Straight Lines

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

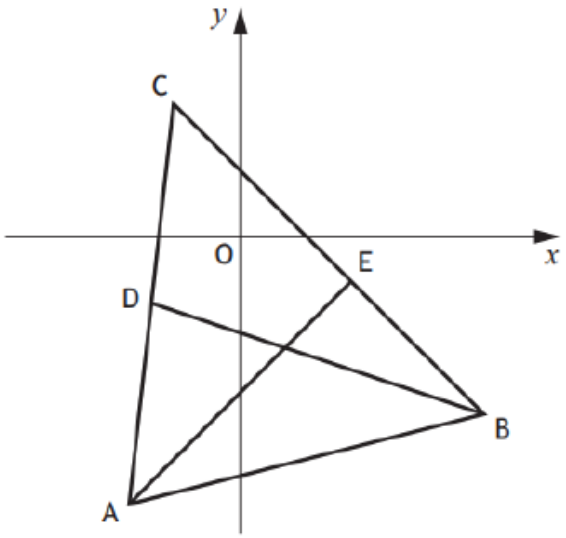
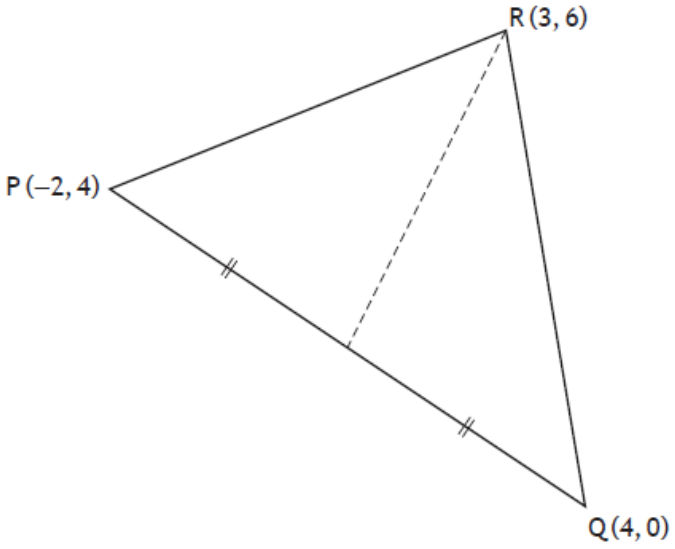
Table of standard derivatives:

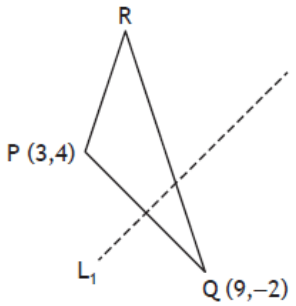
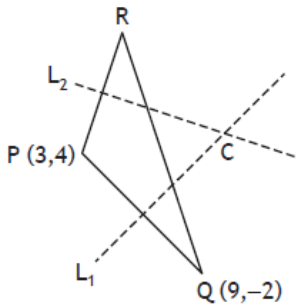
$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

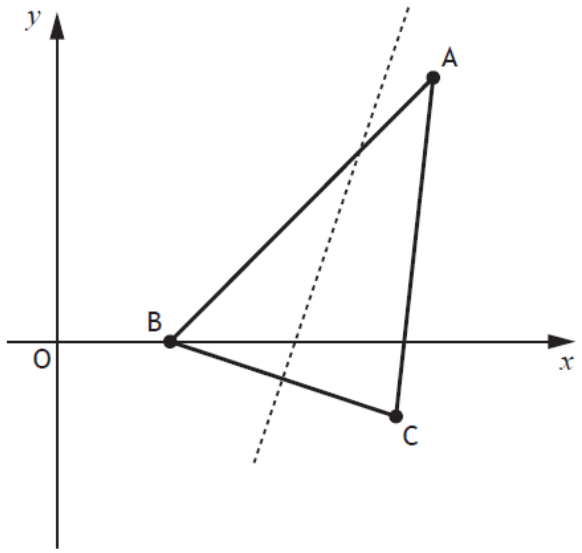
Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Straight Lines

