

## Angle between Vectors

### FORMULAE LIST

#### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x-a)^2 + (y-b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

#### Scalar Product:

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

#### Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

#### Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

#### Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$

## Essential Skills 30

The skills in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

### Angle between Vectors

Calculate the angle between each pair of vectors:



1.  $\underline{u} = \underline{i} + 2\underline{j} + 3\underline{k}$  and  $\underline{v} = 4\underline{i} + \underline{j} + 5\underline{k}$
2.  $\underline{u} = 4\underline{i} + 2\underline{k}$  and  $\underline{v} = -2\underline{i} + 2\underline{j} + 2\underline{k}$
3.  $\underline{u} = 3\underline{i} - 2\underline{j} + \underline{k}$  and  $\underline{v} = 5\underline{j} - \underline{k}$
4.  $\underline{u} = \underline{i} + \underline{j} + \underline{k}$  and  $\underline{v} = \underline{i} + 2\underline{j} - 2\underline{k}$
5.  $\underline{u} = 2\underline{i} + 3\underline{j}$  and  $\underline{v} = -\underline{i} + 2\underline{j} + \underline{k}$
6.  $\underline{u} = 3\underline{i} + \underline{k}$  and  $\underline{v} = 2\underline{i} + 2\underline{j} - 7\underline{k}$
7.  $\underline{u} = 5\underline{i} + \underline{j} - \underline{k}$  and  $\underline{v} = 2\underline{i} - \underline{j} + 2\underline{k}$
8.  $\underline{u} = 2\underline{j} - 6\underline{k}$  and  $\underline{v} = 3\underline{i} - 2\underline{j} + 2\underline{k}$
9.  $\underline{u} = 7\underline{i} + \underline{j} - \underline{k}$  and  $\underline{v} = 2\underline{i} - \underline{j} + 3\underline{k}$
10.  $\underline{u} = \underline{i} - 7\underline{j}$  and  $\underline{v} = \underline{i} + 4\underline{j} + \underline{k}$

### APPLYING QUESTION



- (a) C divides the line joining A (2, 1, -1) and B (8, 4, 8) in the ratio 2:1.  
Find the coordinates of C.
- (b) D has coordinates (9, -3, 11) and C divides  $\overline{DE}$  in the ratio 3:1  
Find the coordinates of E.
- (c) Hence, calculate the size of angle BCE.

## Essential Skills 30 - Answers

1	30°
2	105°
3	125.2°
4	78.9°
5	63.0°
6	92.4°
7	63.3°
8	127.8°
9	68.0°
10	154.2°
AQ	(a) $C(6,3,5)$ (b) $E(5,5,3)$ (c) 122°

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