Polynomials

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2\sin A \cos A$$

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A\end{aligned}$$

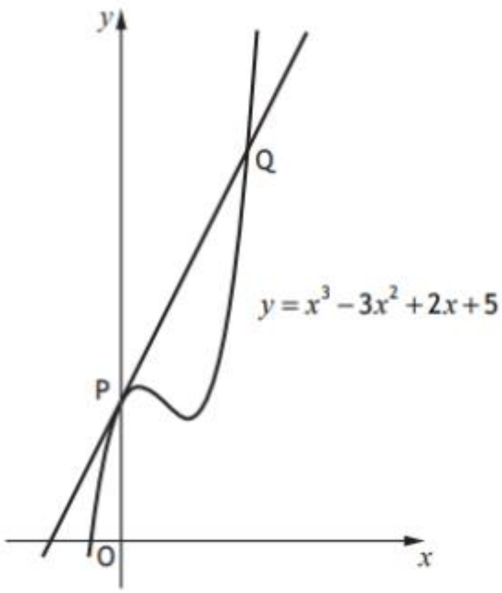
Table of standard derivatives:

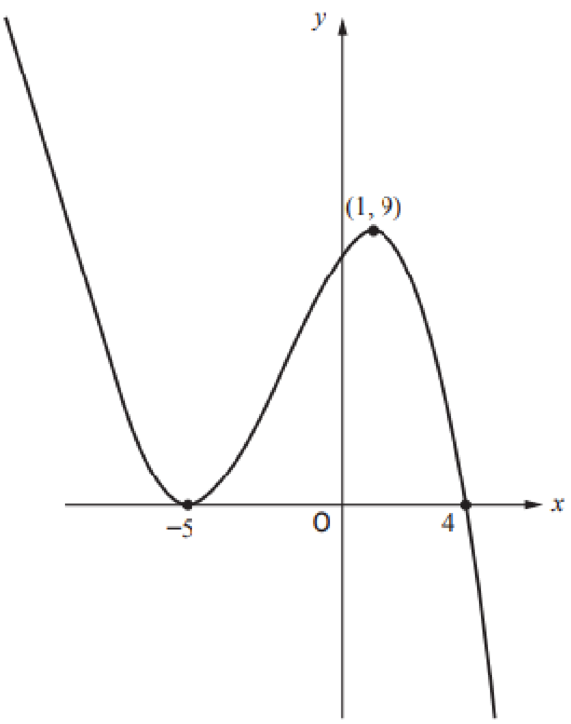
$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Polynomials

2019 P2 Q10	(a) Show that $(x+3)$ is a factor of $3x^4+10x^3+x^2-8x-6$. (b) Hence, or otherwise, factorise $3x^4+10x^3+x^2-8x-6$ fully.	2 5
2018 P1 Q7	The curve with equation $y=x^3-3x^2+2x+5$ is shown on the diagram.  (a) Write down the coordinates of P, the point where the curve crosses the y-axis . (b) Determine the equation of the tangent to the curve at P. (c) Find the coordinates of Q, the point where this tangent meets the curve again.	1 2 3
2018 P1 Q7(a)	(a) (i) Show that $(x-2)$ is a factor of $2x^3-3x^2-3x+2$. (ii) Hence, factorise $2x^3-3x^2-3x+2$ fully.	2 2
2018 P1 Q15	A cubic function, f, is defined on the set of real numbers. • $(x+4)$ is a factor of $f(x)$ • $x=2$ is a repeated root of $f(x)$ • $f'(-2)=0$ • $f'(x)>0$ where the graph with equation $y=f(x)$ crosses the y-axis Sketch a possible graph of $y=f(x)$ on the diagram in your answer booklet.	4

2017 P2 Q2	(a) Show that $(x-1)$ is a factor of $f(x)=2x^3-5x^2+x+2$. (b) Hence, or otherwise, solve $f(x)=0$.	2 3
2016 P1 Q15	The diagram below shows the graph with equation $y=f(x)$, where $f(x)=k(x-a)(x-b)^2$.  (a) Find the values of a, b and k.	3
2016 P2 Q3	(a) (i) Show that $(x+1)$ is a factor of $2x^3-9x^2+3x+14$. (ii) Hence solve the equation $2x^3-9x^2+3x+14=0$.	2 3
2015 P1 Q3	Show that $(x+3)$ is a factor of $x^3-3x^2-10x+24$ and hence factorise $x^3-3x^2-10x+24$ fully.	4