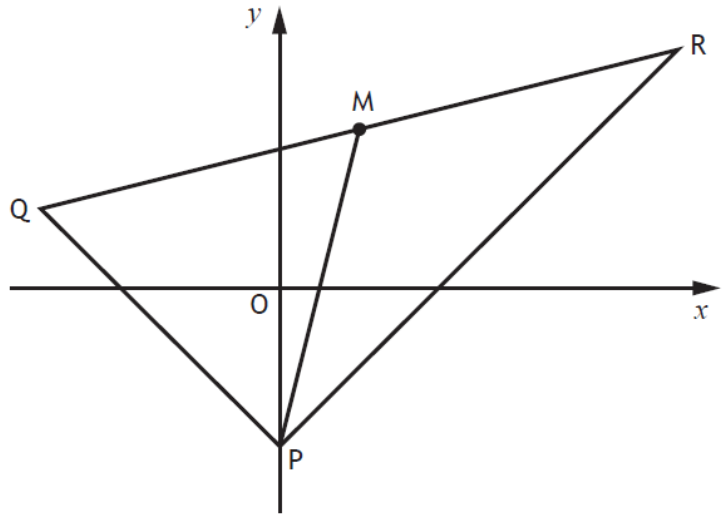
2019 P2 Q1	Triangle ABC has vertices A(–5,–12), B(11,–8) and C(–3,6).  (a) Find the equation of the median BD. (b) Find the equation of the altitude AE. (c) Find the coordinates of the point of intersection of BD and AE.	3 3 2
2019 P1 Q5	The line, L, makes an angle of 30° with the positive direction of the x-axis. Find the equation of the line perpendicular to L, passing through (0,–4).	4
2018 P1 Q1	PQR is a triangle with vertices P (–2, 4), Q (4, 0) and R (3, 6).  Find the equation of the median through R.	3

2018 P1 Q8	A line has equation $y-\sqrt{3}x+5=0$. Determine the angle this line makes with the positive direction of the x-axis.	2
2018 P2 Q5	PQR is a triangle with P(3,4) and Q(9,–2).  (a) Find the equation of L_1, the perpendicular bisector of PQ. The equation of L_2, the perpendicular bisector of PR is $3y+x=25$.  (b) Calculate the coordinates of C, the point of intersection of L_1 and L_2.	3 2
2017 P1 Q7	A (–3, 5), B (7, 9) and C (2, 11) are the vertices of a triangle. Find the equation of the median through C.	3
2017 P1 Q11	A and B are the points (–7, 2) and (5, a). AB is parallel to the line with equation $3y-2x=4$. Determine the value of a.	3

2017 P2 Q1	Triangle ABC is shown in the diagram below. The coordinates of B are (3,0) and the coordinates of C are (9,–2). The broken line is the perpendicular bisector of BC.  (a) Find the equation of the perpendicular bisector of BC. (b) The line AB makes an angle of 45° with the positive direction of the x-axis. Find the equation of AB. (c) Find the coordinates of the point of intersection of AB and the perpendicular bisector of BC.	4 2 2
2016 P1 Q1	Find the equation of the line passing through the point $(-2, 3)$ which is parallel to the line with equation $y + 4x = 7$.	2

2016 P2 Q1

PQR is a triangle with vertices $P(0,-4)$, $Q(-6,2)$ and $R(10,6)$.



