $(x - 12)^2 + (y + 5)^2 = 25$.
- The three centres lie on a parabola whose axis of symmetry is shown by the broken line through A.




(*a*) (i) State the coordinates of A and find the length of the line OA. **2**

(ii) Hence find the equation of the circle with centre B. **3**

(*b*) The equation of the parabola can be written in the form $y = px(x + q)$.
Find the values of $p$ and $q$. **2**

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2003 P2 Q1

$f(x) = 6x^3 - 5x^2 - 17x + 6$.

(*a*) Show that $(x - 2)$ is a factor of $f(x)$.

(*b*) Express $f(x)$ in its fully factorised form. **4**

---

2004 P1 Q2

$f(x) = x^3 - x^2 - 5x - 3$.

(*a*) (i) Show that $(x + 1)$ is a factor of $f(x)$.

(ii) Hence or otherwise factorise $f(x)$ fully. **5**

(*b*) One of the turning points of the graph of $y = f(x)$ lies on the $x$-axis.
Write down the coordinates of this turning point. **1**

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2004 P1 Q8

(*a*) Write $x^2 - 10x + 27$ in the form $(x + b)^2 + c$. **2**

(*b*) Hence show that the function $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ is always increasing. **4**

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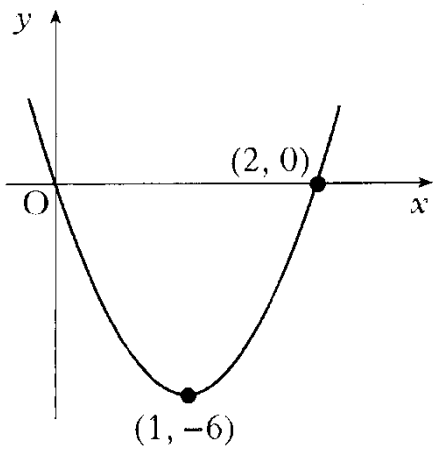
2004 P1 Q11

The diagram shows a parabola passing through the points (0, 0), (1, −6) and (2, 0).




(*a*) The equation of the parabola is of the form $y = ax(x - b)$.
Find the values of $a$ and $b$. **3**

(*b*) This parabola is the graph of $y = f'(x)$.
Given that $f(1) = 4$, find the formula for $f(x)$. **5**

---

2004 P2 Q3

Prove that the roots of the equation $2x^2 + px - 3 = 0$ are real for all values of $p$. **4**

---

2005 P1 Q8

A function $f$ is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where $x$ is a real number.

(*a*) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully. **5**

(*b*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the $x$- and $y$-axes. **2**

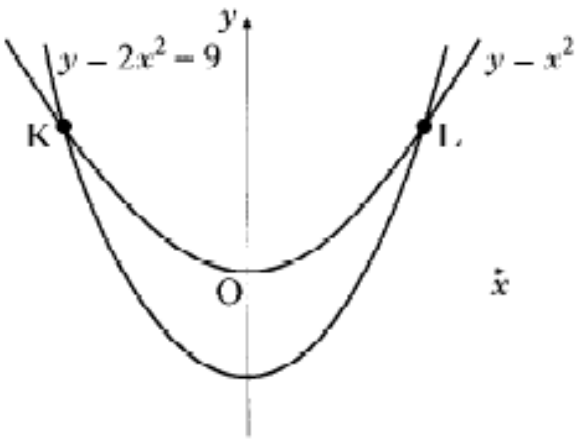
(*c*) Find the greatest and least values of $f$ in the interval $-2 \leq x \leq 2$. **5**

---

2005 P2 Q5

The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown.