2014 P1 Q22	For the polynomial $6x^3 + 7x^2 + ax + b$, • $x + 1$ is a factor • 72 is the remainder when it is divided by $x - 2$. (*a*) Determine the values of a and b. **4** (*b*) Hence factorise the polynomial completely. **3**	
2013 P2 Q3a	(*a*) Given that $(x - 1)$ is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic fully.	4
2012 P1 Q21	(*a*) (i) Show that $(x - 4)$ is a factor of $x^3 - 5x^2 + 2x + 8$. (ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully. (iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$.	6
2011 P2 Q2c	(*c*) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$. (ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully.	5
2010 P1 Q22	(*a*) (i) Show that $(x - 1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$. (ii) Hence factorise $f(x)$ fully. (*b*) Solve $2x^3 + x^2 - 8x + 5 = 0$. (*c*) The line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point G. Find the coordinates of G. (*d*) This tangent meets the curve again at the point H. Write down the coordinates of H.	5 1 5 1
2009 P2 Q3	(*a*) (i) Show that $x = 1$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$. (ii) Hence factorise $x^3 + 8x^2 + 11x - 20$ fully. (*b*) Solve $\log_2(x + 3) + \log_2(x^2 + 5x - 4) = 3$.	4 5

2008 P1

21. A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

(*a*) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. 6

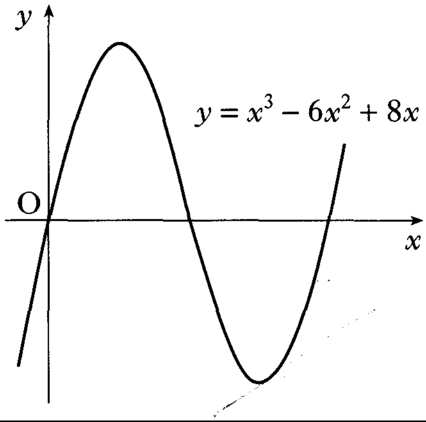
(*b*) (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$.

(ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. 5

(*c*) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve. 4

2008 P1

22. The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.

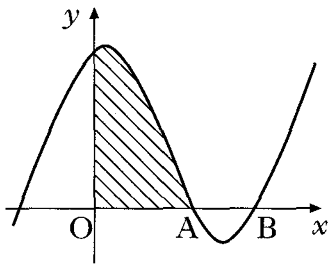


(*a*) Find the coordinates of the points on the curve where the gradient of the tangent is -1. 5

(*b*) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A. 2

2007 P1

8. The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$.



(*a*) Show that the graph cuts the x-axis at (3, 0). 1

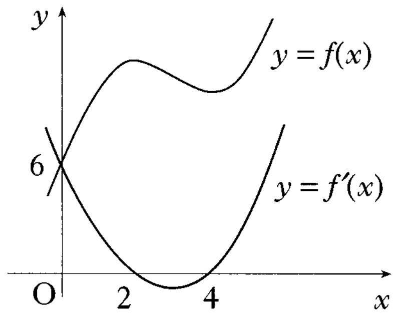
(*b*) Hence or otherwise find the coordinates of A. 3

2007 P2

10. The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.

Both graphs pass through the point (0, 6).

The graph of $y = f'(x)$ also passes through the points (2, 0) and (4, 0).



(*a*) Given that $f'(x)$ is of the form $k(x - a)(x - b)$:

(i) write down the values of a and b;

(ii) find the value of k. 3

2005 P1

8. A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.

(*a*) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully. **5**

(*b*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. **2**

(*c*) Find the greatest and least values of f in the interval $-2 \leq x \leq 2$. **5**

2005 P2

11. (*a*) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$. **1**

(*b*) Hence find the range of values of p for which all the roots of the cubic equation are real. **7**

2004 P1

2. $f(x) = x^3 - x^2 - 5x - 3$.

(*a*) (i) Show that $(x + 1)$ is a factor of $f(x)$.

