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Heinemann

Higher Maths Text Book

Worked Solutions

Ex 13 P
Component form of a scalar product

The Component Form of the Scalar Product

The scalar product can also be calculated as follows:

$$\underline{\boldsymbol{a}}.\underline{\boldsymbol{b}} = a_1b_1 + a_2b_2 + a_3b_3 \quad \text{where } \underline{\boldsymbol{a}} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \underline{\boldsymbol{b}} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.$$

This is given in the exam.

Ex 13P Questions 1a-f, 2a-c, 3a, 4a-c

Worked solutions courtesy of Mr R Milton

1 a $a=\begin{pmatrix}4\\3\\6\end{pmatrix}$ $b=\begin{pmatrix}7\\3\\2\end{pmatrix}$

$a.b = a_1b_1 + a_2b_2 + a_3b_3$

$a=\begin{pmatrix}a1\\a2\\a3\end{pmatrix}$ $b=\begin{pmatrix}b1\\b2\\b3\end{pmatrix}$

$a.b = 4\times7 + 3\times3 + 6\times2$

$= 28 + 9 + 12$

$= \underline{49}$ ✓

b $p.q = p^1q_1 + p^2q^2 + p^3q^3$

$= 5\times6 + 5\times2 + 6\times-3$

$= 30 + 10 - 18$

$= \underline{22}$ ✓

(c) $\vec{AB} \cdot \vec{AC} = 1 \times 8 + (-4 \times 9) + (-2 \times -5)$

$= 8 - 36 + 10$

$= 18 - 36$

$= \underline{-18}$ ✓

(d) $m \cdot n = 2 \times 4 + 5 \times 2 + 7 \times 2$

$= 8 + 10 + 14$

$= \underline{32}$ ✓

(e) $f \cdot g = 4 \times 8 + 2 \times -6 + -3 \times 3$

$= 32 - 12 - 9$

$= 32 - 21$

$= \underline{11}$ ✓

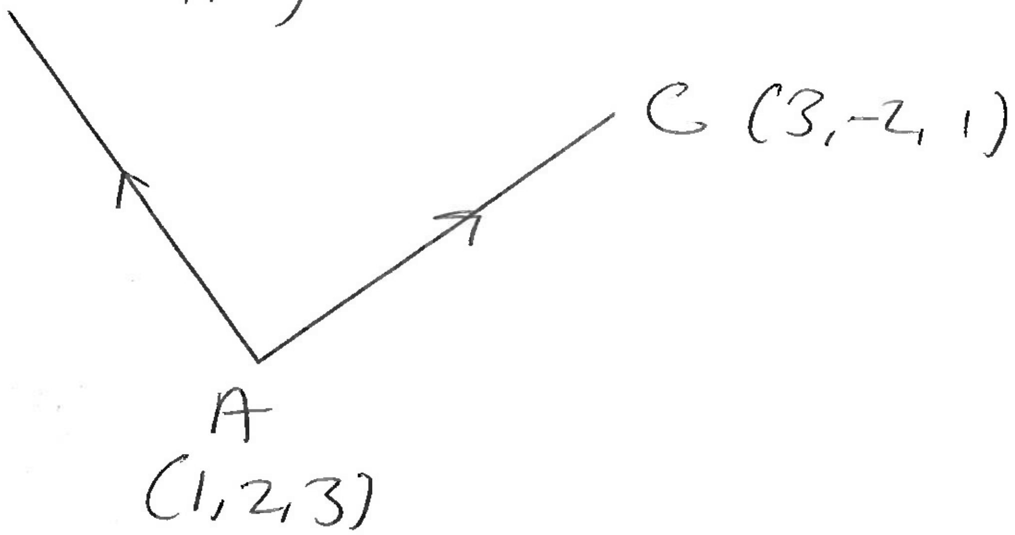
(f) $s \cdot t = 1 \times 1 + (1 \times -1) + (-1 \times -1)$

$= 1 - 1 + 1$

$= 2 - 1$

$= \underline{1}$ ✓

② B (2,4,6)



ⓐ (i) $\vec{AB} = b - a = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \underline{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}}$ ✓