Calculate the area enclosed between the curves. **8**

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2005 P2 Q11

(*a*) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$. **1**

(*b*) Hence find the range of values of $p$ for which all the roots of the cubic equation are real. **7**

---

2006 P1 Q8

(*a*) Express $2x^2 + 4x - 3$ in the form $a(x + b)^2 + c$. **3**

(*b*) Write down the coordinates of the turning point on the parabola with equation $y = 2x^2 + 4x - 3$. **1**

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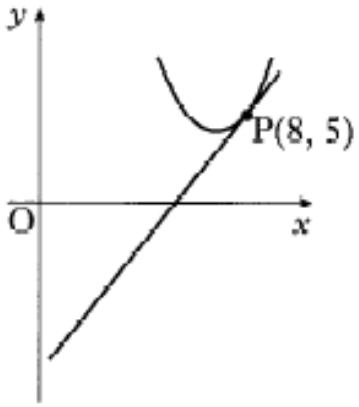
2006 P2 Q2

Find the value of $k$ such that the equation $kx^2 + kx + 6 = 0$, $k \neq 0$, has equal roots. **4**

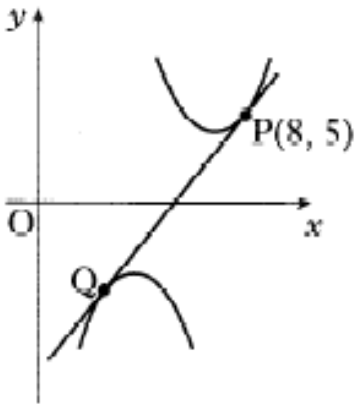
2006 P2 Q3

The parabola with equation $y = x^2 - 14x + 53$ has a tangent at the point P(8, 5).

(*a*) Find the equation of this tangent. **4**




(*b*) Show that the tangent found in (*a*) is also a tangent to the parabola with equation $y = -x^2 + 10x - 27$ and find the coordinates of the point of contact Q. **5**

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2007 P1 Q4

Find the range of values of $k$ such that the equation $kx^2 - x - 1 = 0$ has no real roots. **4**

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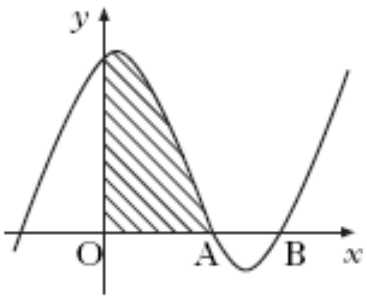
2007 P1 Q8

The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$.

(*a*) Show that the graph cuts the $x$-axis at (3, 0). **1**

(*b*) Hence or otherwise find the coordinates of A. **3**

(*c*) Find the shaded area. **5**

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2007 P2 Q3

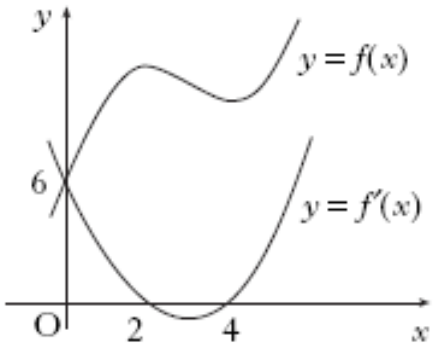
Show that the line with equation $y = 6 - 2x$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 4y - 7 = 0$ and find the coordinates of the point of contact of the tangent and the circle. **6**

2007 P2 Q10

The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.

Both graphs pass through the point (0, 6).

The graph of $y = f'(x)$ also passes through the points (2, 0) and (4, 0).




(*a*) Given that $f'(x)$ is of the form $k(x - a)(x - b)$:

(i) write down the values of $a$ and $b$;

(ii) find the value of $k$. **3**

(*b*) Find the equation of the graph of the cubic function $y = f(x)$. **4**

---

2008 P1 Q10

Here are two statements about the roots of the equation $x^2 + x + 1 = 0$:

(1) the roots are equal;

(2) the roots are real.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

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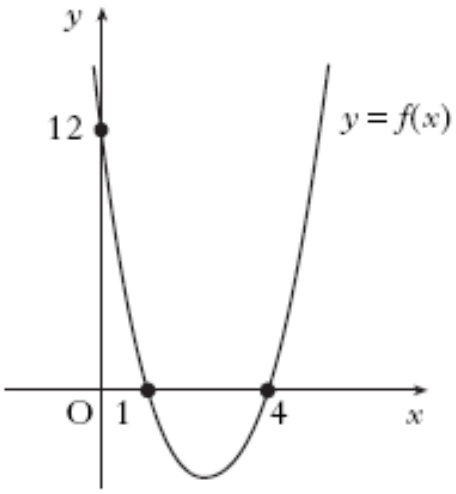
2008 P1 Q13

The diagram shows part of the graph of a quadratic function $y = f(x)$.