(a) (i) State the coordinates of M, the midpoint of QR. 1

(ii) Hence find the equation of PM, the median through P. 2

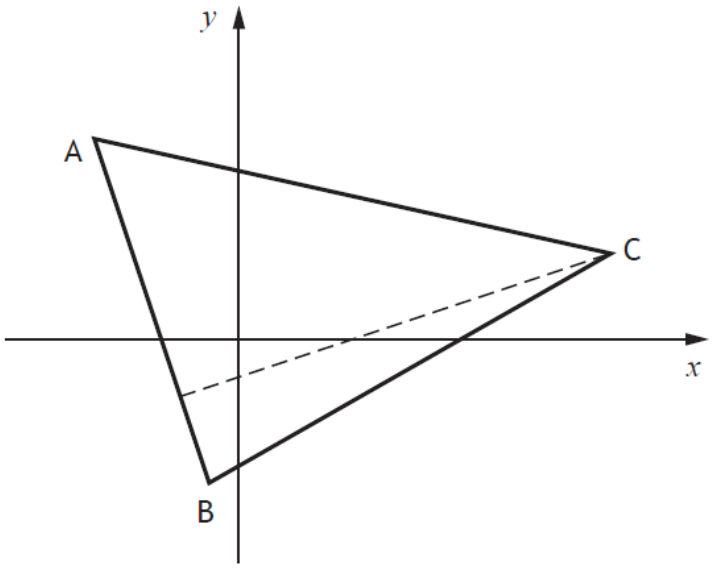
(b) Find the equation of the line, *L*, passing through M and perpendicular to PR. 3

(c) Show that line *L* passes through the midpoint of PR. 3

2015 P2 Q1

The vertices of triangle ABC are $A(-5,\ 7)$, $B(-1,\ -5)$ and $C(13,\ 3)$ as shown in the diagram.

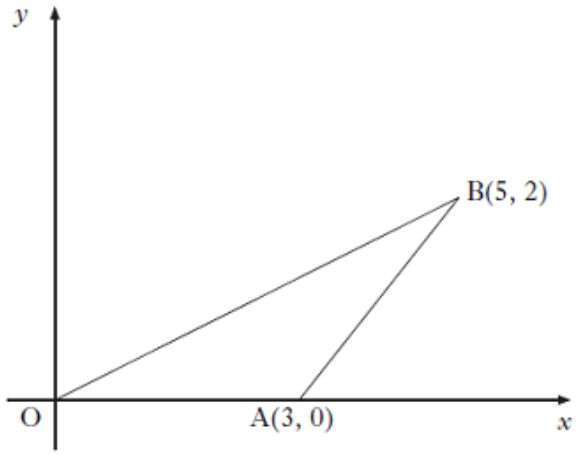
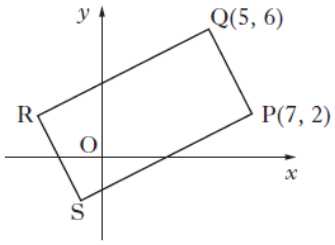
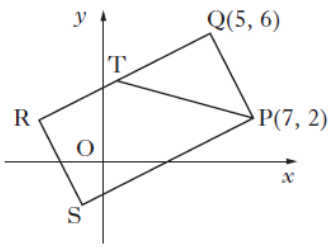
The broken line represents the altitude from C.

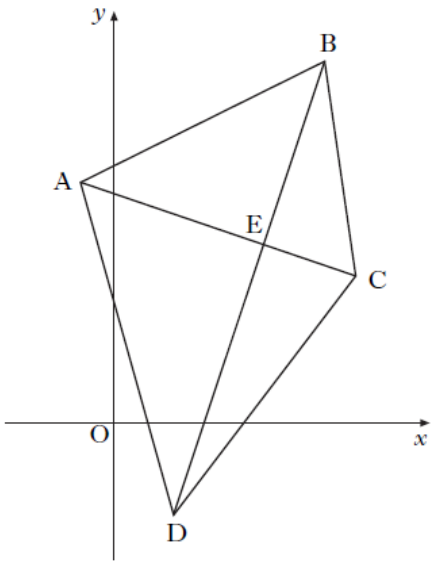
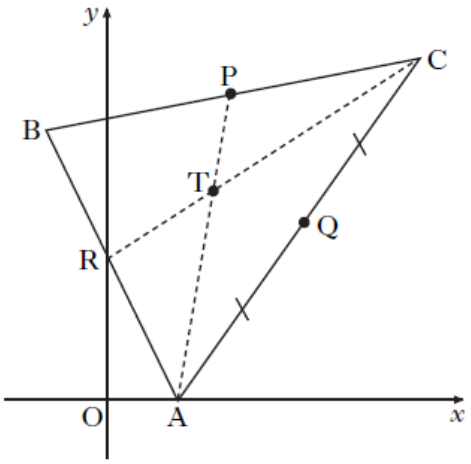


