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2004

Worked Solutions

With courtesy to the SQA and author(s)

Higher -2004 – Paper 1 answers

<table>
<tr><td>1.
(a)</td><td>

$x + 3y = -1 \longrightarrow 2x + 6y = -2$

$2x + 5y = 0 \qquad\quad \underline{2x + 5y = 0}$

y = -2

x = 5

Thus point of intersection : **B (5 , -2)**

$m_{AB} = \frac{-2-4}{5-7} = \frac{-6}{-2} = 3$
</td></tr>
<tr><td>(b)</td><td>

Line perpendicular to AB has gradient **-1/3 (m1 x m2 = -1)**

For x + 3y = -1, $y = -\frac{1}{3}x - \frac{1}{3}$ so m = -1/3

For 2x + 5y = 0, $y = -\frac{2}{5}x$ so m = -2/5

Therefore AB can is perpendicular to x + 3y = -1 only.
</td></tr>
<tr><td>2.
(a)</td><td>

-1 | 1 -1 -5 -3
    | -1 2 3
     1 -2 -3 **0**

As remainder is zero (x+1) is a factor of f(x)
</td></tr>
<tr><td>(ii)</td><td>

f(x) = (x + 1) (x^2 – 2x – 3)
= (x + 1) (x + 1) (x - 3)
= **$(x + 1)^2$ (x + 3)**
</td></tr>
<tr><td>(b)</td><td>

(-1, 0) (repeated root)
</td></tr>
<tr><td>3.</td><td>

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$x = \pm\tan^{-1}(\sqrt{3})$

x = 60^0 or 240^0 or 120^0 or 300^0

x $= \frac{\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}$
</td></tr>
</table>

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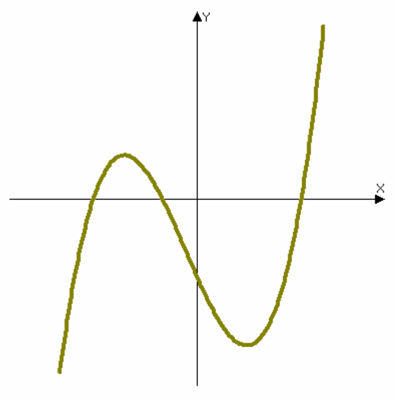
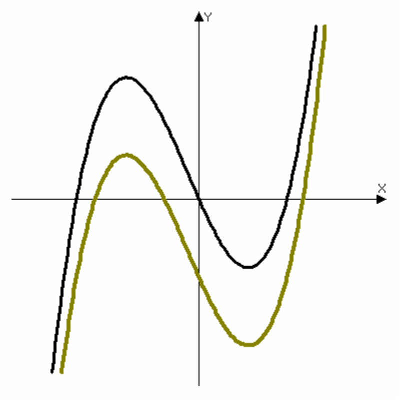
<table>
<tr><td>4.</td><td>For -g(x): (a, -2) → (a , 2) (0,1) → (0 , -1) (b,3) → (b , -3)
 </td></tr>
<tr><td>5.
(a)</td><td>$\overrightarrow{AB} = \begin{pmatrix}-1\\8\\3\end{pmatrix} - \begin{pmatrix}-3\\4\\7\end{pmatrix} = \begin{pmatrix}2\\4\\-4\end{pmatrix} = 2\overrightarrow{BC}$ $\overrightarrow{BC} = \begin{pmatrix}0\\10\\1\end{pmatrix} - \begin{pmatrix}-1\\8\\3\end{pmatrix} = \begin{pmatrix}1\\2\\-2\end{pmatrix}$
As AB = 2BC , AB and BC are parallel and as B is a common point A,B and C must be collinear.</td></tr>
<tr><td>(b)</td><td>$\overrightarrow{AD} = 4\overrightarrow{AB}$
$d - a = 4\overrightarrow{AB}$
$d = 4\overrightarrow{AB} + a$
$d = 4\begin{pmatrix}2\\4\\-4\end{pmatrix} + \begin{pmatrix}-3\\4\\7\end{pmatrix} = \begin{pmatrix}5\\20\\-9\end{pmatrix}$
So D is the point (5, 20, -9)</td></tr>
<tr><td>6.</td><td>$y = 3\sin x + \cos 2x$
$\frac{dy}{dx} = 3\cos x - 2\sin 2x$</td></tr>
</table>

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7.	$\int_0^2 (4x+1)^{\frac{1}{2}} dx$ $= \left[\frac{(4x+1)^{\frac{3}{2}}}{4\left(\frac{3}{2}\right)} \right]$ $= \left[\frac{(4x+1)^{\frac{3}{2}}}{6} \right]_0^2$ $= \left[\frac{(9)^{\frac{3}{2}}}{6} \right] - \left[\frac{(1)^{\frac{3}{2}}}{6} \right]$ $= \frac{27}{6} - \frac{1}{6} = \frac{26}{6} = \frac{13}{3}$
8. **(a)** **(b)**	$x^2 - 10x + 27$ $=(x - 5)^2 + 27 - 25$ **$=(x - 5)^2 + 2$** $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ $g^1(x) = x^2 - 10x + 27 = (x-5)^2 + 2$ For function to be ever increasing $g^1(x)$ must always be positive. In this case that occurs as $(x-5)^2$ must be positive and so $g^1(x)$ can never be less than 2. Thus the function is ever increasing.
9.	$\log_2(x+1) - 2\log_2(3) = 3$ $\log_2(x+1) - \log_2(3)^2 = 3$ $\log_2\left(\frac{x+1}{9}\right) = 3$ $\frac{x+1}{9} = 2^3$ $x+1 = 72$ $x = 71$

