

Scalar Product

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

| $f(x)$ | $f'(x)$ |
|-----------|--------------|
| $\sin ax$ | $a \cos ax$ |
| $\cos ax$ | $-a \sin ax$ |

Table of standard integrals:

| $f(x)$ | $\int f(x) dx$ |
|-----------|----------------------------|
| $\sin ax$ | $-\frac{1}{a} \cos ax + c$ |
| $\cos ax$ | $\frac{1}{a} \sin ax + c$ |

Essential Skills 21

The skills in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed



Scalar Product (Non-Calculator)

Calculate the scalar product in each:

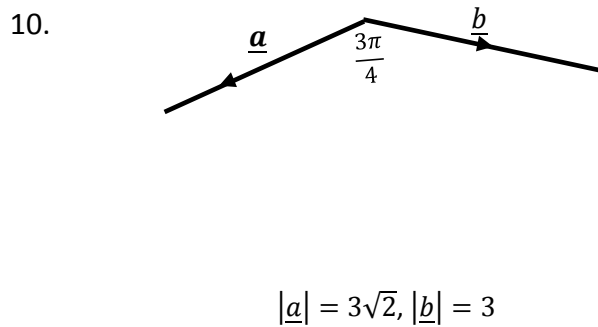
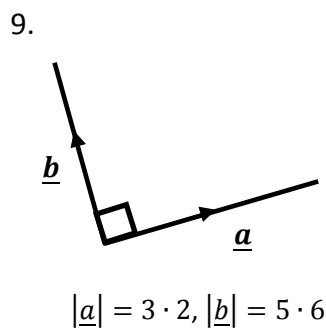
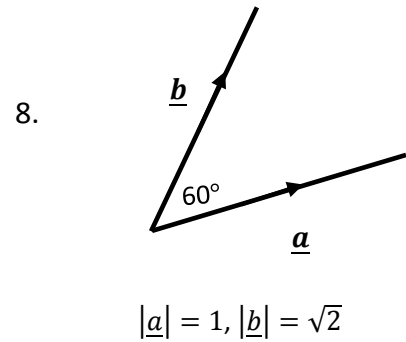
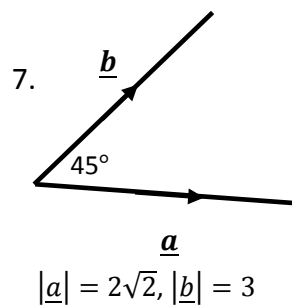
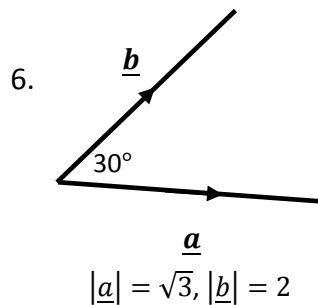
1. $\underline{a} = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}, \underline{b} = \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$

2. $\overrightarrow{FG} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \overrightarrow{FE} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$

3. $\underline{u} = 4\underline{i} - 2\underline{j} - \underline{k}, \underline{v} = 6\underline{i} + 2\underline{j} + 3\underline{k}$

4. $\underline{u} = 3\underline{j} - \underline{k}, \underline{v} = \underline{i} + \underline{j} - \underline{k}$

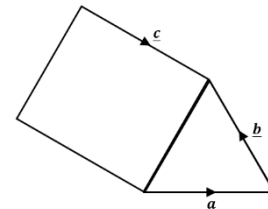
5. A(3, 2, 0), B(4, 0, 5) & C(1, -2, 6) Find $\overrightarrow{AB} \cdot \overrightarrow{AC}$



APPLYING QUESTIONS

1. The diagram shows an equilateral triangle of side 3 with an adjoining rectangle of length 5 units.

Calculate $\underline{a} \cdot (\underline{a} + \underline{b} + \underline{c})$



2. Vectors $\underline{u} = 4\underline{i} + 5\underline{j} - 3\underline{k}$ & $\underline{v} = \underline{i} + 7\underline{j} + m\underline{k}$ are perpendicular.

Find the value of m .



Essential Skills 21 - Answers

| | |
|----|---|
| 1 | -7 |
| 2 | -2 |
| 3 | 17 |
| 4 | 4 |
| 5 | 36 |
| 6 | 3 |
| 7 | 6 |
| 8 | $\frac{\sqrt{2}}{2}$ |
| 9 | 0 |
| 10 | -9 |
| AQ | (1) $\frac{9 + 15\sqrt{3}}{2}$ (2) $m = 13$ |

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