The graph has an equation of the form $y = k(x - a)(x - b)$.




What is the equation of the graph?

A $y = 3(x - 1)(x - 4)$

B $y = 3(x + 1)(x + 4)$

C $y = 12(x - 1)(x - 4)$

D $y = 12(x + 1)(x + 4)$

---

2008 P2 Q16

$2x^2 + 4x + 7$ is expressed in the form $2(x + p)^2 + q$.

What is the value of $q$?

A 5

B 7

C 9

D 11

---

2008 P2 Q21

A function $f$ is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

(*a*) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. **6**

(*b*) (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$.

(ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. **5**

(*c*) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve. **4**

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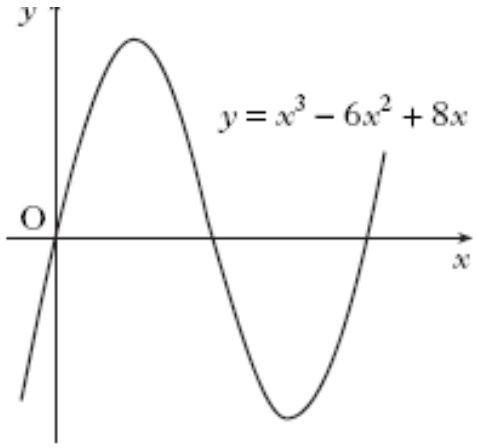
2008 P2 Q22

The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.




(*a*) Find the coordinates of the points on the curve where the gradient of the tangent is –1. **5**

(*b*) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A. **2**

---

2008 P2 Q4

(*a*) Write down the centre and calculate the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$. **2**

(*b*) A second circle has equation $(x - 4)^2 + (y - 6)^2 = 26$.

Find the distance between the centres of these two circles and hence show that the circles intersect. **4**

(*c*) The line with equation $y = 4 - x$ is a common chord passing through the points of intersection of the two circles.

Find the coordinates of the points of intersection of the two circles. **5**

---

2009 P1 Q9

The line with equation $y = 2x$ intersects the circle with equation $x^2 + y^2 = 5$ at the points J and K.

What are the $x$-coordinates of J and K?

A $x_J = 1, x_K = -1$

B $x_J = 2, x_K = -2$

C $x_J = 1, x_K = -2$

D $x_J = -1, x_K = 2$

---

2009 P1 Q12

A function $f$ is given by $f(x) = 2x^2 - x - 9$.

Which of the following describes the nature of the roots of $f(x) = 0$?

A No real roots

B Equal roots

C Real distinct roots

D Rational distinct roots

---

2009 P2 Q3

(*a*) (i) Show that $x = 1$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$.

(ii) Hence factorise $x^3 + 8x^2 + 11x - 20$ fully. **4**

(*b*) Solve $\log_2(x + 3) + \log_2(x^2 + 5x - 4) = 3$. **5**

---

2010 P1 Q5

When $x^2 + 8x + 3$ is written in the form $(x + p)^2 + q$, what is the value of $q$?

A $-19$

B $-13$

C $-5$

D $19$

---

2010 P1 Q6

The roots of the equation $kx^2 - 3x + 2 = 0$ are equal.

What is the value of $k$?

