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Heinemann

Higher Maths Text Book

Worked Solutions

Ex 13E
Multiplication by a Scalar

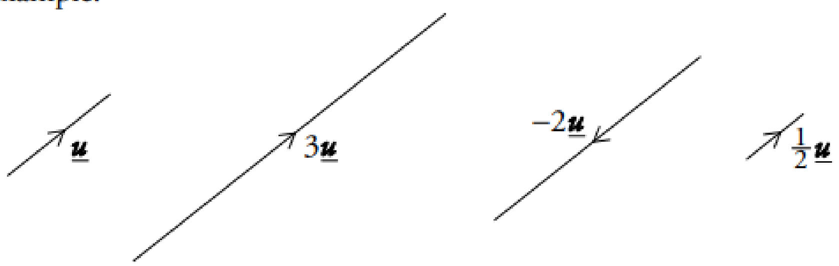
Multiplication by a Scalar

EF

A vector $\underline{u}$ which is multiplied by a scalar $k > 0$ will give the result $k\underline{u}$. This vector will be k times as long, i.e. the magnitude will be $k|\underline{u}|$.

Note that if $k < 0$ this means that the vector $k\underline{u}$ will be in the opposite direction to $\underline{u}$.

For example:



$$\text{If} \quad \underline{u} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad \text{then} \quad k\underline{u} = \begin{pmatrix} ka \\ kb \\ kc \end{pmatrix}.$$

Each component is multiplied by the scalar.

EXAMPLES

1. Given $\underline{v} = \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix}$, find $3\underline{v}$.

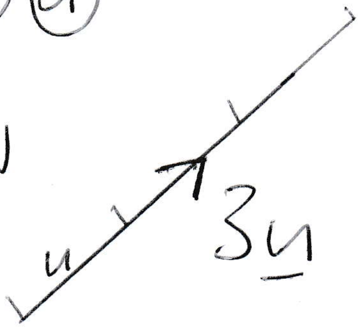
$$3\underline{v} = 3\begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ 15 \\ -9 \end{pmatrix}.$$

Ex 13E Questions 1-5

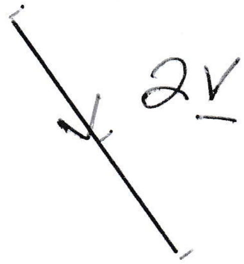
Worked solutions courtesy of Mr R Milton

1 (a)

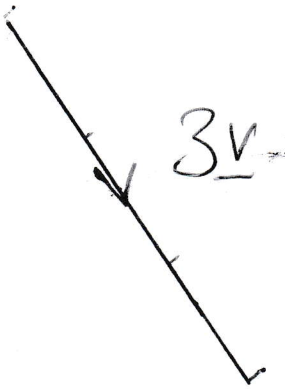
(i)



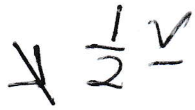
(ii)



(iii)



(iv)



(b) (i) $\underline{u} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \Rightarrow 3\underline{u} = \begin{bmatrix} 9 \\ 6 \end{bmatrix}$

(ii) $2\underline{v} = 2\begin{bmatrix} 2 \\ -2 \end{bmatrix} \Rightarrow 2\underline{v} = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$

(iii) $3\underline{v} = \begin{bmatrix} 6 \\ -6 \end{bmatrix}$ (iv) $\frac{1}{2}\underline{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(c) (i) $k\underline{u} = \begin{bmatrix} 3k \\ 2k \end{bmatrix}$ (ii) $k\underline{v} = \begin{bmatrix} 2k \\ -2k \end{bmatrix}$

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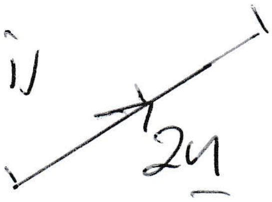
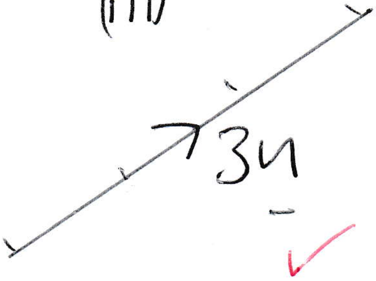
2 (i)

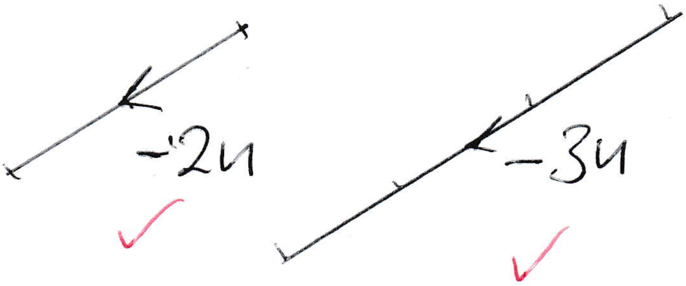
(ii)



(iii)



(iv)



3 (a) $3\underline{u} = \begin{bmatrix} -24 \\ 18 \end{bmatrix}$ (b) $4\underline{v} = \begin{bmatrix} 48 \\ 40 \end{bmatrix}$

(c) $\frac{1}{2}\underline{u} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$ (d) $\frac{3}{4}\underline{v} = \begin{bmatrix} 9 \\ 7.5 \end{bmatrix}$

(e) $3u + 4v = \begin{bmatrix} -24 \\ 18 \end{bmatrix} + \begin{bmatrix} 48 \\ 40 \end{bmatrix} = \begin{bmatrix} 24 \\ 58 \end{bmatrix}$

(f) $3u - 4v = \begin{bmatrix} -24 \\ 18 \end{bmatrix} - \begin{bmatrix} 48 \\ 40 \end{bmatrix} = \begin{bmatrix} -72 \\ -22 \end{bmatrix}$

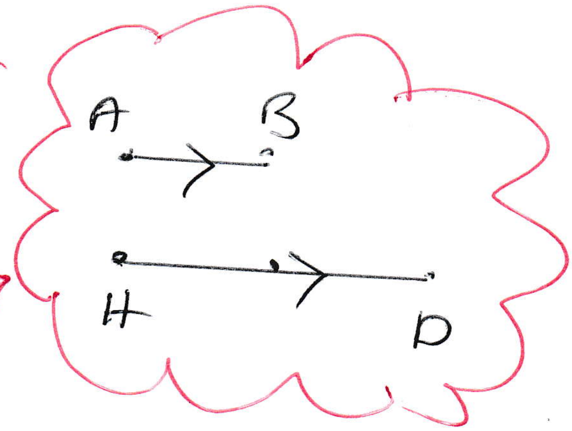
(4) $r = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$\underline{p}\ (k=-1)$ $\quad$ $\underline{s}\ (k=-3)$

$\underline{r}\ (k=2)$ $\quad$ $\underline{t}\ (k=1.5)$

(5) (a) $\overrightarrow{AE} = 4\overrightarrow{AI}$

(b) $\overrightarrow{HD} = 2\overrightarrow{AB}$



(c) $\overrightarrow{GL} = \frac{1}{4}\overrightarrow{GC}$

(d) $\overrightarrow{LM} = \frac{1}{2}\overrightarrow{MC}$

(e) $\overrightarrow{FB} = 2\overrightarrow{FM}$

(f) $\overrightarrow{GJ} = \frac{3}{4}\overrightarrow{GC}$

(g) $2\overrightarrow{GL} + 2\overrightarrow{MI} = \overrightarrow{GA}$

(h) $3\overrightarrow{EK} + \overrightarrow{HI} = 2\overrightarrow{ED} + \frac{1}{2}\overrightarrow{CA}$