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Questions 141 - 160

Answers Included!

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between **a** and **b**

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

141 Show that $(x-4)$ is a factor of x^3-5x^2+2x+8. Hence, fully factorise and solve x^3-5x^2+2x+8.	
142 Solve $6-x-x^2<0$	
143 Before a forest fire was brought under control, the spread of the fire was described by a law of the form $A=A_0e^{kt}$ where A_0 is the area covered by the fire when it was first detected and A is the area covered by the fire t hours later. If it takes 1.5 hours for the area of the forest fire to double, find the value of the constant k.	
144 Solve $2\sin(2x-60)^\circ=1$ for $0\leq x\leq 360$.	
145 Using $75°=45°+30°$, show that $sin75°=\frac{\sqrt{6}+\sqrt{2}}{4}$.	
146 If $y=3x^{-2}+2x^{\frac{3}{2}}$, $x>0$, determine $\frac{dy}{dx}$.	
147 The parabola with equation $y=x^2-14x+53$ has a tangent at the point P(8, 5). Find the equation of this tangent.	
148 Find $\int\frac{(x^2-2)(x^2+2)}{x^2}\,dx$, $x\neq 0$	
149 The curve $y=f(x)$ is such that $\frac{dy}{dx}=4x-6x^2$. The curve passes through the point (-1, 9). Express y in terms of x.	
150 Express $3cosx°+4sinx°$ in the form $kcos(x-a)°$ Hence, solve $3cosx°+4sinx°=5$	

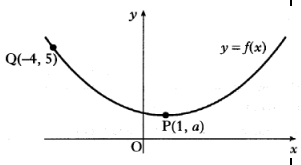
151

$f(x) = 8x^2 - 5$ and $g(x) = 5 + x$

Find $f\big(g(x)\big)$ and $g\big(f(x)\big)$.

152

The diagram shows the graph of a function $y = f(x)$.
Sketch the graphs of:
$y = f(x - 4)$ and $y = 2 + f(x - 4)$.



153

A(0, -3, 5), B(7, -6, 9) and C(21, -12, 17).
Show that A, B and C are collinear, stating the ratio AB:BC.

154

P is the point (-1, 2, -1) and Q is (3, 2, -4). Write down PQ in component form. Calculate the length of $\overrightarrow{PQ}$.
Find the components of a unit vector which is parallel to $\overrightarrow{PQ}$.

155

Prove the identity:

$$cos^2Q tan^2Q = 1 - cos^2Q$$

156

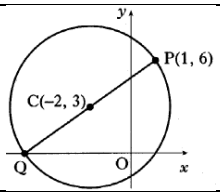
The point A has coordinates (7, 4). The straight lines with equations $x + 3y + 1 = 0$ and $2x + 5y = 0$ intersect at B.
Find the gradient of AB.

157

A triangle has vertices A(5, 5), B(-10, 0) and C(0, -10). Find the equation of the altitude from A.

158

A circle has centre C(-2, 3) and passes through P(1, 6). Find the equation of the circle.

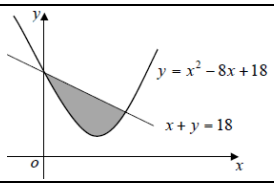


159

A sequence is defined by the recurrence relation $u_{n+1} = 0.8u_n + 12,\ \ u_0 = 4$.
State why this sequence has a limit and find this limit.

160

Calculate the area between the line $y = x + y = 18$ and the curve $y = x^2 - 8x + 18$.



Answers Only

Ques 1 - 20

1. $f(x) = (x-1)(2x+5)(x-1)$
2. $(x+4)^2 - 13$ Min T.P at $(-4, -13)$
3. 2
4. $x = \frac{2\pi}{3}$ for $\frac{\pi}{2} \leq x \leq \pi$
5. $\sin(x+a) = \frac{4}{5}sinx + \frac{3}{5}cosx$
6. $\frac{dy}{dx} = 12x^2 + 10x - 3$
7. Max T.P at $(-1, 17)$ and Min T.P. at $(3, -15)$
8. $\frac{-2x^{-3}}{3} + \frac{1}{5}sin5x + C$
9. $y = 4x^2 - 3x - 3$
10. $k = 2$ and $a = 30°$
11. $x \leq 3$
12. $a = 2 \quad b = 3$
13. 1: 2
14. 26
15. $sin60 = \frac{\sqrt{3}}{2} \quad tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$
16. $m = -1$
17. $y = -3x + 10$
18. $(x+7)^2 + (y-6)^2 = 36$
19. $u_{12} = 8.7$
20. Area = 32 square units

