

Differential Equations

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product:

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$

Essential Skills 25

The skills in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed



Differential Equations

Find the equations of the curves (y or $f(x)$) that satisfy each of the following conditions:

1. $\frac{dy}{dx} = 6x + 5$, passing (2, 21)
2. $\frac{dy}{dx} = 4x - 4$, passing (-1, 6)
3. $f'(x) = x^2$, where $f(3) = 13$
4. $f'(x) = 3x^2 - 6$, where $f(-1) = 8$
5. $\frac{dy}{dx} = 6x^2 + 8x + 5$, passing (-2, -12)
6. $f'(x) = 2(2 - 3x)$, where $f(1) = 1$
7. $\frac{dy}{dx} = \frac{9}{2}x^2 - 6x$, passing (2, 3)
8. $\frac{dy}{dx} = \frac{4}{x^3}$, passing (1, 1)
9. $\frac{dy}{dx} = 9(3x - 5)^2 + 5$, passing (2, 6)
10. $f'(x) = 6\cos 2x$, where $f\left(\frac{\pi}{12}\right) = \frac{5}{2}$



APPLYING QUESTIONS

1. The gradient of a tangent to a curve at each point (x, y) is given by $\frac{dy}{dx} = 3x(2x - 1)$.

If the curve passes through the point $(-1, 10)$, find its equation.

2. The velocity of an object is given by $\frac{ds}{dt} = 9\sqrt{t} - 12$, where s is the distance in metres and t is the time in seconds.

Find an expression for the displacement s , given that when $t = 0, s = 2$.

Essential Skills 25 - Answers

1	$y = 3x^2 + 5x - 1$
2	$y = 2x^2 - 4x$
3	$f(x) = \frac{1}{3}x^3 + 4$
4	$f(x) = x^3 - 6x + 3$
5	$y = 2x^3 + 4x^2 + 5x - 2$
6	$f(x) = 4x - 3x^2$
7	$y = \frac{3}{2}x^3 - 3x^2 + 3$
8	$y = 3 - \frac{2}{x^2}$
9	$y = (3x - 5)^3 + 5x - 5$
10	$f(x) = 3\sin 2x + 1$
AQ	(1) $y = 2x^3 - \frac{3}{2}x^2 + \frac{27}{2}$ (2) $s = 6\sqrt{t^3} - 12t + 2$

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