(ii) Hence or otherwise factorise $f(x)$ fully. **5**

(*b*) One of the turning points of the graph of $y = f(x)$ lies on the x-axis. **1**

Write down the coordinates of this turning point.

2003 P2

1. $f(x) = 6x^3 - 5x^2 - 17x + 6$.

(*a*) Show that $(x - 2)$ is a factor of $f(x)$.

(*b*) Express $f(x)$ in its fully factorised form. **4**

2002W P1

5. Given that $(x - 2)$ and $(x + 3)$ are factors of $f(x)$ where $f(x) = 3x^3 + 2x^2 + cx + d$, find the values of c and d. **5**

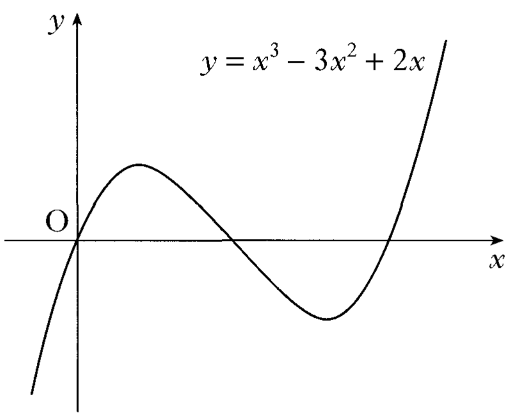
2001 P2

1. (*a*) Given that $x + 2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of k. **3**

(*b*) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when k takes this value. **2**

2000 P2

1. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.



(*a*) Find the equation of the tangent to this curve at the point where $x = 1$. **5**

(*b*) The tangent at the point (2, 0) has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. **5**

Specimen 2 P1	**1.** Show that $x = 2$ is a root of the equation $y = 2x^3 + x^2 - 13x + 6 = 0$ and hence, or otherwise, find the other roots.	4
Specimen 1 P1	**3.** (*a*) Show that $(x - 1)$ is a factor of $f(x) = x^3 - 6x^2 + 9x - 4$ and find the other factors. (*b*) Write down the coordinates of the points at which the graph of $y = f(x)$ meets the axes.	3 1

Answers

2019 P2 Q10

(a)

•[1] use −3 in synthetic division or in evaluation of quartic	•[1] $\begin{array}{r\|rrrrr} -3 & 3 & 10 & 1 & -8 & -6 \\ \hline & 3 & & & & \end{array}$ or $3\times(-3)^4+10\times(-3)^3+(-3)^2$ $-8\times(-3)-6$
•[2] complete division/evaluation and interpret result	•[2] $\begin{array}{r\|rrrrr} -3 & 3 & 10 & 1 & -8 & -6 \\ & & -9 & -3 & 6 & 6 \\ \hline & 3 & 1 & -2 & -2 & 0 \end{array}$ Remainder $=0\;\therefore (x+3)$ is a factor or $f(-3)=0\;\therefore (x+3)$ is a factor

(b) $(x+3)(x-1)(3x^2+4x+2)$

2018 P1 Q7

(a) $P\ (0,5)$

(b) $y = 2x+5$

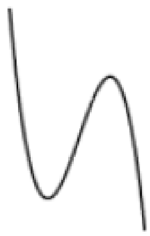
(c) $Q\ (3,11)$

2018 P1 Q7(a)

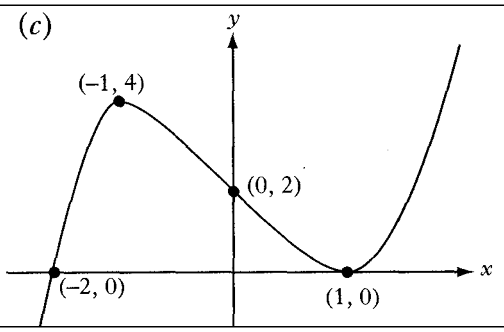
(a) (i) *Use 2 in synthetic division or in cubic evaluation*

(ii) $(x-2)(2x-1)(x+1)$

2018 P1 Q15

•[1] root at $x=-4$ identifiable from graph	•[1]
•[2] stationary point touching x-axis when $x=2$ identifiable from graph	•[2]
•[3] stationary point when $x=-2$ identifiable from graph	•[3]
•[4] identify orientation of the cubic curve and $f'(0)>0$ identifiable from graph	•[4] 