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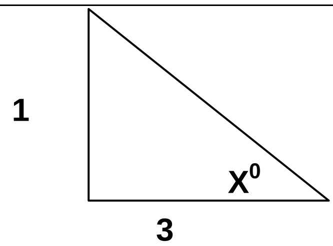
10.



$$\sin x = \frac{1}{\sqrt{10}}$$

$$\cos x = \frac{3}{\sqrt{10}}$$

DEA = 2x + 90^0

$$\begin{aligned}
&\cos(2x+90)\\
&= \cos 2x \bullet \cos 90 - \sin 2x \bullet \sin 90\\
&= 0 - \sin 2x\\
&= 0 - 2\sin x \cos x\\
&= 0 - 2 \bullet \frac{1}{\sqrt{10}} \bullet \frac{3}{\sqrt{10}}\\
&= 0 - \frac{6}{10}\\
&= -\frac{3}{5}
\end{aligned}$$

11.

$y = ax^2 - abx$

For (2 , 0) : $0 = 4a - 2ab$ For (1 , -6) : $-6 = a - ab$

Simultaneous equations so; ab = a + 6 and 0 = 4a -2(a + 6)
0 = 4a – 2a -12
0 = 2a -12
a = 6 and so b =2

Equation is y = 6x (x – 2)

(b)

$$\begin{aligned}
&f(x)^1 = 6x^2 - 12x\\
&f(x) = 2x^3 - 6x^2 + c\\
&4 = 2 - 6 + c\\
&c = 8\\
&f(x) = 2x^3 - 6x^2 + 8
\end{aligned}$$

2004 - Higher - Paper 2

1. (a)

$$x - 2y = 0$$
$$y = \tfrac{1}{2}x$$
$$m = \tfrac{1}{2}$$

→

$$m = \tan\theta$$
$$\tfrac{1}{2} = \tan a$$
$$a = \tan^{-1}(\tfrac{1}{2})$$
$$a = 26\cdot6^0$$

(b)

$$m_{OB} = \tan(56\cdot6)^0$$
$$m_{OB} = 1\cdot5$$

→

$$\theta = 30 + 26\cdot6$$
$$\theta = 56\cdot6$$

2. (a)

$$\overrightarrow{QP} = \begin{pmatrix}1\\3\\-1\end{pmatrix} - \begin{pmatrix}2\\0\\-1\end{pmatrix} = \begin{pmatrix}-1\\3\\-2\end{pmatrix}$$

$$\overrightarrow{QR} = \begin{pmatrix}-3\\1\\2\end{pmatrix} - \begin{pmatrix}2\\0\\1\end{pmatrix} = \begin{pmatrix}-5\\1\\1\end{pmatrix}$$

(b)

$$\cos PQR = \frac{QP.QR}{|QP||QR|}$$
$$\cos PQR = \frac{6}{\sqrt{14}\times\sqrt{27}}$$
$$PQR = \cos^{-1}(0.3086.....)$$
$$PQR = 72.0^0$$

$$|QP| = \sqrt{-1^2 + 3^2 + -2^2} = \sqrt{14}$$
$$|QR| = \sqrt{-5^2 + 1^2 + 1^2} = \sqrt{27}$$
$$QR.QP = (-1\times-5) + (3\times1) + (-2\times1)$$
$$QR.QP = 5 + 3 - 2$$
$$QR.QP = 6$$

3. $2x^2 + px - 3 = 0$

$b^2 - 4ac = p^2 - 4 \times 2 \times (-3)$
$= p^2 + 24$

As p^2 is positive for all values of p, the discriminant is always positive.
Thus roots are real for all values of p.