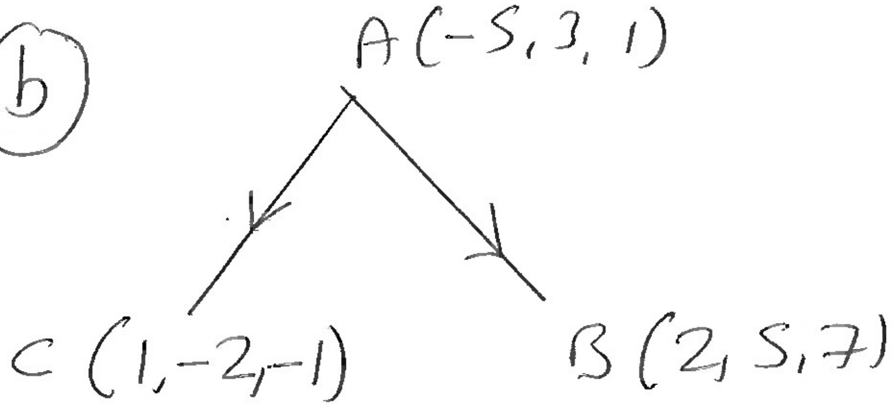
$\vec{AC} = c - a = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \underline{\begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}}$ ✓

(ii) $\vec{AB} \cdot \vec{AC} = (1 \times 2) + (2 \times -4) + (3 \times -2)$

$= 2 - 8 - 6$

$= \underline{-12}$ ✓

(b)



(i) $\overrightarrow{AB} = b - a = \begin{pmatrix} 2 \\ 5 \\ 7 \end{pmatrix} - \begin{pmatrix} -5 \\ 3 \\ 1 \end{pmatrix} = \underline{\begin{pmatrix} 7 \\ 2 \\ 6 \end{pmatrix}}$ ✓

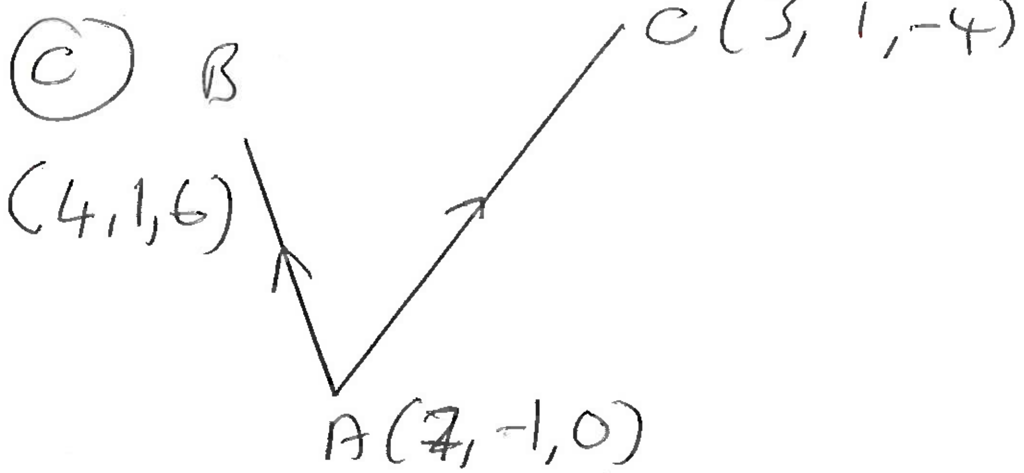
$\overrightarrow{AC} = c - a \quad \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} - \begin{pmatrix} -5 \\ 3 \\ 1 \end{pmatrix} = \underline{\begin{pmatrix} 6 \\ -5 \\ -2 \end{pmatrix}}$ ✓

(ii) $\overrightarrow{AB} \cdot \overrightarrow{AC} = 7 \times 6 + (2 \times -5) + (6 \times -2)$

$= 42 - 10 - 12$

$= \underline{20}$ ✓

(c)



(i) $\vec{AB} = b - a = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix} - \begin{pmatrix} 7 \\ -1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ 2 \\ 6 \end{pmatrix}$ ✓

$\vec{AC} = c - a = \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix} - \begin{pmatrix} 7 \\ -1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} -4 \\ 2 \\ -4 \end{pmatrix}$ ✓