Ques 21 - 40

21. $f(x) = (x-1)(x+2)(x-1)$
22. $q = 5$
23. $x = 2$
24. $x = \frac{\pi}{6}, \frac{11\pi}{6}$
25. $cos2x = \frac{-3}{5}$
26. $f'(x) = 3x(4-3x^2)^{\frac{-3}{2}}$
27. $Max\ T$.P. at (-1, 4) and Min T.P. at (1, 0)
28. $\frac{8}{3}x^{\frac{3}{2}} - \frac{1}{2}x^{-2} + C$
29. $y = \frac{-1}{3}cos3x + \frac{7}{6}$
30. $\sqrt{2}\sin(x - \frac{\pi}{4})$
31. $f(g(x)) = 3x^2 - 5 \quad g(f(x)) = 9x^2 + 6x - 1$
32. $a = 3 \quad b = 3$
33. $k = 4$
34. $\overrightarrow{DE} = 3\overrightarrow{EF}$ so $\overrightarrow{DE}$ and $\overrightarrow{EF}$ are parallel. E is a common point so D,E,F are collinear.
35. Proof.
36. $3y + 5x = -13$
37. $m_{PS} = \frac{7}{4}$
38. $4y + 5x = 71$
39. $L = -400$
40. Area = $12\frac{3}{20}$ square units.

Ques 41 - 60

41. $Y = 3(x-1)(x-4)$
42. $x > 3 \quad x < -2$
43. $log_a 5$
44. $x = 0°, 60°, 300°, 360°$
45. $\sin(p+q) = \frac{2+2\sqrt{5}}{3\sqrt{5}}$
46. $\frac{dy}{dx} = 6x^5 + 24x^2$
47. $y = 6x - 18$
48. $-2\cos(2x+3) + C$
49. $9\frac{1}{3}$
50. $\sqrt{13}\sin(x - 303.7)$
51. 0.5
52. Correct shape, Min T.P at (-4, -4) Max T.P. at (-1, 1)
53. $\begin{pmatrix} 8 \\ -4 \\ -5 \end{pmatrix}$
54. $x = 1$
55. Proof
56. $m = \frac{1}{\sqrt{3}}$
57. $3y - x - 8 = 0$
58. Centre $(-4, -2)$ radius=$\sqrt{58}$
59. $u_3 = 29$

Ques 61 - 80

61. $(x-1)(x+4)(x+5)$
62. $k = \frac{9}{8}$
63. -4
64. $x = 60°, 132°, 228°, 300°$
65. $cos2a = \frac{7}{25}$
66. 14π
67. $y = -x + 1$
68. $\frac{x^{-3}}{-9} + C$
69. 1
70. $5\cos(x + 306.9°)$
71. $f(g(x)) = x^2 + 8x + 19 \quad g(f(x)) = x^2 + 7$
72. $q = 13$
73. 3:2
74. $\frac{9}{2}$
75. $120° = \frac{3\pi}{4}$ and $\frac{2\pi}{3} = 120°$
76. 5 units
77. $y = 4x + 4$
78. $J(-1, -2) \quad K(1, 2)$
79. $l = 100/3$
80. $Area = \frac{27}{4}$square units

60. Area = $57\frac{1}{6}$ square units

Ques 81 - 100

81. $K = -2$ and $t = -5$
82. $x > 0 \quad x < -4$
83. $x = \frac{3}{8}$
84. $x = 30°, 90°, 150°$
85. $\sin(a+b) = \frac{63}{65}$
86. $f'(x) = -\frac{1}{5}x^{-\frac{6}{5}}$
87. $y = 4x - 2$
88. $\frac{(2x-1)^{\frac{3}{2}}}{3} + C$
89. $\frac{2}{3}$
90. $\sqrt{34}\cos(x - 59.0)°$
91. $f(g(x)) = 9x^2 - 24x + 17 \quad g(f(x)) = 3x^2 - 1$
92. $a = 4 \quad b = 2 \quad c = 1$
93. 3:2
94. $\theta = 72°$
95. **-1**
96. $y = \frac{1}{4}x + \frac{13}{4}$
97. $y = \frac{3}{2}x - 3$
98. Centre $(-1, -2)$ Radius $\sqrt{32}$
99. $L = \frac{28}{3}$
100. $Area = 9\ square\ units$

Ques 101 - 120

1014
102. $q = -9$
103 $x = 2y^3$
104. $x = 20°, 100°, 140°, 220°, 260°, 340°$
105. $sin2x = \frac{4}{5}$
106. 1
107 Max T.P. when $x = \frac{1}{3}$
108. $x^3 + x^2 + c$
109. $f(x) = 2x + \frac{1}{x} + 5$
110. $2.5\cos(x + 306.9)°$
111. $x \neq 3 \ and \ x \neq 4$
112 Correct shape drawn and labelled with (0,3), (3,1), (5, 3)
113. $Q(3, 1, -2)$
114. t=-3
115. Proof.
116. $m = 1$
117. $y = -2x + 2$
118. $y = -\frac{3}{2}x + 6$
119. $u_4 = 3$
120. 9/8