(a) Show that the equation of the altitude from C is $x - 3y = 4$. 4

(b) Find the equation of the median from B. 3

(c) Find the coordinates of the point of intersection of the altitude from C and the median from B. 2

2015 P1 Q9	A, B and C are points such that AB is parallel to the line with equation $y + \sqrt{3}x = 0$ and BC makes an angle of $150°$ with the positive direction of the x-axis. Are the points A, B and C collinear?	3
2014 P2 Q1	A(3, 0), B(5, 2) and the origin are the vertices of a triangle as shown in the diagram.  (*a*) Obtain the equation of the perpendicular bisector of AB. (*b*) The median from A has equation $y + 2x = 6$. Find T, the point of intersection of this median and the perpendicular bisector of AB. (*c*) Calculate the angle that AT makes with the positive direction of the x-axis.	4 2 2
2013 P2 Q2	The diagram shows rectangle PQRS with P(7, 2) and Q(5, 6).  (*a*) Find the equation of QR. (*b*) The line from P with the equation $x + 3y = 13$ intersects QR at T.  Find the coordinates of T. (*c*) Given that T is the midpoint of QR, find the coordinates of R and S.	3 3 3

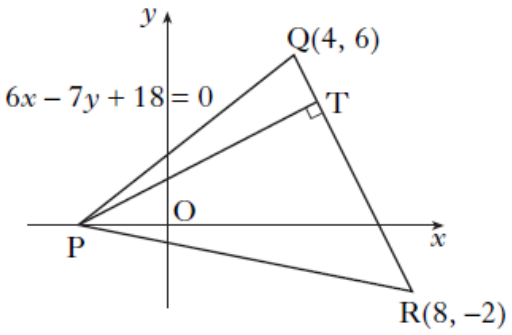
2012 P1 Q 23	(*a*) Find the equation of ℓ_1, the perpendicular bisector of the line joining P(3, –3) to Q(–1, 9).	4
	(*b*) Find the equation of ℓ_2 which is parallel to PQ and passes through R(1, –2).	2
	(*c*) Find the point of intersection of ℓ_1 and ℓ_2.	3
	(*d*) Hence find the shortest distance between PQ and ℓ_2.	2
2011 P1 Q21	A quadrilateral has vertices A(–1, 8), B(7, 12), C(8, 5) and D(2, –3) as shown in the diagram.  (*a*) Find the equation of diagonal BD.	2
	(*b*) The equation of diagonal AC is $x + 3y = 23$. Find the coordinates of E, the point of intersection of the diagonals.	3
	(*c*) (i) Find the equation of the perpendicular bisector of AB. (ii) Show that this line passes through E.	5
2010 P1 Q21	Triangle ABC has vertices A(4, 0), B(–4, 16) and C(18, 20), as shown in the diagram opposite. Medians AP and CR intersect at the point T(6, 12).  (*a*) Find the equation of median BQ.	3
	(*b*) Verify that T lies on BQ.	1
	(*c*) Find the ratio in which T divides BQ.	2

2009 P1 Q21

Triangle PQR has vertex P on the x-axis, as shown in the diagram.

Q and R are the points (4, 6) and (8, –2) respectively.

The equation of PQ is $6x - 7y + 18 = 0$.



(*a*) State the coordinates of P. 1

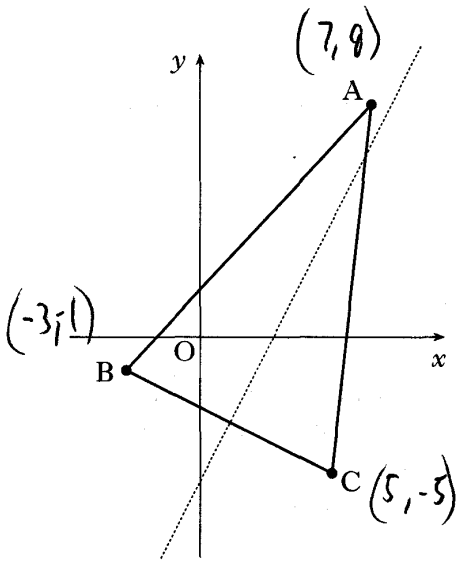
(*b*) Find the equation of the altitude of the triangle from P. 3

(*c*) The altitude from P meets the line QR at T. Find the coordinates of T. 4

2008 P2

1. The vertices of triangle ABC are A(7, 9), B(–3, –1) and C(5, –5) as shown in the diagram.

The broken line represents the perpendicular bisector of BC.