A $-\frac{9}{8}$

B $-\frac{8}{9}$

C $\frac{8}{9}$

D $\frac{9}{8}$

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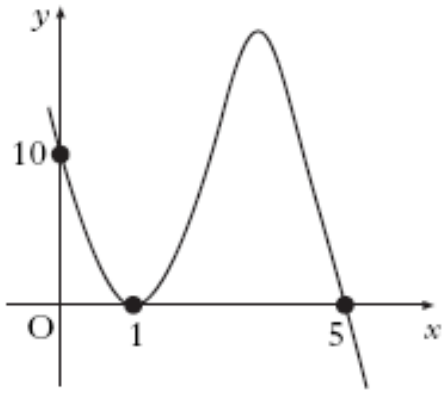
2010 P1 Q16

The diagram shows the graph with equation $y = k(x-1)^2(x+t)$.




What are the values of $k$ and $t$?

| | $k$ | $t$ |
|---|---|---|
| A | −2 | −5 |
| B | −2 | 5 |
| C | 2 | −5 |
| D | 2 | 5 |

---

2010 P1 Q18

What is the solution of $x^2 + 4x > 0$, where $x$ is a real number?

A $-4 < x < 0$

B $x < -4,\ x > 0$

C $0 < x < 4$

D $x < 0,\ x > 4$

---

2010 P1 Q22

(*a*) (i) Show that $(x-1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$.

(ii) Hence factorise $f(x)$ fully. **5**

(*b*) Solve $2x^3 + x^2 - 8x + 5 = 0$. **1**

(*c*) The line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point G.

Find the coordinates of G. **5**

(*d*) This tangent meets the curve again at the point H.

Write down the coordinates of H. **1**

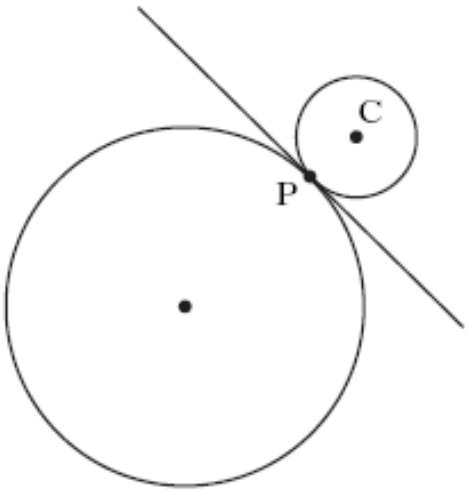
---

2010 P2 Q3

(*a*) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$.

(ii) Find the coordinates of the point of contact, P. **5**

(*b*) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (*a*) and a second smaller circle with centre C.




The line $y = 3 - x$ is a common tangent at the point P.

The radius of the larger circle is three times the radius of the smaller circle.

Find the equation of the smaller circle. **6**

---

2011 P1 Q5

If $x^2 - 8x + 7$ is written in the form $(x - p)^2 + q$, what is the value of $q$?

A –9

B –1

C 7

D 23

---

2011 P1 Q7

A function $f$ is defined on the set of real numbers by $f(x) = x^3 - x^2 + x + 3$.

What is the remainder when $f(x)$ is divided by $(x - 1)$?

A 0

B 2

C 3

D 4

---

2011 P1 Q9

The discriminant of a quadratic equation is 23.

Here are two statements about this quadratic equation:

(1) the roots are real;

(2) the roots are rational.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

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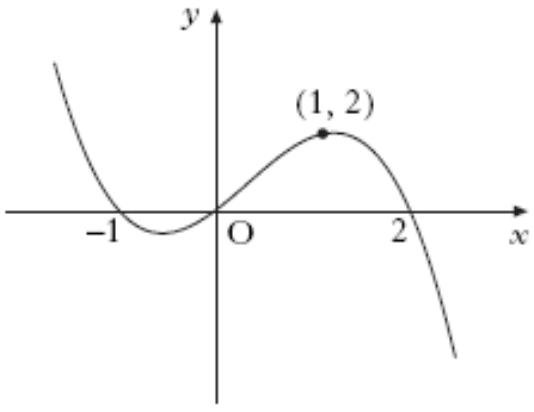
2011 P1 Q17

The diagram shows the graph of a cubic.




