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Stationary Points

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Essential Skills 5

The skills in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Stationary Points

Find the co-ordinates and determine the nature of the stationary points:

1. $y = x^3 - 3x^2$

2. $f(x) = x^3 - 12x$

3. $f(x) = x^3 + 9x^2 + 24x - 18$

4. $y = 2x^3 - 7x^2 + 4x + 4$

5. $y = 2x^3 - 3x^2 - 36x + 17$

6. $f(x) = x^2(2x - 3)$

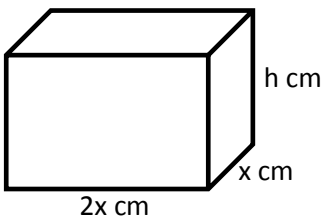
7. $f(x) = x^3 - 2x^2 - 4x + 1$

8. $y = (x - 1)(x - 2)^2$

9. $y = x(27 - x^2)$

10. $f(x) = 2x^2(2 - x^2)$

APPLYING QUESTIONS



1. An open top box measures x cm by $2x$ cm and has a depth of h cm. The outer surface has an area of 216cm².

(a) Show that the volume of the cuboid is given by $V(x) = 72x - \frac{2}{3}x^3$

(b) Find the value of x for which the volume is a maximum and calculate the volume.

2. A function f is defined by $f(x) = x(x^2 - 3)$, where $0 \leq x \leq 3$.

Find the maximum and minimum values of f.

Essential Skills 5 - Answers

1	$maximum\ (0,0), \quad minimum\ (2,-4)$
2	$maximum\ (-2,16), \quad minimum\ (2,-16)$
3	$maximum\ (-4,-34), \quad minimum\ (-2,-38)$
4	$maximum\ \left(\frac{1}{3},\frac{125}{27}\right), \quad minimum\ (2,0)$
5	$maximum\ (-2,61), \quad minimum\ (3,-64)$
6	$maximum\ (0,0), \quad minimum\ (1,-1)$
7	$maximum\ \left(-\frac{2}{3},\frac{67}{27}\right), \quad minimum\ (2,-7)$
8	$maximum\ \left(\frac{4}{3},\frac{4}{27}\right), \quad minimum\ (2,0)$
9	$maximum\ (-3,-54), \quad minimum\ (3,54)$
10	$maximum\ (-1,2)\ \&\ (1,2), \quad minimum\ (0,0)$
AQ	$(1)(a) proof\ (b) x=6: V=288\ cm^3\ \ (2)\ max\ 18\ @\ x=3, \min -2\ @\ x=1$

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