(*a*) Show that the equation of the perpendicular bisector of BC is $y = 2x - 5$. 4

(*b*) Find the equation of the median from C. 3

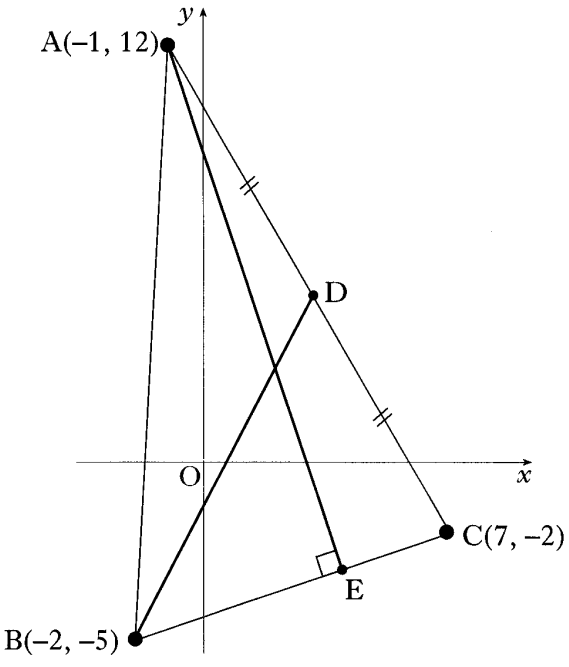
(*c*) Find the coordinates of the point of intersection of the perpendicular bisector of BC and the median from C. 3

2007 P1

1. Find the equation of the line through the point (–1, 4) which is parallel to the line with equation $3x - y + 2 = 0$. 3

2006 P1

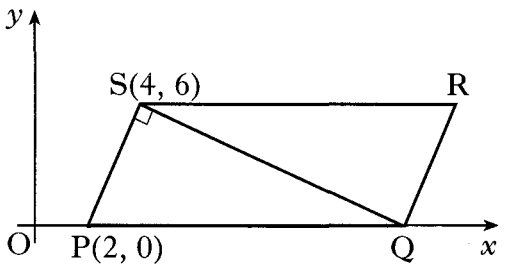
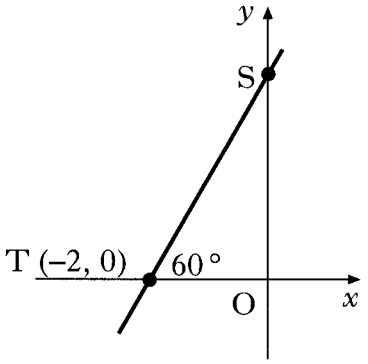
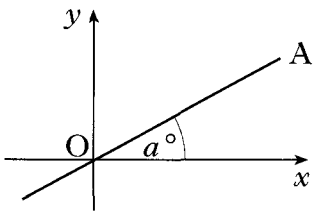
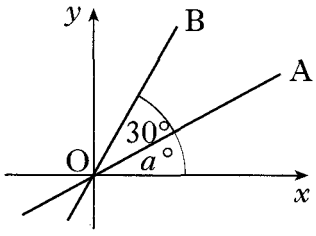
1. Triangle ABC has vertices A(–1, 12), B(–2, –5) and C(7, –2).