What is the equation of this cubic?

A $y = -x(x + 1)(x - 2)$

B $y = -x(x - 1)(x + 2)$

C $y = x(x + 1)(x - 2)$

D $y = x(x - 1)(x + 2)$

---

2011 P1 Q18

If $f(x) = (x - 3)(x + 5)$, for what values of $x$ is the graph of $y = f(x)$ above the $x$-axis?

A $-5 < x < 3$

B $-3 < x < 5$

C $x < -5, x > 3$

D $x < -3, x > 5$

---

2011 P2 Q2

Functions $f$, $g$ and $h$ are defined on the set of real numbers by

- $f(x) = x^3 - 1$
- $g(x) = 3x + 1$
- $h(x) = 4x - 5$.

(*a*) Find $g(f(x))$. **2**

(*b*) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$. **1**

(*c*) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$.

(ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully. **5**

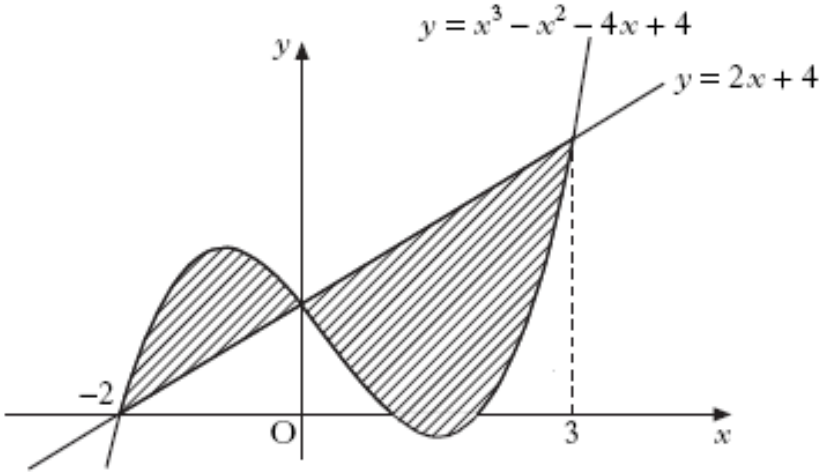
(*d*) Hence solve $g(f(x)) + xh(x) = 0$. **1**

---

2011 P2 Q4

The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$.

The curve and the line intersect at the points $(-2, 0)$, $(0, 4)$ and $(3, 10)$.




Calculate the total shaded area. **10**

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2011 P2 Q7

Circle $C_1$ has equation $(x+1)^2+(y-1)^2=121$.

A circle $C_2$ with equation $x^2+y^2-4x+6y+p=0$ is drawn inside $C_1$.

The circles have no points of contact.

What is the range of values of $p$? **9**

---

2012 P1 Q3

If $x^2-6x+14$ is written in the form $(x-p)^2+q$, what is the value of $q$?

A $-22$

B $5$

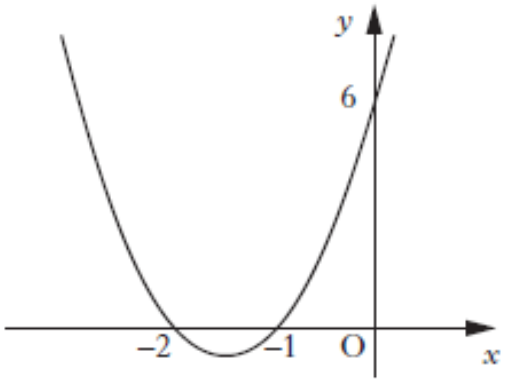
C $14$

D $50$

---

2012 P1 Q13

A parabola intersects the axes at $x=-2$, $x=-1$ and $y=6$, as shown in the diagram.




What is the equation of the parabola?

A $y=6(x-1)(x-2)$

B $y=6(x+1)(x+2)$

C $y=3(x-1)(x-2)$

D $y=3(x+1)(x+2)$

---

2012 P1 Q19

Solve $6-x-x^2<0$.