Ques 121 - 140

121. $y = -x(x+1)(x-2)$
122. $x < -5 \ and \ x > 3$
123. 1
124. $x = 60°, 120°, 240°, 300°$
125. Proof.
126. 12
127. $y = 2x - 12$
128. $-\frac{(1-6x)^{\frac{1}{2}}}{3} + C$
129. $y = 2x^3 - 2x^2 + 3x + 2$
130. $10\cos(x + 36.9)°$
131. $p(x) = 3 - \frac{3}{x}$ and $p(q(x)) = x$
132. $y = -2f(x)$ passing through (-6, 0), (1, 14), (3, 0) and $y = f(x-3)$ passing through (-3, 0), (4, -7), (6, 0)
133. $\overrightarrow{QR} = 2\overrightarrow{PQ}$ and Q is a common point so P, Q, R are collinear.
134. 5
135. Proof.
136. $y = -\frac{1}{2}x + \frac{13}{2}$
137. $y = 6x + 9$
138. $y = \frac{1}{4}x - \frac{7}{2}$
139. $L = -50$
140. Area = $\frac{20}{3}$ square units

Ques 141 - 160

141. $(x-4)(x-2)(x+1)$
142. $x < -3 \ and \ x > 2$
143. $k = 0.46$
144. $x = 45°, 105°, 225°, 285°$
145. Proof.
146. $\frac{dy}{dx} = -6x^{-3} + 3x^{\frac{1}{2}}$
147. $y = 2x - 11$
148. $\frac{x^3}{3} + 4x^{-1} + C$
149. $y = 2x^2 - 2x^3 + 5$
150. $x = 53.1°, 413.1°$
151. $f(g(x)) = 8x^2 + 80x + 195 \quad g(f(x)) = 8x^2$
152. $y = f(x-4)$ passing through $(0,5), (5,a)$ $y = 2 + f(x-4)$ passing through $(0,7), (5, a+2)$
153. AB:BC = 1:2
154. Unit vector = $\begin{pmatrix} \frac{4}{5} \\ 0 \\ \frac{-3}{5} \end{pmatrix}$
155. Proof.
156. m=3
157. $y = x$
158. $(x+2)^2 + (y-3)^2 = 18$
159. $L = 60$
160. $Area = \frac{343}{6}$ square units

Ques 161 - 180	Ques 181 - 200
161. $(x+2)(x-2)(x-2)$ 162. $b^2-4ac=-24$ 163. $x=\frac{43}{15}$ 164. $x=\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}$ 165. $Proof.$ 166. $\frac{dy}{dx}=-12sinxcos^3x$ 167. $Max\ T.P$ at (-1, 17). Min T.P at (3, -15). 168. $2x^2+\frac{1}{x}+C$ 169. $0.363\ radians$ 170. $25\sin(x-1.287)$ (in radians) 171. $f(g(x))=\frac{-x+2}{x+1}\quad g(f(x))=\frac{1}{3x}$ 172. $y=-g(x)\ passes\ through\ (a,-2),(0,-1),(b,-3)$ $y=3-g(x)$ passes through $(a,5),(0,2),(b,0)$ 173. $\lvert\boldsymbol{f}+\boldsymbol{g}\rvert=\sqrt{66}$ 174. $\theta=50.9$ 175. Proof. 176. $y=\sqrt{3}x+2\sqrt{3}$ 177. $y=-3x+9$ 178. Point of contact is $(1,4)$ 179. $u_9=9\quad u_{10}=6.8$ 180. Area = 36 square units	181. $3(3x+1)(x+2)(x-1)$ 182. $-1<x<\frac{1}{3}$ 183. $x=71$ 184. $x=45°,105°,165°,225°,285°,345°$ 185. $sin2a=\frac{15}{17}$ 186. $\frac{dy}{dx}=2xcos(x^2-3)$ 187. $Min\ T.P\ at\ (0,0)\ \ Max\ T.P\ at\ (2,4)$ 188. $\frac{(2x+9)^6}{12}+C$ 189. $\frac{13}{3}$ 190. $2\sin(x+\frac{\pi}{3})$ 191. $x\neq-8\ ,\quad x\neq2$ 192. $y=2f(x)+1$ passes through $(0,1),(2,7),(5,1)$ 193. $\overrightarrow{PQ}=\begin{pmatrix}8\\-4\\1\end{pmatrix}\quad\lvert\overrightarrow{PQ}\rvert=9$ 194. $k=\frac{6}{7}$ 195. Proof. 196. $y=3x+7$ 197. $3y=4x-11$ 198. $P(1,-2)\quad Q(3,4)$ 199. $u_1=16\quad u_2=20\quad u_3=21$ 200. Area = $\frac{9}{2}$ square units

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