2017 P2 Q2	(a) $\begin{array}{r\|rrrr} 1 & 2 & -5 & 1 & 2 \\ & & 2 & -3 & -2 \\ \hline & 2 & -3 & -2 & 0 \end{array}$ Remainder $= 0$ $\therefore (x-1)$ is a factor (b) $x = -\frac{1}{2}, 1, 2$	
2016 P1 Q15	(a) $a = 4, b = -5, k = -\frac{1}{12}$	
2016 P2 Q3	(a) (i) Use synthetic division or substitution (ii) $x = -1,\ 2,\ 3.5$	
2015 P1 Q3	$\begin{array}{r\|rrrr} -3 & 1 & -3 & -10 & 24 \\ & & -3 & & \\ \hline & 1 & & & \end{array}$ $(x+3)(x-4)(x-2)$	
2014 P1 Q22	(a) $a = -1$ or $b = -2$ (b) $(x+1)(6x^2 + x - 2)$ $(x+1)(3x+2)(2x-1)$	

2013 P2 Q3a	$(x-1)(x^2+4x+5)$ with valid reason $b^2-4ac=16-20<0$, so does not factorise.	
2012 P1 Q21	$(x-4)(x^2-x-2)$ $(x-4)(x-2)(x+1)$ $-1, 2, 4$	
2011 P2 Q2c	(ii) $(x-1)(3x+1)(x+2)$	
2010 P1 Q22	(ii) $(2x+5)(x-1)^2$ (b) $x = 1, -\frac{5}{2}$ (d) $H(-\frac{5}{2}, -8)$	
2009 P2 Q3	(a) $(x-1)(x+4)(x+5)$ (b) $x=1$ *or* $x=-4$ *or* $x=-5$ **Stated explicitly here** $x=1$ *only*	
2008 P1 Q21	(*a*) (–1, 4) maximum (1, 0) minimum (*b*) (i) $x = 1$, $f(x) = 0$ so $(x-1)$ is a factor (ii) $(x-1)(x-1)(x+2)$ (*c*)  	

2008 P1 Q22	(a) (1,3), (3,-3) (b) (1,3)	
2007 P1 Q8	(*a*) To cut the x-axis, $y = 0$. So $0 = x^3 - 4x^2 + x + 6$ $= (x - 3)(x^2 - x - 2)$ $= (x - 3)(x - 2)(x + 1)$ So graph cuts x-axis at $x = -1, 3, 2$. (*b*) (2,0)	
2007 P2 Q10	(*a*) (i) $a = 2,\ b = 4$ (ii) $k = \frac{3}{4}$	
2005 P1 Q8	(*a*)$(x - 3)(2x - 3)(x + 1)$ (*b*)$(-1,0)$, $(\frac{3}{2}, 0)$, $(3, 0)$ (*c*)greatest value = 9 least value = −35	
2005 P2 Q11	(*a*)$f(-1) = -1 + p - p + 1 = 0$ (*b*)$p \leq -1,\ p \geq 3$	
2004 P1 Q2	$(x + 1)(x + 1)(x - 3)$ $(-1, 0)$	
2003 P2 Q1	(*b*) $(x - 2)(2x + 3)(3x - 1)$	
2002W P1 Q5	$c = -19,\ d = 6$	

2001 P2 Q1	(a) $k = -5$ (b) $x = -2, \frac{1}{2}, 1$	
2000 P2 Q1	(a) $x + y = 1$ (b) $(-1, -6)$	
Spec 2 P1 Q1	$\begin{array}{c\|cccc} 2 & 2 & 1 & -13 & 6 \\ & & 4 & 10 & -6 \\ \hline & 2 & 5 & -3 & 0 \end{array}$ remainder = 0 $\Rightarrow x = 2$ is a root $2x^2 + 5x - 3 = 0 \Rightarrow x = \frac{1}{2}, -3$	
Spec 1 P1 Q3	(a) $f(1) = 0, (x - 4), (x - 1)$ (b) $(1,0), (4,0), (0, -4)$	