A $-3<x<2$

B $x<-3, x>2$

C $-2<x<3$

D $x<-2, x>3$

---

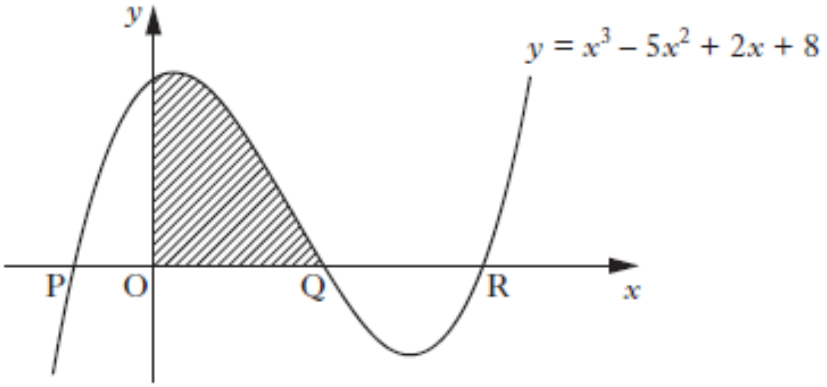
2012 P1 Q21

(*a*) (i) Show that $(x-4)$ is a factor of $x^3 - 5x^2 + 2x + 8$.

(ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.

(iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$. **6**

(*b*) The diagram shows the curve with equation $y = x^3 - 5x^2 + 2x + 8$.




The curve crosses the $x$-axis at P, Q and R.

Determine the shaded area. **6**

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2013 P1 Q3

Calculate the discriminant of the quadratic equation $2x^2 + 4x + 5 = 0$.

A –32

B –24

C 48

D 56

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2013 P1 Q6

What is the remainder when $x^3 + 3x^2 - 5x - 6$ is divided by $(x-2)$?

A 0

B 3

C 4

D 8

---

2013 P1 Q13

A function $f$ is defined on a suitable domain by $f(x) = \dfrac{x+2}{x^2-7x+12}$.

What value(s) of $x$ cannot be in this domain?

A 3 and 4

B $-3$ and $-4$

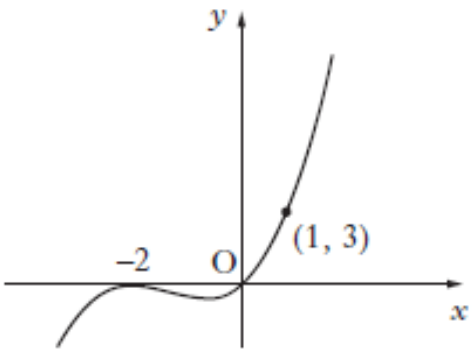
C $-2$

D 0

---

2013 P1 Q17

The diagram shows a curve with equation of the form $y = kx(x + a)^2$, which passes through the points $(-2, 0)$, $(0, 0)$ and $(1, 3)$.




What are the values of $a$ and $k$?

| | $a$ | $k$ |
|---|---|---|
| A | $-2$ | $\frac{1}{3}$ |
| B | $-2$ | 3 |
| C | 2 | $\frac{1}{3}$ |
| D | 2 | 3 |

---

2013 P1 Q19

Solve $1 - 2x - 3x^2 > 0$, where $x$ is a real number.

A $x < -1$ or $x > \frac{1}{3}$

B $-1 < x < \frac{1}{3}$

C $x < -\frac{1}{3}$ or $x > 1$

D $-\frac{1}{3} < x < 1$

---

2013 P1 Q21

Express $2x^2 + 12x + 1$ in the form $a(x + b)^2 + c$. **3**

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2013 P2 Q3

(*a*) Given that $(x - 1)$ is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic fully. **4**

(*b*) Show that the curve with equation

$$y = x^4 + 4x^3 + 2x^2 - 20x + 3$$

has only one stationary point.

Find the $x$-coordinate and determine the nature of this point. **5**

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