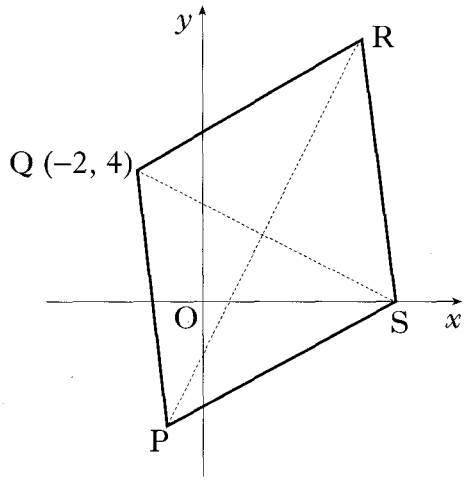
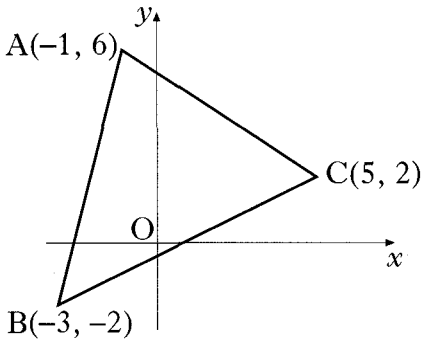
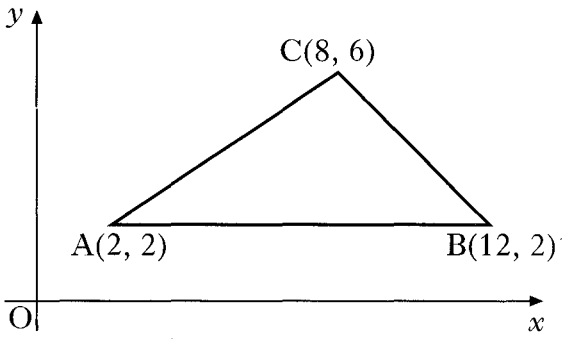


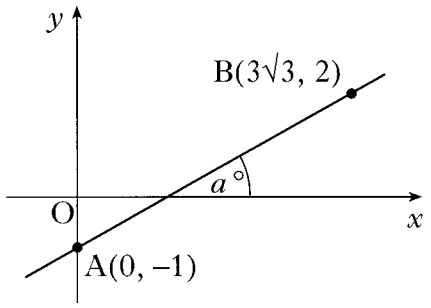
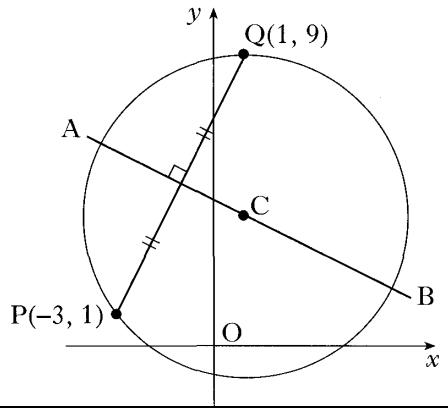
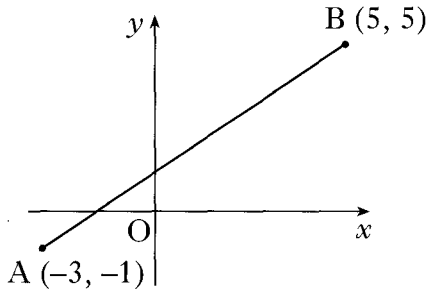
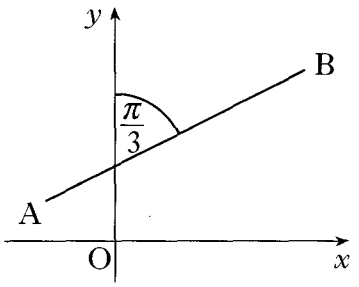
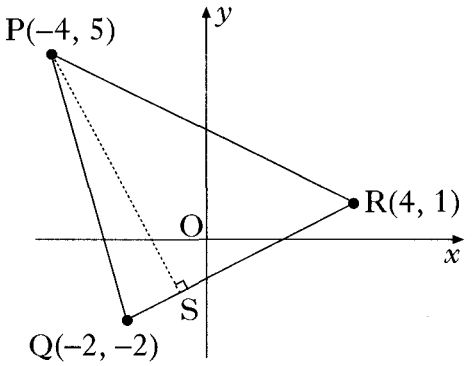
(*a*) Find the equation of the median BD. 3