(ii) $\vec{AB} \cdot \vec{AC} = (-3 \times -4) + (2 \times 2) + (6 \times -4)$

$= 12 + 4 - 24$

$= 16 - 24$

$= -8$ ✓

(3) $P(2, 5, -4)$

$Q(3, 4, 3)$

$R(-4, 5, 1)$

(a) $\overrightarrow{PQ} \cdot \overrightarrow{PR}$

$$\overrightarrow{PQ} = q - p = \begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 7 \end{pmatrix}$$

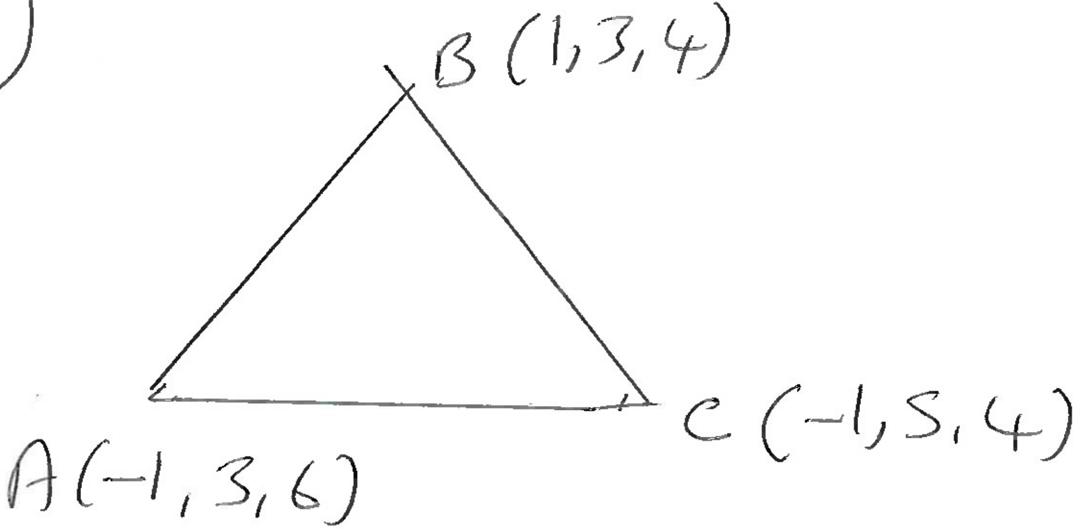
$$\overrightarrow{PR} = r - p = \begin{pmatrix} -4 \\ 5 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \\ 5 \end{pmatrix}$$

$$\overrightarrow{PQ} \cdot \overrightarrow{PR} = (1 \times -6) + (-1 \times 0) + (7 \times 5)$$

$$= -6 + 0 + 35$$

$$= 29$$ ✓

4



(a) (i) $\overrightarrow{AB} = b - a = \begin{pmatrix}1\\3\\4\end{pmatrix} - \begin{pmatrix}-1\\3\\6\end{pmatrix} = \underline{\begin{pmatrix}2\\0\\-2\end{pmatrix}}$ ✓

(ii) $\overrightarrow{BC} = c - b = \begin{pmatrix}-1\\5\\4\end{pmatrix} - \begin{pmatrix}1\\3\\4\end{pmatrix} = \underline{\begin{pmatrix}-2\\2\\0\end{pmatrix}}$ ✓

(iii) $\overrightarrow{AC} = c - a = \begin{pmatrix}-1\\5\\4\end{pmatrix} - \begin{pmatrix}-1\\3\\6\end{pmatrix} = \underline{\begin{pmatrix}0\\2\\-2\end{pmatrix}}$ ✓

(b) $\overrightarrow{AB} \cdot \overrightarrow{AC} = (2 \times 0) + (0 \times 2) + (-2 \times -2)$

$= 0 + 0 + 4$

$= \underline{4}$ ✓

$\overrightarrow{BC} \cdot \overrightarrow{BA} = (-2 \times -2) + (2 \times 0) + (0 \times 2)$

$= 4 + 0 + 0$

$= \underline{4}$ ✓

NOTE

$\overrightarrow{BA} = -\overrightarrow{AB}$

$= \begin{pmatrix}-2\\0\\2\end{pmatrix}$

(c) BECAUSE 3 SIDES AND 3 ANGLES ARE EQUAL