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Completing the Square

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Essential Skills 7

The skills in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Completing the Square

Write in the form $a(x+b)^2+c$:

1. $3x^2+6x+1$
2. $2x^2+12x-3$
3. $5x^2-10x-7$
4. $3x^2-18x+4$
5. $4x^2+24x+3$
6. $2x^2-20x-5$
7. $3-8x-x^2$
8. $5+16x-8x^2$
9. $2x^2-8x-2$
10. $3x^2+9x+1$

APPLYING QUESTION

(a) Write $2x^2-20x+54$ in the form $a(x+b)^2+c$

(b) Hence show that $y=\frac{2}{3}x^3-10x^2+54x-4$ is always increasing.

Essential Skills 7 - Answers

1	$3(x+1)^2-2$
2	$2(x+3)^2-21$
3	$5(x-1)^2-12$
4	$3(x-3)^2-23$
5	$4(x+3)^2-33$
6	$2(x-5)^2-55$
7	$19-(x+4)^2$
8	$13-8(x-1)^2$
9	$2(x-2)^2-10$
10	$3(x+\frac{3}{2})^2-\frac{23}{4}$
AQ	$(a)\ 2(x-5)^2+4 \qquad (b)\ \frac{dy}{dx}>0\ for\ all\ x, always\ increasing$

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