(*b*) Find the equation of the altitude AE. 3

(*c*) Find the coordinates of the point of intersection of BD and AE. 3

2006 P2	**1.** PQRS is a parallelogram. P is the point (2, 0), S is (4, 6) and Q lies on the x-axis, as shown. The diagonal QS is perpendicular to the side PS.  (*a*) Show that the equation of QS is $x + 3y = 22$. (*b*) Hence find the coordinates of Q and R.	4 2
2005 P1	**1.** Find the equation of the line ST, where T is the point (–2, 0) and angle STO is 60 °. 	3
2004 P1	**1.** The point A has coordinates (7, 4). The straight lines with equations $x + 3y + 1 = 0$ and $2x + 5y = 0$ intersect at B. (*a*) Find the gradient of AB. (*b*) Hence show that AB is perpendicular to only one of these two lines.	3 5
2004 P2	**1.** (*a*) The diagram shows line OA with equation $x - 2y = 0$. The angle between OA and the x-axis is a °. Find the value of a.  (*b*) The second diagram shows lines OA and OB. The angle between these two lines is 30 °. Calculate the gradient of line OB correct to 1 decimal place. 	3 1
2003 P1	**1.** Find the equation of the line which passes through the point (–1, 3) and is perpendicular to the line with equation $4x + y - 1 = 0$.	3

2002W P1	**1.** (*a*) Find the equation of the straight line through the points A(–1, 5) and B(3, 1).	2
	(*b*) Find the size of the angle which AB makes with the positive direction of the x-axis.	2
2002W P2	**1.** The diagram shows a rhombus PQRS with its diagonals PR and QS. PR has equation $y = 2x - 2$. Q has coordinates (–2, 4). (*a*) (i) Find the equation of the diagonal QS. (ii) Find the coordinates of T, the point of intersection of PR and QS. (*b*) R is the point (5, 8). Write down the coordinates of P. 	6 2
2002 P2	**1.** Triangle ABC has vertices A(–1, 6), B(–3, –2) and C(5, 2). Find (*a*) the equation of the line p, the median from C of triangle ABC. (*b*) the equation of the line q, the perpendicular bisector of BC. (*c*) the coordinates of the point of intersection of the lines p and q. 	3 4 1
2001 P1	**1.** Find the equation of the straight line which is parallel to the line with equation $2x + 3y = 5$ and which passes through the point (2, –1).	3
2001 P2	**7.** Triangle ABC has vertices A(2, 2), B(12, 2) and C(8, 6). (*a*) Write down the equation of l_1, the perpendicular bisector of AB. (*b*) Find the equation of l_2, the perpendicular bisector of AC. (*c*) Find the point of intersection of lines l_1 and l_2. (*d*) Hence find the equation of the circle passing through A, B and C. 	1 4 1 2

2000 P1	**3.** Find the size of the angle $a°$ that the line joining the points A(0, –1) and B($3\sqrt{3}$, 2) makes with the positive direction of the x-axis. 	3
2000 P2	**2.** (a) Find the equation of AB, the perpendicular bisector of the line joining the points P(–3, 1) and Q(1, 9). 	4
Specimen 2 P1	**2.** A and B are the points (–3, –1) and (5, 5). Find the equation of the perpendicular bisector of AB. 	4
Specimen 2 P1	**4.** The line AB makes an angle of $\frac{\pi}{3}$ radians with the y-axis, as shown in the diagram. Find the exact value of the gradient of AB. 	2
Specimen 1 P1	**1.** P(–4, 5), Q(–2, –2) and R(4, 1) are the vertices of triangle PQR as shown in the diagram. Find the equation of PS, the altitude from P. 	3
Specimen 1 P2	**1.** ABCD is a parallelogram. A, B and C have coordinates (2, 3), (4, 7) and (8, 11). Find the equation of DC.	3