4. (a) For limit $-1 < k < 1$

(b)

$$L = kL + 3$$
$$5 = 5k + 3$$
$$5k = 2$$
$$k = \tfrac{2}{5}$$

2004 - Higher - Paper 2

5.
(a)

$y = 6x^2 - x^3$

$y^1 = 12x - 3x^2$

$12 = 12x - 3x^2$ $\longrightarrow$

$-12 + 12x - 3x^2 = 0$

$3(-4 + 4x - x^2) = 0$

$3(2 - x)(x - 2) = 0$

$x = 2$

(b) **If x = 2 at P then y = 6(2)^2 – (2)^3 = 24 - 8 = 16 so P(2 , 16)**

y – b = m(x – a)
y – 16 = 12 (x - 2)
y = 12x – 24 + 16
y = 12x – 8

6.
(a)

$y = 3\cos x + 5\sin x$

$k\cos(x - a)$

$k(\cos x \cos a + \sin x \sin a)$

$k\cos x \cos a + k \sin x \sin a$

$k \sin a = 5$

$k \cos a = 3$ $\longrightarrow$

$\tan a = \frac{5}{3}$

$a = \tan^{-1}(\frac{5}{3}) = 59.0^0$

$k = \sqrt{5^2 + 3^2} = \sqrt{34}$

$$\boxed{3\cos x + 5\sin x = \sqrt{34}\cos(x - 59)^0}$$

(b)

$\sqrt{34}\cos(x - 59)^0 = 4$

$\cos(x - 59)^0 = \frac{4}{\sqrt{34}}$

$x - 59 = \cos^{-1}\left(\frac{5}{\sqrt{34}}\right)$

$x - 59 = 46.7^0 OR 313.3^0$

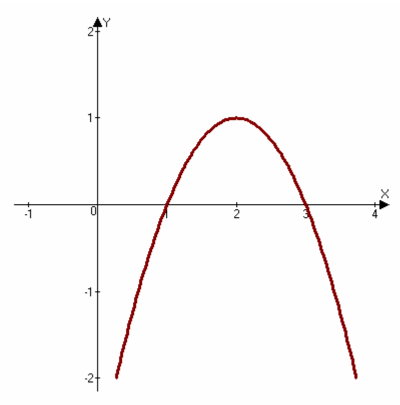
$x = 46.7 + 59 = 105.7^0$

$x = 313.3 + 59 = 372.3^0 = 12.3^0$

As 105.7^0 is out with domain, x = 12.3^0

7.

	1-	1	1+	3	3+
f'(x)	-ve	0	+ve	0	-ve



8. (a)

A (6 , 1)   P (5 , -1)

$$m_{AP} = \frac{-1-1}{5-6} = \frac{-2}{-1} = 2$$

Gradient of tangent = -1/2

Equation of Tangent at P:

$$y-b=m(x-a)$$
$$y+1=-\tfrac{1}{2}(x-5)$$
$$2y+2=-x+5$$
$$2y+x=5-2$$
$$2y+x=3$$

(b)

B(-5 , -1)   x + 2y = 3 so x = 3 – 2y

Subbing into circle's equation:

$$x^2+y^2+10x+2y+6=0$$
$$(3-2y)^2+y^2+10(3-2y)+2y+6=0$$
$$9-12y+4y^2+y^2+30-20y+2y+6=0$$
$$5y^2-30y+45=0$$
$$5(y^2-6y+9)=0$$
$$(y-3)(y-3)=0$$
$$y=3$$

This repeated root proves that there is only one point of contact and so line must be a tangent.

(c) P(5 , -1) and Q (3, -3)

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$PQ = \sqrt{8^2 + 4^2}$$
$$PQ = \sqrt{80} = 4\sqrt{5}$$

9. Surface Area = $2x^2$ + (2 x 2Xh) + (2 x X x h)
= $2x^2$ + 4Xh + 2Xh
= $2x^2$ + 6Xh

$$12 = 2x^2 + 6xh$$
$$12 = 2x(x + 3h)$$

SO $$\frac{6}{x} = x + 3h$$

$$3h = \frac{6}{x} - x$$
$$h = \frac{2}{x} - \frac{x}{3} = \frac{6 - x^2}{3x}$$

$$V(x) = \frac{6 - x^2}{3x} \times 2x^2$$
$$V(x) = \frac{2x(6 - x^2)}{3}$$

(b)

$$V(x) = 4x - \tfrac{2}{3}x^3$$
$$V'(x) = 4 - 2x^2$$
$$0 = 4 - 2x^2$$ ← **Maximum occurs when V'(x) = 0**
$$2x^2 = 4$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

As – ve square root impossible for length, length is sqr root 2.

10. (a)

$$A_t = A_o \exp-(0.002t)$$
$$600 = A_0 \exp(-0.002 \times 1000)$$
$$A_0 = \frac{600}{\exp(-2)}$$
$$A_0 = 4433.4mg$$

(b) $0.5 = 1 \times \exp(-0.002t)$

$-0.002t = \ln(0.5)$

$t = \dfrac{\ln(0.5)}{-0.002}$

$t = 346.6\,years$

11. $\int_1^3 (2x - \frac{1}{2}x^2)dx$

$\left[x^2 - \frac{1}{6}x^3 \right]_1^3$

$\left[3^2 - \dfrac{3^3}{6} \right] - \left[1^2 - \dfrac{1^3}{6} \right]$

$9 - \frac{27}{6} - \frac{5}{6} = \frac{54}{6} - \frac{32}{6}$

$\dfrac{22}{6} = 3\frac{2}{3}$

Now must deduct rectangle which is under section of graph but not part of glass area.

Rectangle = (3 - 1) x 1.5 = 3

So area of glass = 2/3 m^2